THE VALUATION OF HOUSES IN AN UNCERTAIN WORLD WITH SUBSTANTIAL TRANSACTION COSTS

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ABSTRACT

This paper presents a dynamic model of residential real estate valuation that takes into account the substantial transaction costs and the uncertain time paths of rents and prices. By temporarily postponing decisions, buyers and sellers obtain information about future rents and prices and may avoid transactions that are costly to reverse. These are powerful arguments for inertia. Another implication is that there may be a large wedge between the buyers' and sellers' valuation of houses.

The Intrinsic Value of a House

The intrinsic value of a stock is the present value of its cash flow. The same is true of a house. The intrinsic value V is the present value of the cash flow C_t, discounted by the required rate of return R:

$$V = \frac{C_1}{(1+R)^1} + \frac{C_2}{(1+R)^2} + \frac{C_3}{(1+R)^3} + \dots$$

The cash flow from a house is the rent the buyer would pay to live in this house minus the expenses associated with home ownership. If you would pay \$30,000 a year to rent a house, home ownership implicitly gives you \$30,000 that you otherwise pay to someone else. On the other hand, as a homeowner, you must pay property taxes, insurance, maintenance, and some utilities that you would not have to pay if you were a renter. If these expenses are \$10,000 a year, your implicit net annual income is C_1 = \$30,000 - \$10,000 = \$20,000. To determine the present value, we must project this implicit income into the future. As with stocks, the constant growth model provides a simple and insightful starting point. If the cash flow grows as a rate $g < R$, then the present value formula simplifies to $V = C_1/(R - g)$. For example, if the cash flow is \$20,000, growing at 5% a year, and the required rate of return is 10%, this house's intrinsic value is V = \$20,000/(0.10 - 0.05) = \$400,000.

Uncertainty

As with stocks, uncertainty is a key fact of life. One enormous difference between stocks and real estate is that the transaction costs are trivial for the former and substantial for the latter. This combination of substantial uncertainty and high transaction costs has interesting implications for the valuation of houses. A household considering the purchase of a home faces uncertainty regarding both the potential future rent savings from home ownership and changes in the cost of buying a house. Assume that the net cash flow C from home ownership can be described by a geometric Brownian motion equation:

$$\frac{dC}{C} = \square dt + \square dz \qquad (1)$$

where □ is the trend rate of growth of rent and dz is the increment of a standard Wiener process. The cost P of buying a house also evolves according to geometric Brownian motion:

$$\frac{dP}{P} = \square dt + \square dz \qquad (2)$$

The correlation coefficient between rent C and price P is □.

The house costs P and the expected present value of the cash flow is V = C/(R - □). The simple investment rule of buying or selling when V is larger or smaller than P is inappropriate in an uncertain world when investments are costly to reverse. Because purchases and sales are expensive to undo, there is a potential benefit from postponing transactions until price/rent conditions are decisively favorable.

Dynamic programming can be used to determine the value F[C, P] of the purchase option. A natural assumption is that the option value is homogeneous of degree one: F[C, P] = Pf[C/P]. The substitution of the requisite partial derivatives into the differential equation given by Ito's Lemma yields

$$f\left[C/P\right] = A_1\left(C/P\right)^a \qquad (3)$$

where

$$a = 0.5 + \frac{\square\square\square}{\square} + \sqrt{\square 0.5 + \frac{\square\square\square}{\square}\square^2 + \frac{2\left(R\square\square\right)}{\square}} > 1 \qquad \square = \square^2 - 2\square s\square + \square^2$$

For the homeowner, the value G[C, P] of the house includes the cash flow C and the value of the option to sell if the rent/price ratio falls sufficiently. We assume this value to be homogeneous of degree one, G[C, P] = Pg[C/P]. In the region where the home is held,

$$g\left[C/P\right] = B_2\left(C/P\right)^b + \frac{C/P}{R\square\square} \qquad (4)$$

where

$$b = 0.5 + \frac{\square\square\square}{\square}\square\sqrt{\square 0.5 + \frac{\square\square\square}{\square}\square^2 + \frac{2\left(R\square\square\right)}{\square}} < 0$$

A household waiting to buy will do so when the rent/price ratio rises to the threshold $\square_1$; a homeowner waiting to sell will do so when the rent/price ratio falls to the threshold $\square_2$. At C/P = $\square_1$, the value of the buy option is equal to the value of owning the house net of the purchase price: F[C, P] = G[C, P] - P. At C/P = $\square_2$, the value of the sell option is equal to the value of the buy option plus the sale price net of the proportional sales cost □, G[C, P] = F[C, P] - (1 - □)P. Also equating the first derivatives, we have

$$f\left[\square_1\right] = g\left[\square_1\right]\square 1 \qquad (5)$$

$$f\square\left[\square_1\right] = g\square\left[\square_1\right] \qquad (6)$$

$$g\left[\square_2\right] = f\left[\square_2\right] + \left(1\square\square\right) \qquad (7)$$

$$g\square\left[\square_2\right] = f\square\left[\square_2\right] \qquad (8)$$

The substitution of the differential equations (3) and (4) into the value-matching and smooth-pasting conditions (5) - (8) gives these four equations,

$$0 = A_1 \square_1^{a} \square B_2 \square_1^{b} \square \frac{\square_1}{R \square \square} + 1$$

$$0 = aA_1 \square_1^{a \square 1} \square bB_2 \square_1^{b \square 1} \square \frac{1}{R \square \square}$$

$$0 = A_1 \square_2^{a} \square B_2 \square_2^{b} \square \frac{\square_2}{R \square \square} + \left(1 \square \square\right)$$

$$0 = aA_1 \square_2^{a \square 1} \square bB_2 \square_1^{b \square 1} \square \frac{1}{R \square \square}$$

which can be solved for the thresholds $\square_1$ and $\square_2$ and the differential-equation parameters A_1 and B_2.

Illustrative Calculations

Consider this case: $\square = 0.05$, $\square = 0.10$, $\square = 0.05$, $\square = 0.20$, $\square = 0.50$, $R = 0.10$, and $\square = 0.08$. Rent has a 5% trend growth rate and 10% standard deviation; price has a 5% trend growth rate and 20% standard deviation. The correlation between rent and price is 0.50. The required rate of return is 10% and the sale transaction cost is 8% (including brokerage commission, legal fees, and fixup costs). The present value is $V = C/(0.10 - 0.05) > P$ iff $C/P > 0.05$. If there were no transaction costs, the house should be bought or sold when the rent-price ratio is above or below 0.05. With an 8% sales cost, the threshold rent-price ratios work out to be $\square_1 = 0.069$ and $\square_2 = 0.034$. A household that is waiting to buy should do so when the rent-price ratio rises to 0.069; a homeowner should sell when the rent-price ratio falls below 0.034. A buyer requires a rent-price ratio 38% larger than 0.05 because a purchase has the additional cost of extinguishing the possibility of buying at terms that are even more favorable and also less likely to incur future transaction costs. A seller requires a rent-price ratio 32% below 0.05 because a sale incurs transaction costs and extinguishes the possibility of selling at more favorable terms that are less likely to be reversed sufficiently to persuade the seller to buy again. These effects are substantial. A rent-price ratio of 0.05 implies that a house with a \$20,000 cash flow is worth \$20,000/0.05 = \$400,000. The 0.069 threshold implies that a buyer is only willing to pay \$20,000/0.069 = \$290,000; the 0.034 threshold implies that a homeowner isn't willing to sell for less than \$20,000/0.069 = \$588,000.

Conclusion

Homebuyers should consider that, if they buy, the rent-price ratio may subsequently fall, causing them to regret their purchase, and, if they wait, the rent-price ratio may rise, giving them the opportunity to buy at more favorable terms that are less likely to be reversed in the future. Homeowners should consider the possibility that, if they sell now, the rent-price ratio may rise, causing them to regret their sale, and, if they wait, the rent-price ratio may fall, giving them the opportunity to sell at more favorable terms that are less likely to be reversed. Thus uncertainty and transaction costs are powerful arguments for inertia and create a wedge between the buyers' and sellers' valuation of houses. This may explain why—unlike the stock market—so many real estate transactions seem motivated by socio-demographic necessity (marriage, divorce, relocation), rather than purely economic calculations. In the absence of this wedge, households would shift back and forth between renting and buying the same way they jump in and out of stocks. Because of this wedge, homeowners typically sell because they have to, not because they consider the sale price high enough to make renting more attractive than homeownership.