# AMERICAN INSTITUTE FOR DECISION SCIENCES

**WESTERN REGION**

*NATIONAL OFFICERS*

University Plaza
Atlanta, Georgia 30303

*President*

D. Clay Whybark
Indiana University

*President-Elect*

Norman L. Chervany
University of Minnesota

*WESTERN REGIONAL OFFICERS*

*President*

Michael Parent
College of Business Administration
Utah State University
Logan Utah 84322

*President-Elect*

Pieter Vandenberg
School of Business
San Diego State University
San Diego, California 92182

*Vice Presidents*

*for Programs and Meetings*

Michael Geurts
College of Business
Brigham Young University
Provo, Utah 84602

*and*

Terry Lee
College of Business
Brigham Young University
Provo, Utah 84602

*for Membership*

Bert Steece
University of Oregon
Eugene, Oregon

*for Special Arrangements*

Daniel Brooks
College of Business Administration
Arizona State University
Tempe, Arizona 85281

*Secretary/Treasurer*

William R. Sherrard
School of Business
San Diego State University
San Diego, California 92182

*Executive Secretary*

Helen Beaver
School of Business
San Diego State University
San Diego, California 92182

*PAST REGIONAL PRESIDENTS*

1979-80 Steven D. Wood
Arizona State University

1978-79 Jim D. Barnes
California State College, Bakersfield

1977-78 Paul Baum
California State University, Northridge

1976-77 James E. Reinmuth
University of Oregon

1975-76 William T. Newell
University of Washington

1974-75 Charles P. Bonini
Stanford University

1973-74 William R. Sherrard
San Diego State University

1972-73 Robert Childress
University of Southern California

1971-72 Manfred W. Hopfe
California State University, Sacramento

# AMERICAN INSTITUTE FOR DECISION SCIENCES

Western Regional Conference
Sacramento, California
May 6 - 7, 1971

## ADVANCE REGISTRATION

PLEASE PREREGISTER. We can save time for you and us if we have this form completed and your check by April 15, 1971.

Name ______________________________

Organization ______________________________

Address ______________________________

______________________________ (Zip) __________

PREREGISTRATION FEE
(Includes Dinner May 6)

AIDS Member $10 Non-member $16 FEE $ __________

## HOTEL REGISTRATION

Please reserve ____ room(s) of the type checked below for the nights indicated at the Host of Sacramento Airport Hotel. (Prices include 5% California tax. Check out time: 12 noon.)

| Single | Double | Nights |
|---|---|---|
| ( ) Twin $15.23 | ( ) Twin $19.95 | ( ) 5 May |
| ( ) Queen $17.33 | ( ) Queen $22.05 | ( ) 6 May |
| | | ( ) 7 May |

NOTE: If double is desired, please indicate roommate's name and address to assist in coordination.

Roommate's Name ____________________ Address ____________________

## TOTAL REMITTANCE

Make total remittance payable to:
"AIDS Western Regional 1971"

Mail to:

Professor Manfred W. Hopfe
School of Business Administration
Sacramento State College
6000 Jay Street
Sacramento, California 95819

Preregistration Fee $ ________

Hotel (Price X Nights) $ ________

TOTAL $ ________

# AMERICAN INSTITUTE FOR DECISION SCIENCES

DECISION SCIENCES

EDUCATIONAL INNOVATIONS AND TECHNIQUES

Western Regional Conference

Sacramento, California

May 6 - 7, 1971

"DECISION SCIENCES - EDUCATIONAL INNOVATIONS AND TECHNIQUES"

This is the organizational meeting of the Western Region of the American Institute for Decision Sciences. The enthusiasm, as well as the quality of the papers and panel discussions, publicly demonstrates the expertise of AIDS members.

In the medieval university the teacher either read from a book which the students did not possess or have access to, or lectured from his personal knowledge. After the invention of the printing press, the process remained the same, even though the students had relatively the same information as the teacher. With the development of the electronic media the process all too frequently remained the same. The teacher remained at the front of the room with students seated facing him.[1] Our meeting demonstrates that AIDS members have developed more stimulating methodology.

Welcome to the challenge!

Manfred W. Hopfe
Michael J. Dottarar
Arthur N. Jensen
V. Paul Jeppesen

School of Business Administration
Sacramento State College

[1] Blackburn, John L., "After the Furor..." NASPA (mimeographed)

# AMERICAN INSTITUTE FOR DECISION SCIENCES

Western Regional Conference
Sacramento, California
May 6 - 7, 1971

## P R O G R A M

| | | |
|---|---|---|
| Thursday May 6 | 12:00 - 1:20 p.m. | Late registration and confirmation of attendance. (Pre-registrants pick up meeting packet at AIDS Registration Desk in hotel lobby.) |
| | 1:30 - 3:15 p.m. | Panel Discussion (Camellia Room "A")<br>EDUCATIONAL INNOVATIONS AND TECHNIQUES<br><br>"Integrated Courses in Quantitative Analysis and Organizational Behavior"<br>Rick Hesse (Chairman)<br>University of Southern California<br><br>"Interactive Learning Cells in Classical Statistics"<br>Lee Cooper<br>University of California, Los Angeles<br><br>"New Programs and Techniques for Decision Analysis"<br>Jack Moore<br>Stanford University |
| | 3:15 - 3:45 p.m. | Contributed Paper (Camellia Room "A")<br><br>"Conceptual Problems with a Basic EOQ Model"<br>Ted Weston<br>Colorado State University |
| | 3:45 - 5:00 p.m. | Student Group Presentation (Camellia Room "A")<br>THE EDUCATIONAL VALUE OF STUDENT CHAPTERS: AN EXAMINATION OF THE QUESTION<br><br>Joe Feigo (Chairman)<br>California State Polytechnic College, Pomona |

| | | |
|---|---|---|
| | 6:00 - | Dinner (Camellia Room "A") |
| | | Speakers: |
| | | Professor Dennis E. Grawoig, President of AIDS, Georgia State University |
| | | Professor George W. Summers, President-elect of AIDS, University of Arizona |
| | 8:00 - ? | No host social (Suite 104) |
| Friday May 7 | 8:15 - 9:45 a.m. | Contributed Papers (Garden Room) |
| | | "Decision Making: A Case for Determinism" John Stockdale (Chairman) Sacramento State College |
| | | "A Case of Decision Making Under Conditions of Uncertainty" Harry Costis Fresno State College |
| | | "A Model of Optimization of Decision Science Curriculum" Joe Feigo California State Polytechnic College, Pomona |
| | 10:00 - 11:30 a.m. | Organizational Meeting (Garden Room) |
| | 12:00 noon | Check-out (1:00 p.m. latest) |

ABSTRACTS OF PAPERS PRESENTED

(If you are interested in receiving a copy of a working paper, please correspond directly with the author.)

ABSTRACT

CONCEPTUAL PROBLEMS WITH A BASIC
EOQ MODEL

Frederick C. Weston, Jr.
Colorado State University

This paper presents a discussion of a popular flow receipt production inventory model that is correct only in the limited case where annual demand and daily (or weekly) usage are expressed in a certain specific relationship to each other. For an intermittent producer who employs the standard textbook model but who may produce a product only once or twice a year, and who may not carry inventory throughout the year, the standard model is erroneous and misleading.

This paper describes the standard text model and the conditions under which the model can and cannot be employed. The author then proceeds to develop a revised model that is both consistent and correct for the conditions purported to be represented by the popular textbook model. Finally, the discussion is extended to include the effects of the revised model on a quantity discount or price break situation.

The paper should be of interest to inventory managers and managers of intermittent production operations, in addition to OR practitioners and theoreticians.

ABSTRACT

# DECISION MAKING: THE CASE FOR DETERMINISM

John M. Stockdale
Sacramento State College

The purpose of this presentation is to relate the decision-making science to the other established sciences, and to attempt to classify decision-making techniques into categories. A great deal of confusion exists in this field at present, at least on my part, since so many techniques are used from so many different fields. The use of the term "science" in the title of this organization implies that its philosophical underpinnings are similar to the other sciences. Any attempt to define decision science at this point in time would be a disservice to us all. However, it is not too early to make a beginning even though the attempt will be modest at best.

ABSTRACT

A CASE OF DECISION MAKING
UNDER CONDITIONS OF UNCERTAINTY

Harry G. Costis
Fresno State College


This paper projects the advantages of computer simulation by the Monte Carlo method in order to solve problems of decision making which cannot be solved in an easy and optimal way through the use of standard mathematical techniques. The problem at hand deals with labor levels determination in a hypothetical meat-packing company receiving quantities of meat on a bulk basis at the beginning of the week to be processed and packed during the week. A Monte Carlo computer simulation solution is shown to be advantageous.

# ABSTRACT

## A MODEL OF OPTIMIZATION OF DECISION SCIENCE CURRICULUM

Joseph A. Feigo, California State Polytechnic College

It is felt by the author that mathematical programming can serve as a useful device to develop an individual decision science curriculum suited to specific limitations and capabilities of the interested school. The program proposed here is one of logarithmic programming based upon the premise that the maximation of exposure hours to the decision sciences in business school's curriculum will facilitate the transition of that program to a systems-wide approach to decision making. The model is an attempt to incorporate interrelationships and overlappings in faculty assignment and course offerings on an internal (within the business school) and external (the business school interaction with other campus schools) basis. In addition, the model allows for other real world events such as non-linearity of relationships and factor limitations. The output of the model will provide the maximum exposure in hours of the decision sciences in a school of business curriculum subject to the limitations and constraints faced by that institution. As "by-products" of the analysis, the model produces opportunity costs of the constraints, in effect, the "prices" which must be paid (opportunity-wise) in order to expand the decision science exposure. Finally, the model submits readily to a sensitivity analysis, enabling the administration of the school of business to determine the "area" most capable of being expanded by the school (i.e., the areas being marketing, management, etc.) in terms of decision sciences.

WESTERN REGIONAL MEETING

American Institute of Decision Sciences

Integral Courses in Quantitative Analysis and Organizational Behavior.

Rick Hesse, USC

In the Fall, 1970, the USC undergraduate school combined two courses, Organizational Behavior and Quantitative Methods taught by Pete Reynolds and Rick Hesse. The rationale for this was as follows:

(1) The introductory course in Organizational Behavior had been suffering from a lack of "real" situations - the dynamics were more of games and puzzles than situations where vested interests might be threatened;

Groups who cares

(2) In Quantitative Analysis I, while trying to break the barrier of fear that many business students develop against math, it became apparent that the logic, analysis, feedback and communication, etc., underlying the math, had many behavioral parallels, and that these could be used as tie-ins from the familiar to the unfamiliar.

(3) Looking at both courses, the OB seemed to lack some in content while the QA lacked in process (being straight lecture, limited participation, etc.)

Thus, by combining the two, each could reinforce the other.

The QA provided the content for OB to study and the OB provided group process to help with tackling the QA assignment.

The grading for the two courses was kept separate, but a student had to drop both courses if he were doing poorly in one. Enrollment for both courses was simultaneous, and both courses (combined or separate) were required courses. The same classroom was used for both courses, and although the schedule officially read 1:15 - 2:55 and 3:15 - 4:55 for both courses, the times from 1:15 - 5:00 twice a week were blocked out. This enabled flexibility in programming the class and allowed the professors more freedom in scheduling.

In the first two class periods, the OB part of the course formed 6 - man groups, trying to maintain a heterogeneous mixture of quantitative abilities. The attached table will show a brief outline of the two courses.

There are several integrated exercises during the semester.

(1) Developing a computer language (see attachment). This brings into play all of the elements of communication (encoding, decoding), language structure, computer and algebra. This is done with the basic OB groups and after the languages are developed, their dictionaries are interchanged and the groups must write a simple program to be executed by the originating group.

(2) Group Quizzes - Early in the semester a proficiency quiz is given in algebra (1/2 hour). At the end of the quiz the quiz is given again, with the group getting together to help with the answers. Each individual still fills out his own exam and is free to accept or reject answers from the group.

(3) Homework Groups - Sometimes 15 - 20 minutes are devoted to resolving any difficulties in homework. Since the groups are heterogeneous, some students emerge as teachers and others as learners. The dynamics of the group prevent students from not doing any individual effort. These times are also used for starting homework assignments on computer consoles.

(4) Homogeneous Operations Research Groups are formed on the basis of individual test scores in Quantitative Analysis. These run counter to the OB groups and are concerned with solving two Linear Programming cases. These cases are graded in difficulty, so that the groups with higher averages receive more difficult cases. Part of the requirement of the final reports is a group behavioral analysis.

(5) Tests - QA tests are given during the first period and then reviewed during part of the second period.

## Interface

Usually the period is structured to have

| | |
|---|---|
| Class session I | 1:15 - 2:30 |
| Break | 2:30 - 3:00 |
| Interface | 3:00 - 3:30 |
| Class Session II | 3:30 - 5:00 |

The interface period was designed as an attempt to explicitly bring the two courses together. It failed miserably. What was discovered was that this was a good time for emergent behavior - from both professors and students, in both inter- and intra-action mode.

## Results

(1) Motivation is higher

(2) Integration of course material

(3) Changing of the norm in class

(4) Increased Learning

(5) Changed norm in other classes.

(6) Reinforcement of concepts

The dynamics created by combining the two courses has given much richer data for the OB side to work with. The dynamics of a group working on a quantitative assignment where there is a "right" answer are quite different than a group working on a paper (bull sessions). Being forced to resolve issues of discord, non-confrontation, inter-dependence, etc. are much more real (because of the ever present fact that a grade is riding on the outcome).

## Future

Considering the success of this program, the undergraduate School of Business will add two more courses together in the coming Fall semester to form a Block Program. 80 students will be blocked together in four courses (Business Communications, Finance, Organizational Behavior, and Quantitative Analysis) meeting 5 days a week from 9 - 12 A.M. Two rooms plus a lounge will be reserved so that in essence there will be a small college of 80 students and four professors operating within the Business School. This will be an attempt to teach a completely coordinated and integrated curriculum of four required courses for the first semester of the junior year. If successful, it may lead to a second semester, which would fulfill all the required courses except one and leave the senior year free for majors and electives. In this manner, the professors will be assured that certain required topics have already been covered and he can assume a student's knowledge of it. The four courses will be integrated only as the teachers themselves become integrated with the other subject matter. Hopefully this will be very beneficial to the faculty member, who will be putting in extra hours working with the other professors, and would carry over into other courses he is teaching.

| | | Math | Computers | |
|---|---|---|---|---|
| Phase I: Introduction of Conceptual Foundations of Organizational Behavior; weekly concept quizzes given individually and then taken by the group. | 12 | Algebra Review, Functions, Linear Models, finance models, curve fitting | RAX terminals, library programs, simple FORTRAN (I/O, Assignment, GO TO) | 2 |
| | | ***** T E S T ***** | | 1 |
| Phase II: Cases in OB: Individual or group paper; analysis of group process in Phase I. | 6 | Matrix Algebra<br>Sets<br>Algorithms<br>Linear Programming | IF statements<br>Function Subprograms | 9 |
| | | ***** T E S T ***** | | 1 |
| Phase III: Projects in OB: (for presentation to class in Phase IV) Enterprise projects, critique of some enterprise. | 8 | Calculus: Differential | Subscripted Variables<br>Format | 7 |
| | | ***** T E S T ***** | | 1 |
| Phase IV: Conclusion of course and Final Exam - presentation of Phase III projects, simulation exercise for group. | 3,4 | Calculus: Integral | Do loops | 3 |

Final Exam in week 14 or 15 (returned and critiqued during scheduled final exam session).

PROGRAMS TO AID STUDENTS IN MATH

Rick Hesse, USC
School of Business

Having determined that the greatest derrant to learning higher mathematics is deficiency in algebra, I have tried to take advantage of the computer facilities to aid the student in familiarizing himself with algebra and computers. The following is a brief description of some simple programs and the rationale behind them.

<u>Purpose</u>

1. To Gain Physical Familiarity with the terminal (keypunch).
2. To gain Logical Familiarity with Command structure, how to sign on and off of the terminal.
3. Renew familiarity with graphing.
4. Renew familiarity with the algebraic functions:

    Polynomials
    Exponentials
    Logarithms
    Quotients
    Periodic

5. To allow some creativity and ingenuity as to how the student is to discover what the function is.
6. To allow students to experiment with root finding.
7. To allow students to sample points for curve fitting.
8. To allow students to experiment with determining maximum & minimum.

The program is a simple FORTRAN program to read in a value of X interactively, and to test to aww if it is less than -100. If it is, the program stops, if not, it computes and writes out X, F(X). Each program is stored in object core (there are 10 programs, the only difference in each being the function evaluated).

```
//JOB GØ
      3 CALL FDATA(X)
        WRITE(6,50)
     50 FØRMAT(7X,'X',15X,'F(X)')
        IF(X+100)1,2,2
      2 CALL FWRITE(X,F(X))
        GØ TØ 3
      1 STOP
        END
        FUNCTIØN F(X)
        F=X**2+3*x-4
        RETURN
        END
```

The two subroutines CALL FDATA ( ) AND CALL FWRITE ( ) were originally developed by North American Rockwell and implemented on the RAX system on the IBM 360/30 at the USC School of Business. Although this is designed for terminals and interaction, it is possible to put these programs in a non-interactive terminal mode or a batch mode.

The instructions for assignments are given to the students and are attached.

The ten functions I have used are as follows (the students do not have access to these):

F1: $X^2 + 3x - 4$
F2: $1/(x-3)$
F3: $4x^3+6$
F4: $2 \sin(x)+6$
F5: $4e^{-x}$
F6: $3 \log(1+x)$
F7: $3x^3-9x^2-16x+60$
F8: $(x-6.2)^2$
F9: $10^{x-1} - 3$
F10: $(x-6.1)(x+4.2)(x-17.8)(x)\cdot(x+14)(x+19)$

A typical scenario is as follows:

```
*    /INPUT
*    /INCLUDE F2
*    /END RUN
*    M.0076 ACTIØN IN PRØGRESS
             x                        F(X)
*    5
    -2       5.0000                   0.5000
*            2.0000                  -0.2000
*   -101
   STØP 100
```

* denotes lines typed by the student.

Type in the following, starting in the first column on the type-writer (note the spacing and be careful of the difference between "one" and "el").

```
*   /ID
*   /INPUT
*   /INCLUDE F1
*   /END RUN
                  X              F(X)
*   5
                  5.0000         36.0000               ****
*   -2.0         -2.0000        -6.0000                ****
*   -10                                                ****
    STOP 100
    /OFF
```

* typed in by student

For every value of X you enter a value of X and f(X) will be typed out.

Run the programs F1, F2, F3, F4, F5, and F6 to do the following:

a) enter as many points as you wish for $-100 \leq X \leq 100$

b) graph each function neatly and identify the function as polynomial, exponential, logarithm, periodic or quotient.

c) to STOP, type in -10 for X.

d) hand in both your computer output and your graphs.

e) if there are asterisks in the answer (***********) this means that /X/ is very large; if the function is undefined, you will get an error message that will usually say FLT. PT. DIV. CHECK.

Identify, using the procedure outlined in Assignment No. 2, the functions F1, F3, F5, F7, F8, F9, F10 as to

1) Number of roots

2) Polynomial - highest power
Exponential - base and other constants (a, $c_1$, $c_2$)

3) Graph F7, F8, F9, F10

## ASSIGNMENT No. 4

Fit the appropriate polynomials to the following points using SIMEQ to determine the coefficients.

1) Any four (4) points from F3

2) Any three (3) points from F1

3) Any three (3) points from F8

4) Any four (4) points from F7.

Stage I:

Develop a general computer language using whatever symbols you wish and use the following formulas as examples to help test it out:

$$S = P(1+i)^n \qquad \frac{-1}{x+10} = y$$

The following steps might help you in the definition of your languages. You will have to write a "dictionary" to explain this language to another group, so be precise in your definition. These expressions, unlike algebra, must not contain half spaces up (like in exponents) or down (like in subscripts). Pretend that you are writing them with a typewriter and that you can't roll the carriage up or down.

1. Define numerical constants.

2. Define algebraic variables (what letters, capital, lower case, etc.).

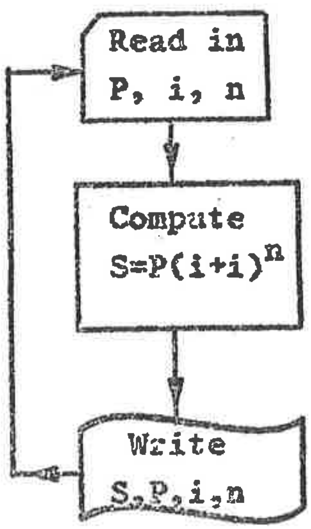
3. Define operations, using a single unique symbol for each.
   - add
   - subtract
   - multiply (be careful: does 2.3 mean 6 or 2.3? axb mean a times b or the variable axb?)
   - divide
   - exponentiate
   - unary minus

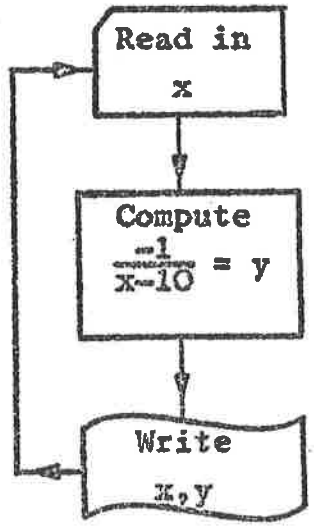
4. Define the order of operations so that the expression will be evaluated correctly.

5. Define a storage operation, so that you can store the intermediate result of some operation or the final result.

Stage II

Further refine your language to include commands and use the following problems to help test it out. (Question: how do you differentiate between commands and variables?)




1. Input Command: Your definition must make it clear

   a) that it is an input command (that it can be uniquely recognized as such);

   b) what variables are to be inputted;

   c) where they are to be found.

2. Output Command: Your definition must make it clear

   a) that it is an output command (that it can be uniquely recognized as such);

   b) what variables or expressions are to be outputted;

   c) where they are to be outputted.

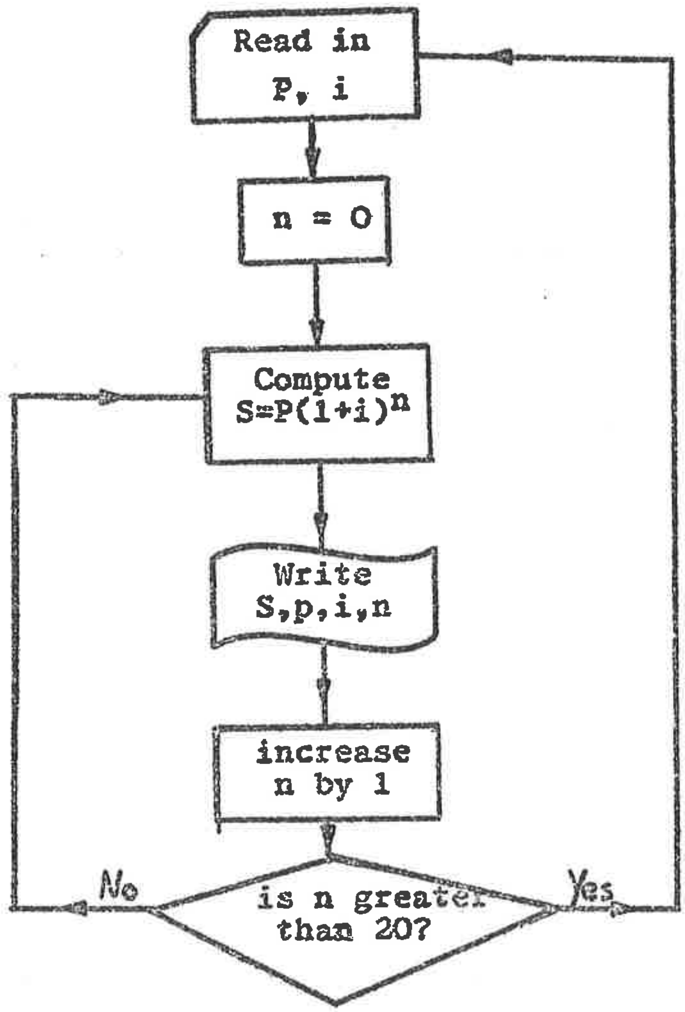
3. Branch Command: This command will change or alter the order in which you are doing the statements of your language. If you want to jump back to or forward to another statement, you must:

   a) identify uniquely the statement to be processed next (the one you wish to jump to) instead of the one that follows;

   b) make clear that this is a branch command.

Stage III:

1. Test and Branch Command: Now further refine your language to include a command that will test if a relation is true or false. If the relation is true, branch to an indicated statement; if it is false, continue on in the normal order of processing statements. Use the following problem to help test out your language:



Papers

# CONCEPTUAL PROBLEMS WITH A BASIC EOQ MODEL

by

FREDERICK S. WESTON, JR.
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO

Presented at

Western Regional Conference
American Institute for Decision Sciences
Sacramento, California
May 6, 1971

## CONCEPTUAL PROBLEMS WITH A BASIC EOQ MODEL

Several reputable sources in both operation research and inventory theory, including Buffa (Ref. 1, p. 496) and Niland (Ref. 4, p. 160), discuss a simplistic economic order quantity (EOQ) model labeled a "flow receipt" or "variable shipment" EOQ model. This model, using a combination of Buffa's and the author's notation, is as follows:

$$TC = \frac{R}{Q}\, Cp + \frac{Q}{2}\, CCh \left( 1 - \frac{r}{p} \right) + R \cdot C$$

where

TC = total cost

R = demand in units for a given time period (here assumed to be 1 year)
Q = economic order quantity (EOQ)
Cp = setup or preparation cost per setup
C = item cost per unit
Ch = carrying cost per time period (consistent with R)
r = usage or consumption rate per sub-time period (day, week or month)
p = production rate per sub-time period (day, week, or month)

The total cost equation for this model states that the total inventory cost for the item in question is the sum of the:

- ordering cost
- inventory carrying cost
- total item cost of units demanded

Thus this model assumes that back orders do not occur (an infinite cost of stocking out), that demand and cost are known and constant, and, the assumption which is often neglected, that the number of ordering cycles per time period is consistent with the number of production cycles per time period. See Figure 1.

Taking the derivative of the total cost equation with respect to the variable Q yield,

$$\frac{dTC}{dQ} = \frac{-R}{Q} \; Cp + \frac{C \cdot Ch}{2} \left( 1 - \frac{r}{p} \right)$$

which, when set equal to zero, will determine the value of Q which will generate the lowest total cost. Thus,

$$\frac{R}{Q} \; Cp = \frac{C \cdot Ch}{2} \left( 1 - \frac{r}{p} \right)$$

and solving for Q yields

$$Q^* = \sqrt{\frac{2RCp}{C \cdot Ch \left( 1 - \frac{r}{p} \right)}}$$

## The Model Illustrated

To illustrate the model, let us assume the following values:

| | | |
|---|---|---|
| R | = | 3600 units demanded per year |
| Cp | = | \$12/order |
| C | = | \$1.00/unit |
| Ch | = | 20%/year |
| r | = | 10 units/day |
| p | = | 50 units/day |

Thus,

$$Q^* = \sqrt{\frac{(2)\ (3600)\ (12)}{(1.00)\ (.20)\ (1 - 10/50)}}$$

$$= 734.85.$$

The total variable cost is equal to total cost less the total item cost of units demanded, or

$$TRC = \frac{R}{Q} \; Cp + \frac{Q}{2} \; CCh \left( 1 - \frac{r}{p} \right)$$

and

$$TRC = \frac{3600}{734.85} + \frac{734.85}{2} \; (1.00)\ (.20)\ (1 - 10/50)$$

$$= 58.79 + 58.79$$

$$= \$117.58.$$

In this model the ordering costs exactly balance the carrying costs. See Figure 2.

At this point, let us pose a question, as if the question were in the context of sensitivity analysis. For example, <u>what if</u> demand were now given as 2400 units per year. Seemingly, no problem exists and we can calculate Q* and TRC as before.

$$Q^* = \sqrt{\frac{(2)\ (2400)\ (12)}{(1.00)\ (.20)\ (\ 1\ -\ 10/50)}}$$

$$= 600$$

and

$$TRC = \frac{2400}{600}\ .12 + \frac{600}{2}\ (1.00)\ (.2)\ (1 - 10/50)$$

$$= 48 + 48$$

$$= \$96.$$

Again it should be noticed that the order costs and carry costs balance each other. The problem with this second example is that <u>the analysis is incorrect</u>--and, to the uninitiated, this problem and inconsistency may be anything but obvious.

Figure 1
Flow Receipt EOQ Model

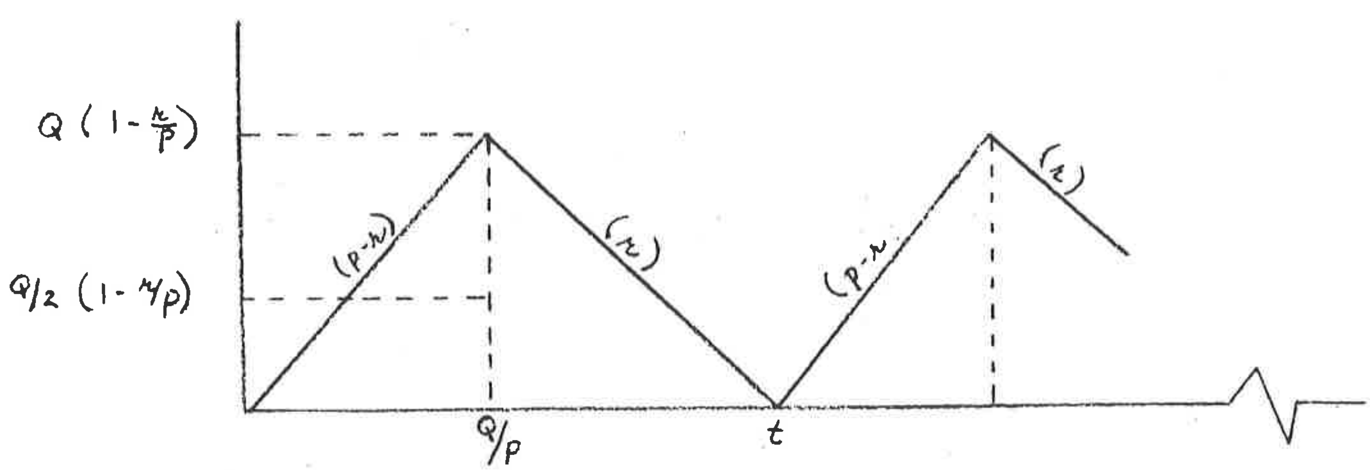
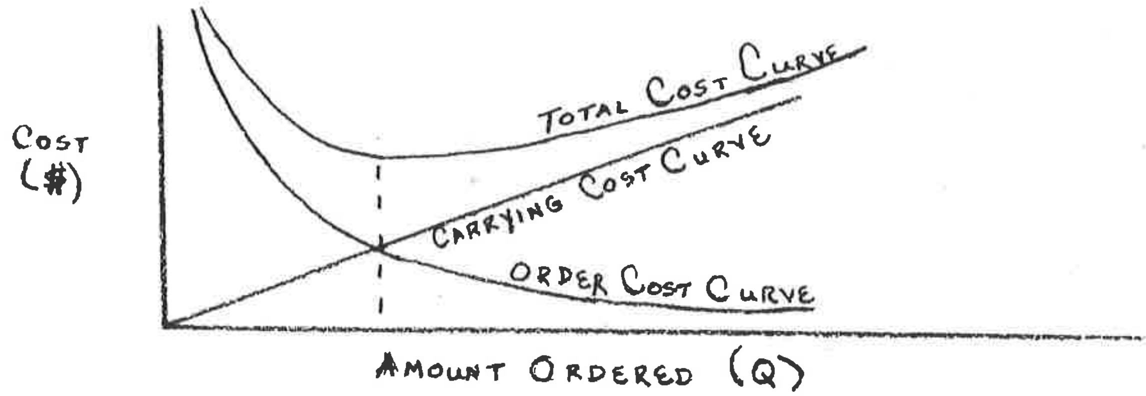


Figure 2
EOQ Cost Curves




Order Cycles and Production Cycles Compared

Let us check the consistency of the model by comparing the number of order cycles with the number of production cycles. For continuity, let us perform the consistency check again using the previous examples. In the first example where

$$R = 3600$$
$$Q^* = 734.85$$

the number of order cycles per year is

$$\frac{R}{Q} = \frac{3600}{734.85} = 4.90.$$

Now, if we assume 360 days per year, then the number of production days per production cycle (See Figure 1) is Q/p and equals 14.70. During each production cycle, inventory will accumulate for Q/p days at a rate of (p - r) per day, where (p - r) is the difference in the production rate versus the usage rate. Thus maximum inventory is

$$I_{max} = \frac{Q}{p}(p - r) = \frac{734.85}{50}(50 - 10) = 587.88.$$

Now,

$$\frac{\frac{Q}{p}(p - r)}{r}$$

is the number of run out days or that number of days required to deplete the inventory from the maximum level to a zero level, and the beginning of another production cycle.

Then "t," total days per cycle, should equal production days plus run out days, or

$$t = \frac{Q}{p} + \frac{\frac{Q}{p}(p - r)}{r}$$

$$= \frac{Q}{p} + \frac{Q}{p}\left(\frac{p - r}{r}\right)$$

$$= \frac{Q}{p}\left[1 + \left(\frac{p - r}{r}\right)\right]$$

$$= \frac{Q}{r}.$$

Substituting for t,

$$t = \frac{734.85}{10} = 73.5 \text{ days.}$$

Again assuming 360 days per year, then

$$\frac{360}{73.5} = 4.90 \text{ production cycles per year}$$

which is the same result that was determined for R/Q. Thus, our first example has been shown to be consistent.

Continuing, in the second example with R = 2400, Q* = 600 and TRC* = 96.0, performing the same check as before yields the following:

$$\frac{R}{Q} = \frac{2400}{600} = 4.0$$

and

$$t = \frac{Q}{r} = \frac{600}{10} = 60 \text{ days per production cycle}$$

and, with 360 days per year, the total cycles per year is 360/60 = 6. This result indicating six production cycles per year with only four ordering (setup) cycles is truly inconsistent.

The inconsistency problem results because the model, as presented in the standard texts, presupposes consistency between R and r. Thus the total cost equation should be adjusted for the fact that the actual number of cycles per year in days (or weeks, months, etc.) may not be consistent with the number of ordering (setup) cycles per year. The necessary adjustment in the model will be seen to occur in the structuring of average inventory and the resulting inventory carrying cost.

<u>The Model Revised</u>

Clearly, the original model as given and as discussed in several texts, is misleading to the user if production is on an intermittent basis. To begin, let

T = total time period consisting of t cycles (<u>not</u> R/Q) expressed in days, weeks, months or whatever unit desired. NOTE: T, r and p must be expressed, or converted, to similar units.

If demand is expressed, for example, annually, and r and p are expressed in days, then

$$\frac{\frac{R}{Q}}{\frac{T}{\frac{Q}{p}\left[1 + \left(\frac{p-r}{r}\right)\right]}} = \frac{\frac{Z}{Q}}{\frac{T_R}{Q}} = \frac{Z}{Q} \cdot \frac{Q}{T_R} = \frac{R}{Q \cdot T}$$

expresses the <u>proportion</u> of time that inventory actually exists. The total relevant cost equation can now be adjusted by this weighting factor, or

$$TRC = \frac{R}{Q}\,Cp + \frac{Q}{2}\,CCh\left(1 - \frac{r}{p}\right)\left[\frac{\frac{R}{Q}}{\frac{T}{\frac{Q}{p}\left[1 + \left(\frac{p-r}{r}\right)\right]}}\right]$$

$$= \frac{R}{Q}\,Cp + \frac{Q}{2}\,CCh\left(1 - \frac{r}{p}\right)\left[\frac{R}{QT} \cdot \frac{Q}{p}\left(1 + \left[\frac{p-r}{r}\right]\right)\right]$$

$$= \frac{R}{Q}\,Cp + \frac{Q}{2}\,CCh\left(1 - \frac{r}{p}\right)\left(\frac{R}{pT}\left[1 + \left(\frac{p-r}{r}\right)\right]\right)$$

$$\frac{dTRC}{dQ} = \frac{-RCp}{Q^2} + \frac{RCCh}{2pT} \left(1 - \frac{r}{p}\right) \left[1 + \left(\frac{p-r}{r}\right)\right] = 0$$

$$Q^* = \sqrt{\frac{2pTCp}{CCh\left(1 - \frac{r}{p}\right)\left[1 + \left(\frac{p-r}{r}\right)\right]}}$$

$$= \sqrt{\frac{2pTCp}{CCh\left(\frac{p-r}{r}\right)}}$$

Now, substituting to determine Q*, yields

$$Q^* = \sqrt{\frac{(2)\ (50)\ (360)\ (12)}{(1.00)\ (.20)\ \left(\frac{50-10}{10}\right)}}$$

$$= \sqrt{540{,}000}$$

$$= 734.85$$

which is exactly the value of Q* determined when R equalled 3600.

The interesting feature of this revised equation for Q* is that R (demand per year) is not included in the equation. The comparison of this new equation for Q* and the previous equation is better illustrated if both numerator and denominator are multiplied by r/p, or

$$Q^* = \sqrt{\frac{2rTCp}{CCh\left(1 - \frac{r}{p}\right)}}$$

This formulation is precisely the form generally found in the literature with the exception of r•T in the numerator in lieu of R. This again expresses the point that when R = r•T, a variation in Q* will result which in turn, will affect total relevant cost (TRC).

The net effect on TRC* can be seen by substituting as follows, first for R = 3600 and then for R = 2400. For R = 3600

$$TRC = \frac{(3600)}{(734.85)}(12) + \frac{(734.85)\ (1.00)\ (.20)}{(2)}(1 - 10/50)\frac{(3600)}{(10)\ (360)}$$

$$= \$58.79 + 58.79$$

$$= \$117.58.$$

And for R = 2400,

$$TRC = \frac{(2400)\ (12)}{(734.85)} + \frac{(734.85)\ (1.00)\ (.2)}{(2)} + (1 - 10/50)\ \frac{(2400)}{(10)\ (3600)}$$

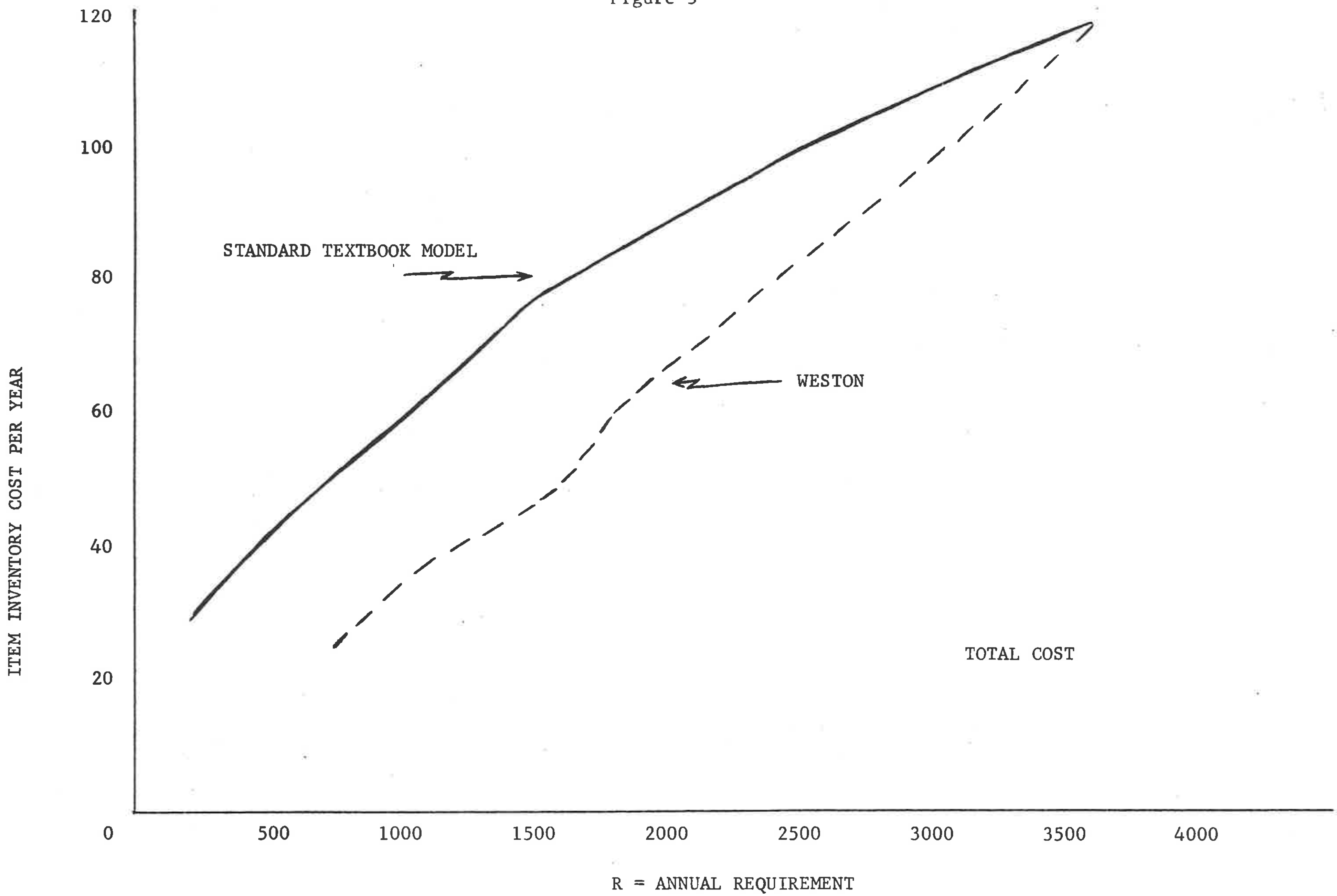
$$= \$39.19 + 39.19$$

$$= \$78.38$$

This is a considerable reduction ($96.00 - 78.38 or 18 %) in the total relevant inventory cost for the item in question for one year. It should be apparent that a manufacturer who produces several products on an intermittent basis, but does not necessarily produce each product throughout the year, that an 18% reduction in inventory cost, applied to many products, could result in considerable savings. Of course, this savings will vary depending on the severity of the inconsistency between R and r·T, and demand and cost factors.

For the example cited, Figure 3 illustrates that savings will increase in proportion to the difference in d and r·T. It is important to note that while the curves intersect at R = 3600, they do not continue beyond R = 3600 with the situation reversed. In other words, the author's formulation does not become suboptimal beyond an annual requirement in excess of r·T units. This is because a demand (R) level in excess of r·T units per year would be inconsistent--that is, at the given consumption rate (r), all units would not be consumed within a year's time. Thus demand in excess of r·T units is nonsensical if r is assumed a constant or an expected value.

Figure 3



ITEM INVENTORY COST PER YEAR
120
100
80
60
40
20
0
500
1000
1500
2000
2500
3000
3500
4000
STANDARD TEXTBOOK MODEL
WESTON
TOTAL COST
R = ANNUAL REQUIREMENT

Effect of Revised Model on Quantity Discount

Let us begin this aspect of our discussion by assuming that our perspective is from the point of view of a producer. Before a producer will offer a quantity discount, the total relevant inventory cost with the discount should be less than or equal to the total relevant inventory cost without discount, or

$$TC_{w/disc} \leq TC_{w/o\ disc.}$$

Having already determined Q* for this model to be

$$Q^* = \sqrt{\frac{2rTCp}{CCh\ (1 - \frac{r}{p})}},$$

the value of Q* can be substituted into left and right hand sides of the TC equation as follows:

$$\frac{RCp}{KQ^*} + \frac{KQ^*ChR}{2Tr}\ (1 - \frac{r}{p})\ C\ (1 - d) - Rcd \leq \frac{RCp}{Q^*} + \frac{Q^*ChCR}{2rT}$$

$$\frac{RCp}{K\sqrt{\frac{2rTCp}{CCh\ (1 - \frac{r}{p})}}} + \frac{K\sqrt{\frac{2rTCp}{CCh\ (1 - \frac{r}{p})}}}{2} \cdot \frac{c\ (1 - d)\ ChR}{rT}\ (1 - \frac{r}{p}) - Rcd$$

$$\leq \frac{RCp}{\sqrt{\frac{2rTCp}{CCh\ (1 - \frac{r}{p})}}} + \sqrt{\frac{2rTCp}{CCh\ (1 - \frac{r}{p})}} \cdot \frac{CChR}{2rT}\ (1 - \frac{r}{p})$$

This equality reduces to

$$\frac{1}{K} + K\ (1 - d) \leq 2 + d\sqrt{\frac{2crT}{CpCh\ (1 - \frac{r}{p})}}$$

Now letting X equal the constant $\sqrt{\frac{2crT}{CpCh\ (1 - \frac{r}{p})}}$ reduces the inequality to $\frac{1}{K} + K\ (1 - d) \leq 2 + dX$. This inequality can be solved for K by the process of completing the square, or

$$K \leq (2 + dX) + \sqrt{\frac{(2 + dX)^2 - 4(1 - d)}{2(1 - d)}}$$

Now, since K is the constant by which Q* must be multiplied to obtain the minimum order quantity necessary to receive the discount, tables of K can be prepared for various discount levels given

- -the inventory carrying cost (Ch) expressed as a percentage
- -setup (ordering) cost (Cp)
- -usage rate per time period (r)
- -production rate per time period (p)
- -total time period (T) consistent with demand

This table does allow for the fact that R (demand) may not equal $r \cdot T$. Thus this table of breakeven values for K would appear in general form as in Figure 4.

Applying this analysis to the example cited earlier where R = 3600 generates a K value of 2.1742, the breakeven table would appear as in Figure 5. This value for K would be interpreted to mean that the maximum quantity that should be produced in one time consistent with the discount offered would be

$$D_Q = K_t \cdot Q^*$$

where $D_Q$ is the maximum amount to produce and $K_t$ is the "table" value of K. Here,

$$D_Q = (2.1742)(734.85)$$
$$= 1597.7$$

or approximately 1600.

Performing the same analysis employing the standard textbook flow receipt model generates

$$D_Q = (2.0269)(600)$$
$$= 1216.1$$

or a difference of almost 400 units of production.

Figure 4
Quantity Discount Breakeven Table

| | Breakeven value for K discount | | | | | |
|---|---|---|---|---|---|---|
| €·r·T | .01 | .02 | .03 | .04 | .05 | etc. |

Figure 5
Application of the Quantity Discount Breakeven Table

| | Breakeven value for $K_d$ |
|---|---|
| €·r·T | .01 |
| 3600 | 2.1742 |

## Conclusion

This paper has attempted to point out the standard flow receipt EOQ model found in most textbooks is incorrect for the situation where the number of ordering (setup) cycles per year is not consistent with the number of production cycles per year. This inconsistency exists where demand (R) is not equal to usage rate per sub-time period times the number of sub-time periods per macro-time period (year). When the inconsistency is applied to a quantity discount situation, the end result from a producer's point of view is more production runs than necessary with a resultant higher total cost; from a purchaser's point of view, the result is the possibility of not accepting discounts when in fact discounts should be accepted.

## References


(1) Buffa, Elwood S. Operations Management. New York: John Wiley & Sons, 1968.

(2) Fetter, Robert B. and Dalleck, Winston C. Decision Models for Inventory Management. Homewood, Illinois: Richard D. Irwin, Inc., 1961.

(3) Hadley, G. and Whitin, T. M. Analysis of Inventory Systems. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963.

(4) Niland, Powell. Production Planning, Scheduling and Inventory Control. London: The Macmillan Company, 1970.

(5) Starr, M. K. and Miller, D. W. Inventory Control: Theory and Practice. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962

(6) Wagner, Harvey M. Principles of Operations Research. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1969.

## A CASE OF DECISION MAKING UNDER CONDITIONS OF UNCERTAINTY

THE PROBLEM:

A meat packing company receives meat in bulk quantities at the beginning of a given time period (say a week) and processes, packs and eventually ships the finishing product to wholesalers. Meat supply is a random variable and follows a normal distribution with known mean and standard deviation. Workers are hired prior to meat delivery for a given number of hours of work during the following period, week, usually for 40 hours. In addition the workers are guaranteed the following overtime schedules:

> for the first 8 hours of overtime, the basic wage rate increases by fifty percent (time and a half)
>
> for more than 8 hours of overtime work the basic wage rate increases by 100 percent (double)

The company is interested in minimizing total overtime and idle costs considering that idle labor costs occur whenever meat delivery falls short the packing capability of the workers for the period that they were hired.

In the case of overtime costs increasing by 50 percent a solution can be reached by minimizing the expected total cost by setting the derivative of the cost function equal to zero and solving for the control variable "labor". In the case of double overtime, however, if the same procedure is followed no easy solution could be reached as the derivative would involve an equation with two unknowns.

The purpose of this paper is to demonstrate that in the second case of the increase of overtime wage rate by 100 percent, a computer simulated solution by Monte Carlo can be obtained and that this method of solution provides certain additional advantages. The problem will be solved first by the cost minimization method by the use of conventional calculus and then for both cases the computer simulated solution will be presented. Consider the mathematical cases first.

## A. OVERTIME WAGE RATE INCREASES BY 50 PERCENT

Notation: Let $\underline{a}$ be the wage rate in dollars per pound of meat

Let $\underline{b}$ be the wage rate per pound of meat for the first 8 overtime hours, or $b = 3/2(a)$

Let $\underline{c}$ be the wage rate per pound of meat for overtime hours beyond the first 8 hours, or $c = 2a$

Let $\underline{M}$ be the random variable "meat" in pounds

Let $\underline{W}$ be the control variable "workers" n expressed in pounds, in such a manner that if ~~N~~ n workers are hired for say 40 hours and each can pack on the average $\bar{x}$ pounds of meat, then

$$W = \cancel{N}\, n \cdot 40 \cdot \bar{x}$$

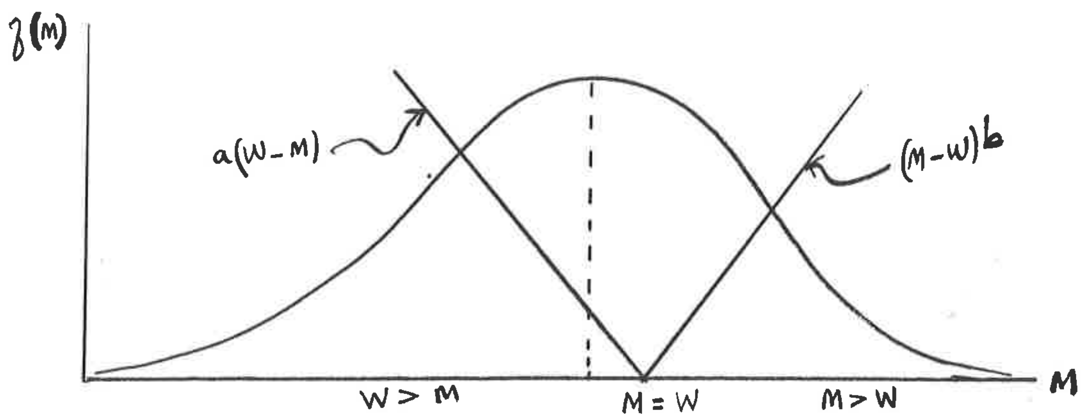
The cost functions for both cases of idle and overtime costs would be:

1. for a given period if $M=W$ there would be no overtime and no idle costs to be minimized.

2. for a given period if $M > W$ the company will incur overtime costs for the first 8 hours or $(M-W)b$

3. for a given period if $M < W$ the company will incur idle costs or $(W-M)a$.

Since the cost functions are linear and the distribution of M is normal, the following diagram shows schematically the relationships:



The total expected idle and overtime cost which needs to be minimized is:

$$E(\text{cost}) = \int_{-\infty}^{W} a(W-M)f(M)\,dM + \int_{W}^{\infty} b(M-W)f(M)\,dM$$

$$= aW\int_{-\infty}^{W} f(M)\,dM - a\int_{-\infty}^{W} M(fM)\,dM + b\int_{W}^{\infty} Mf(M)\,dM - bW\int_{W}^{\infty} f(M)\,dM$$

Since $\int_{-\infty}^{W} f(M)\,dM + \int_{W}^{\infty} f(M)\,dM = 1 \Rightarrow \int_{W}^{\infty} f(M)\,dM = 1 - \int_{-\infty}^{W} f(M)\,dM$

and $\int_{-\infty}^{W} Mf(M)\,dM + \int_{W}^{\infty} Mf(M)\,dM = E(M) \Longrightarrow \int_{W}^{\infty} M(fM)\,dM = E(M) - \int_{-\infty}^{W} Mf(M)\,dM$

Therefore:

$$E(\text{cost}) = aW\int_{-\infty}^{W} f(M)\,dM - a\int_{-\infty}^{W} Mf(M)\,dM + b\left[E(M) - \int_{-\infty}^{W} Mf(M)\,dM\right] - bW\left[1 - \int_{-\infty}^{W} b(M)\,dM\right]$$

If we represent $\int_{-\infty}^{W} f(M)\,dM = F(W)$

and $\int_{-\infty}^{W} Mf(M)\,dM = \underset{-\infty}{\overset{W}{E}}(M)$

Then, $E(\text{cost}) = aW\,F(W) - a\underset{-\infty}{\overset{W}{E}}(M) + b\bar{M} + b\underset{-\infty}{\overset{W}{E}}(M) - bW + bWF(W)$

Simplifying and taking the first derivative of above expression with respect to W, we have:

$$\frac{d\,E(\text{cost})}{d(W)} = (a+b)\,F(W) - b$$

Setting above derivative equal to zero and solving for F(W) we have

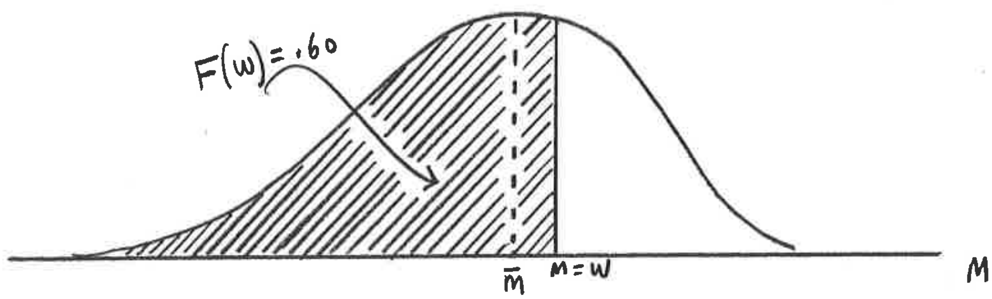
$$F(W) = \frac{b}{b+a}$$

which is the cumulative probability under the normal curve from $-\infty$ to a value of W where the E(cost) is minimum since the second derivative of E(cost) is larger than zero, or $\frac{d^2E(\text{cost})}{dW} = (a+b)$

If we assume now values of a and b, 2 and 3 respectively, then

$$F(W) = \frac{3}{2+3} = .60$$

The cumulative probability under the normal curve at which the cost is minimized is .60 and the following diagram shows the case and gives the value of W at which minimization occurs:




If the random variable M has a mean 40 and a standard deviation 10 ($\mu$ = 40, $\sigma$ = 10), then the optimum strategy for the company would be to hire that number of workers who would be able to pack

$$\mu + .255\sigma = 40+.25(10) = 42.55$$

pounds of meat per week and in doing so in the long run they will be minimizing total idle and overtime costs.

B. OVERTIME WAGE RATE INCREASES BY 100 PERCENT

Consider now the case when the overtime wage rate increases by 100 percent whenever work in overtime exceeds 8 hours per time period, week. The cost functions, still linear, now are:

a. (W-M)a for idle costs

b. (M-W)b for overtime to 8 hours

c. (M-W)2a for overtime past 8 hours

The total expected cost is,

$$E(\text{cost}) = \int_{-\infty}^{W_1} a(W-M)f(M)\,dM + \int_{W_1}^{W_2} b(M-W)f(M)\,dM + \int_{W_2}^{\infty} 2a(M-W)f(M)\,dM \qquad (1)$$

where $W_1$ and $W_1$ correspond to the points of changing of slope of the cost functions and at which the total function is minimized. Optimum solution to this case would be the determination of the cumulative probability at $W_2$ or $F(W_2)$. As before considering that:

$$F(W_1) + F(W_2) + F(W_3) = 1 \Rightarrow F(W_3) = 1 - F(W_1) - F(W_2)$$

and

$$\underset{-\infty}{\overset{W_1}{E}}(M) + \underset{W_1}{\overset{W_2}{E}}(M) + \underset{W_2}{\overset{\infty}{E}}(M) = \bar{M} \Rightarrow \overset{\infty}{E}(M) = \bar{M} - \underset{-\infty}{\overset{W_1}{E}}(M) - \underset{W_1}{\overset{W_2}{E}}(M)$$

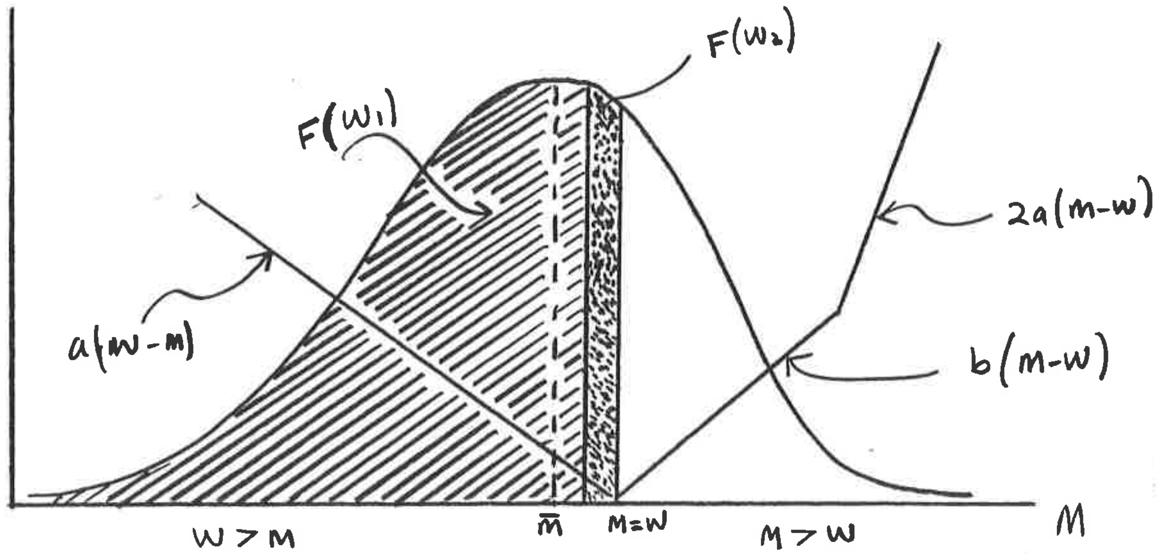
and expanding and collecting terms expression (1) becomes:

$$E(\text{cost}) = W\,F(W_1)\,(a+2a) + (2a-b)\,WF(W_2)\ -\underset{W_1}{\overset{W_2}{E}}(M)\ -2aW-a\underset{-\infty}{\overset{W_1}{E}}(M)+2a\bar{M}-2a\underset{-\infty}{\overset{W_1}{E}}(M)$$

Taking the derivative of E(Cost) with respect to W, we obtain:

$$\frac{d(E(\text{cost}))}{dW} = F(W_1)3a + (2a-b)\,F(W_2) - 2a$$

If we set above expression equal to zero and solve for $F(W_2)$ we would obtain the solution to the problem except that the value of $F(W)$ is functionally related to $F(W_1)$ and cannot utilize the value of $F(W_1)$ previously determined as it did not involve 2a. The following diagram shows the relationships:




It is at this point that the help of simulation may be sought.

C. COMPUTER SOLUTION BY SIMULATION

Since we know the probability distribution of the random variable M we could assume values for W, the labor to be hired, and let the computer, for each value of W considered as constant, compute the cost function,

$$a(W-M)P(M)+b(M-W)P(M) \qquad (2)$$

for all possible values of M the frequency of which is to be determined by either Monte Carlo simulation or by obtaining them directly from tables of the normal distribution given the parameters $\bar{M}$, $\sigma$ and N, in the case of time and a half overtime costs.

For the case of double overtime the function to be evaluated by the computer for given constant values of W and all possible values of M is:

$$a(W-M)P(M)+b(M-W)P(M)+2a(M-W)P(M) \qquad (3)$$

Since we have already assumed values of $\bar{M}$ = 40 and $\sigma$ = 10 for the random variable M in the previous computations, then

$$P(\bar{M}-3\sigma \leq M_i \leq \bar{M}+3\sigma) = .9973$$

$$P(10 \leq M_i \leq 70) = .9973$$

and the domain of the variable $M_i$ is defined in the interval from 10 to 70, and assuming probabilities from normal tables we have:

| $M_i$ | P(M) |
|---|---|
| 10 < 15 | .0062 |
| 15 < 20 | .0166 |
| 20 < 25 | .0440 |
| 25 < 30 | .0919 |
| 30 < 35 | .1498 |
| 35 < 40 | .1915 |
| 40 < 45 | .1915 |
| 45 < 50 | .1498 |
| 50 < 55 | .0919 |
| 55 < 60 | .0440 |
| 60 < 65 | .0166 |
| 65 < 71 | .0062 |
| | 1.0000 |

By using Monte Carlo sampling simulation we assign random digits in each class and then with values of M randomly selected for a given W constant the cost functions of (2) and (3) are computed, which could be expressed as

$$a/2(M-W) \quad \text{and} \quad 5/2(a)(M-W)$$

respectively. Subsequently to obtain the total expected cost $[E(\text{Cost})]$ for each W we compute,

$$\frac{a}{2}\sum_{W=10}^{70}\sum_{M=1}^{N}(M-W) \quad \text{and} \quad \frac{5a}{2}\sum_{W=10}^{70}\sum_{M=1}^{N}(M-W)$$

for (2) and (3) respectively and choose the one with the minimum cost.

For the general case it can be expressed in terms of standard units and be applicable to any parameter normal random variable.

The following table shows the cumulative values and the random digits for the case just described for the application of the Monte Carlo method.

| M | Cumulative | Random digits |
|---|---|---|
| $10 < 15$ | .0062 | 01 |
| $15 < 20$ | .0228 | 2- 3 |
| $20 < 25$ | .0668 | 4- 6 |
| $25 < 30$ | .1587 | 7-16 |
| $30 < 35$ | .3085 | 17-30 |
| $35 < 40$ | .5000 | 31-50 |
| $40 < 45$ | .6915 | 51-69 |
| $45 < 50$ | .8413 | 70-74 |
| $50 < 55$ | .9332 | 75-93 |
| $55 < 60$ | .9772 | 94-97 |
| $60 < 65$ | .9938 | 98-99 |
| $65 < 71$ | 1.0000 | 00 |

An alternative method would be the enumeration and listing of all frequencies of M according to the normal pattern and computation of the costs by programming directly these numbers along with the cost functions without the use of the Monte Carlo method to determine the values of M.

## RESULTS

Using the simulation method with N=1000 the following results were obtained for the values of W from 40 to 50. The simulation started at W=40 since optimum solution should be found in the domain beyond M=40 as it was shown by the calculus method.

1. Solution for the time and a half overtime

It has already been shown using conventional calculus that the optimum solution in this case was at W=42.55 or at a Z=.25 along the standardized M axis. The simulation method obtained the same results as is shown in the following table:

| W | E(Cost) | |
|---|---|---|
| 40 | 19.94024 | |
| 41 | 19.53984 | |
| 42 | 19.33865 | |
| 43 | 19.33167 | ← Optimum, W=43 |
| 44 | 19.51892 | |
| 45 | 19.89044 | |
| 46 | 20.43625 | |
| 47 | 21.14641 | |
| 48 | 22.01096 | |
| 49 | 23.01992 | |
| 50 | 24.16335 | |

By this method the optimum solution is at W=43 since at that value of W E(Cost) is minimum. The difference between 43 and 42.55 should be expected since the variables were discretized because of the use of the computer.

2. Solution for the double overtime

Intuitively we should expect the solution with double overtime to be above W=42 since the slope of the cost function differs now from the previous one and is bigger by an amount equal to the difference of the rates between double overtime and time and a half overtime. The results obtained by simulation for values of W from 40 to 50 are as shown below:

| W | E(Cost) | |
|---|---|---|
| 40 | 34.0340 | |
| 41 | 31.7720 | |
| 42 | 29.9832 | |
| 43 | 28.6492 | |
| 44 | 27.7636 | |
| 45 | 27.2928 | |
| 46 | 27.2304 ← | Optimum W=46 Z=.60 |
| 47 | 27.5276 | |
| 48 | 28.1632 | |
| 49 | 29.1124 | |
| 50 | 30.3520 | |

The optimum solution is at W=46 which corresponds to F(W) = .7257, and Z = .60.

## CONCLUDING REMARKS

This demonstration is not a new method neither a unique one in the use of simulation for solving problems under conditions of uncertainty. The intention is to dramatize the narrow scope of some conventional or classical, so to speak, methods for solving problems of this nature. It was shown that in this case conventional mathematics failed to provide a means which a non-mathematician could use to solve a simple problem as this even if the distribution of the random variable under consideration was known to be normal.

One can visualize the difficulties to be encountered if the random variable follows some unknown pattern for which there may be available a mathematical function or may not. The least we can do in the latter case is to declare ourselves incapable of solving the problem in this way. Furthermore most of the random variables are subject to a constant change or parameters and distribution patterns. We have methods of course in determining the shapes of probability distributions and identification of the changes of the parameters, but this can be done at intervals and only for a specific period of time.

By simulation most of these problems can be effectively solved. It would suffice for the researcher to keep a good constant record of observations of occurrences of values of random variables. By constructing a cumulative distribution of these values the distribution shape and parameter changes at any given time interval can be determined. In addition, the cost functions need not constitute a restraint in the manipulation whenever simulation techniques are to be used, whereas this need not necessarily be true for direct mathematical manipulation of cost functions.

# AIDS WESTERN REGIONAL 1971

CONFERENCE NOTES

# AIDS WESTERN REGIONAL 1971

CONFERENCE NOTES

# AIDS WESTERN REGIONAL 1971

CONFERENCE NOTES

# AIDS WESTERN REGIONAL 1971

## CONFERENCE NOTES

# AIDS WESTERN REGIONAL 1971

CONFERENCE NOTES