OPTIMAL NETWORK CONFIGURATION FOR USAF BASING OPTIONS

Ronald G. McGarvey, Robert S. Tripp, Edward W. Chan, C. Robert Roll, Jr.,
RAND, 1700 Main Street, Santa Monica, CA, 90407. 310 393-0411, ronm@rand.org

ABSTRACT

The shift to a force structured for expeditionary operations has presented the U.S. Air Force with a number of logistical challenges. These operations require such resources as munitions, fuel, engines, avionics and war reserve materiel (WRM). In an expeditionary world, the logistics support processes must be capable of responding to rapidly deployed forces, either by deploying along with the fighting units or by connecting permanently located support processes to the remote forces. In this paper, we present an optimization model that computes a minimum cost location and allocation of WRM storage facilities capable of supporting the entire globe.

1. INTRODUCTION

The end of the Cold War and the associated realignment of power centers placed the United States and its allies in a new environment with vastly different security challenges than the one faced only a decade earlier. In today's environment, the United States Air Force (USAF) has been called upon to make numerous overseas deployments, often on short notice, to carry out missions ranging from humanitarian relief to major combat operations.

Because the nature, timing, and location of these operations have been difficult to predict, the USAF has moved to an expeditionary concept called Air and Space Expeditionary Force (AEF). Rather than attempt to base forces in a large number of permanent overseas installations, forces will instead rapidly deploy from the United States to various Forward Operating Locations (FOLs) – places out of which aircraft operate – on an as-needed basis in the event of a crisis. These forces will be supported by stocks of War Reserve Materiel (WRM), that are to be pre-positioned at Forward Support Locations (FSLs) situated in strategic regions of the world, and shipped to the FOLs as needed (for further information on the AEF concept see [2] [4]). A combat support command and control (CSC2) framework is needed to configure the set of contingency-specific FOLs to the more static set of FSLs. WRM, broadly speaking, consists of heavy items such as kits for constructing temporary bases, general-purpose vehicles, and munitions. WRM is of particular interest because it constitutes the majority of the equipment tonnage that must be transported when the USAF deploys.

This paper describes work conducted to assess potential FSL locations for pre-positioning WRM stocks. Because the future security environment is unpredictable, the selection of FSLs must be evaluated against a collection of scenarios, considering the differing demands on combat support imposed by scenarios ranging from major regional conflicts to humanitarian operations. As a safeguard against geographic uncertainties, these scenarios should be situated across a disparate set of worldwide locations. Time considerations also factor into the analysis. Because the WRM storage warehouses will be operational for many years, the selected network of FSL locations should be capable of supporting the variety of operations that might commence over this time interval.

Facility location problems have been well studied in the literature. However, previous work on this topic does not take into account important aspects of the problem faced by the USAF. An earlier RAND study concluded that a set of five FSLs could support worldwide operations, by placing most of the world within an airlifter's flying range of at least one FSL. In effect, this study solved a *set covering* problem. However, this is an uncapacitated facility location model. It does not consider constraints on the storage space at the FSLs. Nor does it consider constraints on the transportation system, such as those on the number of available transport vehicles in the system, or the vehicle throughput (the number of vehicles that can be loaded or unloaded at a location at a point in time) at FSLs and FOLs. Additionally, since the set covering model does not consider the amount of materiel to be transported, it does not give an allocation of WRM to FSLs. Other studies in the literature (e.g., [1] [3]) have examined the design of distribution networks more similar to our problem. However, these studies also fail to consider restrictions on the fleet of transport vehicles and on the vehicle throughput at FSLs and FOLs. We pose this problem as a mathematical program, the formulation of which appears in Section 2.

2. MATHEMATICAL MODEL

The optimization model is constrained by throughput, storage space, and authorized resources and is driven by the time-phased demand for commodities at FOLs. The output of this optimization is the creation of a network that connects a set of disjointed FSL and FOL nodes together. It allocates resources to a particular FSL and dictates the movement of WRM resources, munitions and personnel from FSLs to FOLs. The model also computes the type and the number of transportation vehicles required to move the materiel to the FOLs. The mathematical description of the model is as follows:

Sets and Set Indices

$i \in \mathcal{I}$	commodities
$\mathcal{AMM}(\mathcal{I})$	munitions; $\mathcal{AMM}(\mathcal{I}) \subseteq \mathcal{I}$
$\mathcal{NAM}(\mathcal{I})$	non-munitions; $\mathcal{NAM}(\mathcal{I}) \subseteq \mathcal{I}$
$j \in \mathcal{J}$	FSL index
$k \in \mathcal{K}$	FOL index
$m \in \mathcal{M}$	mode of transport
$\mathcal{AIR}(\mathcal{M})$	aircraft; $\mathcal{AIR}(\mathcal{M}) \subseteq \mathcal{M}$
$\mathcal{LAN}(\mathcal{M})$	land vehicles; $\mathcal{LAN}(\mathcal{M}) \subseteq \mathcal{M}$
$\mathcal{SEA}(\mathcal{M})$	sea vehicles; $\mathcal{SEA}(\mathcal{M}) \subseteq \mathcal{M}$
$h \in \mathcal{H}$	phase; $\mathcal{H} = \{1, 2, ...\}$
$t \in \mathcal{T}$	time periods which divide up each phase h; $\mathcal{T} = \{1, 2, ...\}$

There is an implicit notation convention such that any parameter or variable defined on both indices k and t reflects the time period t occurring within the unique phase h associated with FOL k. This means that if a location is assumed to act as an FOL in multiple phases, a unique FOL index must be created for that FOL for each associated phase. There is also an implicit assumption throughout the entire model that terms having an index value $t \le 0$ are not considered. Personnel (hereafter referred to as "PAX") constitute a commodity that belongs to neither subset $\mathcal{AMM}(\mathcal{I})$ nor $\mathcal{NAM}(\mathcal{I})$.

Data Parameters

$A_{\aleph j}$ maximum load space, in class $\aleph$ $\left(\mathcal{AIR}(\mathcal{M}) = 1, \mathcal{LAN}(\mathcal{M}) = 2, \mathcal{SEA}(\mathcal{M}) = 3\right)$ equivalent vehicles, at FSL j

$B_{\aleph k}$ maximum unload space, in class $\aleph$ $(\mathcal{AIR}(\mathcal{M})=1, \mathcal{LAN}(\mathcal{M})=2, \mathcal{SEA}(\mathcal{M})=3)$ equivalent vehicles, at FOL k

C_{mh} planned systemwide inventory of mode m vehicles at the beginning of phase h

D_{ikt} cumulative demand, in tons (or PAX), for commodity i at FOL k by time t

$E_{\aleph j}$ minimum square footage needed for an economically feasible FSL at location j for commodity class $\aleph$ $(\mathcal{AMM}(\mathcal{I})=1, \mathcal{NAM}(\mathcal{I})=2)$

$F_{\aleph j}$ maximum potential square feet of storage space at FSL j for commodity class $\aleph$ $(\mathcal{AMM}(\mathcal{I})=1, \mathcal{NAM}(\mathcal{I})=2)$

Δ_j fixed cost incurred to open FSL j with $E_{\aleph j}$ square feet of storage space for commodity class $\aleph$ $(\mathcal{AMM}(\mathcal{I})=1, \mathcal{NAM}(\mathcal{I})=2)$

Θ_{mh} cost of obtaining an additional vehicle of mode m at the beginning of phase h

$\Xi_{\aleph j}$ variable cost per square foot of storage space needed beyond $E_{\aleph j}$ for commodity class $\aleph$ $(\mathcal{AMM}(\mathcal{I})=1, \mathcal{NAM}(\mathcal{I})=2)$ at FSL j

Ψ_{ik} shortfall cost per time unit per ton (or per PAX) of commodity i not fulfilled at FOL k

Ω_{ijkm} cost per ton (or per PAX) of commodity i transported from FSL j to FOL k via mode m

α_m number of time periods necessary to load a mode m vehicle

β_m number of time periods necessary to unload a mode m vehicle

γ_m maximum load in tons per mode m vehicle

ζ_k contingency start date at FOL k

η_k contingency finish date at FOL k

λ_m maximum load in PAX per mode m vehicle

μ_k phase of contingency occurrence associated with FOL k

π_{km} additional time needed following unloading for commodities to reach FOL k via mode m

ρ_m conversion factor for parking space for mode m

σ_m utilization rate, expressed (for airlift) as the average flying hour goal per day divided by 24 hours, for mode m

τ_{jkm} one-way transportation time from FSL j to FOL k (or in opposite direction) via mode m

ϕ_i conversion factor for commodity i from tons to square feet of storage space (= 0 for PAX)

Variables

$n_{\aleph j}$ additional square feet of storage space needed beyond $E_{\aleph j}$ for commodity class $\aleph$ $(\mathcal{AMM}(\mathcal{I})=1, \mathcal{NAM}(\mathcal{I})=2)$ at FSL j

p_{jkmt} number of mode m vehicles tasked to transport solely personnel from FSL j to FOL k, beginning loading on time t

q_{jmh} number of mode m vehicles available at FSL j at the start of time $t=1$ during phase h

r_{mh} additional mode m vehicles obtained at the beginning of phase h

s_{ikt} shortfall below demand, in tons (or PAX), for commodity i at FOL k not fulfilled by time t

u_{jkmt} number of mode m vehicles tasked to transport solely munitions from FSL j to FOL k, beginning loading on time t

v_{jmth} number of mode m vehicles available at FSL j at the end of time t during phase h

w_j binary variable indicating status of FSL j

x_{ijkmt} tons (or PAX) of commodity i sent from FSL j to FOL k via mode m, beginning loading on time t

y_{jkmt} number of mode m vehicles tasked to transport some nonmunitions from FSL j to FOL k, beginning loading on time t

z_{jkmt} number of mode m vehicles tasked to make the return trip from FOL k to FSL j, departing on time t

Problem Formulation

$$\min \sum_j \left(\Delta_j w_j + \Xi_{"1"j} n_{"1"j} + \Xi_{"2"j} n_{"2"j}\right) + \sum_{ijkmt} \Omega_{ijkm} x_{ijkmt} + \sum_{mh} \Theta_{mh} r_{mh} + \sum_{ikt} \Psi_{ik} s_{ikt} \tag{1}$$

Subject to

$$\sum_j q_{jmh} \le \left(C_{mh} + r_{mh}\right) \qquad \forall m,h \tag{2}$$

$$\sum_{k \ni \mu_k = h} [p_{jkmt} + u_{jkmt} + y_{jkmt}] \le v_{jm(t-1)h} \qquad \forall j,m,h; t \ge 2 \tag{3}$$

$$\sum_{k \ni \mu_k = h} \sum_{m \in \mathcal{AIR}(\mathcal{M})} \sum_{n=0}^{\alpha_m - 1} [\rho_m (p_{jkm(t-n)} + u_{jkm(t-n)} + y_{jkm(t-n)})] \le A_{"1"j} \qquad \forall j,t,h \tag{4}$$

$$\sum_{k \ni \mu_k = h} \sum_{m \in \mathcal{LAN}(\mathcal{M})} \sum_{n=0}^{\alpha_m - 1} [\rho_m (p_{jkm(t-n)} + u_{jkm(t-n)} + y_{jkm(t-n)})] \le A_{"2"j} \qquad \forall j,t,h \tag{5}$$

$$\sum_{k \ni \mu_k = h} \sum_{m \in \mathcal{SEA}(M)} \sum_{n=0}^{\alpha_m - 1} [\rho_m (p_{jkm(t-n)} + u_{jkm(t-n)} + y_{jkm(t-n)})] \le A_{"3"j} \qquad \forall j,t,h \tag{6}$$

$$\sum_j \sum_{m \in \mathcal{AIR}(\mathcal{M})} \sum_{n=0}^{\beta_m - 1} [\rho_m (p_{jkm(t-\tau_{jkm}-\alpha_m-n)} + u_{jkm(t-\tau_{jkm}-\alpha_m-n)}) + y_{jkm(t-\tau_{jkm}-\alpha_m-n)})] \le B_{"1"k} \qquad \forall k;\ \zeta_k \le t \le \eta_k \tag{7}$$

$$\sum_j \sum_{m \in \mathcal{LAN}(\mathcal{M})} \sum_{n=0}^{\beta_m - 1} [\rho_m (p_{jkm(t-\tau_{jkm}-\alpha_m-n)} + u_{jkm(t-\tau_{jkm}-\alpha_m-n)}) + y_{jkm(t-\tau_{jkm}-\alpha_m-n)})] \le B_{"2"k} \qquad \forall k;\ \zeta_k \le t \le \eta_k \tag{8}$$

$$\sum_j \sum_{m \in \mathcal{SEA}(\mathcal{M})} \sum_{n=0}^{\beta_m - 1} [\rho_m (p_{jkm(t-\tau_{jkm}-\alpha_m-n)} + u_{jkm(t-\tau_{jkm}-\alpha_m-n)}) + y_{jkm(t-\tau_{jkm}-\alpha_m-n)})] \le B_{"3"k} \qquad \forall k;\ \zeta_k \le t \le \eta_k \tag{9}$$

$$\sum_{j,m}\sum_{n=1}^{t} x_{ijkm(n-\tau_{jkm}-\alpha_m-\beta_m-\pi_{km})} \geq D_{ikt} - s_{ikt} \qquad \forall i,k;\ \zeta_k \leq t \leq \eta_k \tag{10}$$

$$\sum_{k\ni\mu_k=h}\ \sum_{i\in\mathcal{AMM}(\mathcal{I})}\ \sum_{m,t} \phi_i x_{ijkmt} \leq E_{"1"j} w_j + n_{"1"j} \qquad \forall j,h \tag{11}$$

$$n_{"1"j} \leq \left(F_{"1"j} - E_{"1"j}\right) w_j \qquad \forall j \tag{12}$$

$$\sum_{k\ni\mu_k=h}\ \sum_{i\in\mathcal{NAM}(\mathcal{I})}\ \sum_{m,t} \phi_i x_{ijkmt} \leq E_{"2"j} w_j + n_{"2"j} \qquad \forall j,h \tag{13}$$

$$n_{"2"j} \leq \left(F_{"2"j} - E_{"2"j}\right) w_j \qquad \forall j \tag{14}$$

$$\sum_{k,m,t} x_{"PAX"jkmt} \leq \left(\sum_{k,t} D_{"PAX"kt}\right) w_j \qquad \forall j \tag{15}$$

$$\begin{aligned}\sum_{k\ni\mu_k=h}\ \sum_{j}\Bigg(&\left[\sum_{t}\tau_{jkm}\left(p_{jkmt}+u_{jkmt}+y_{jkmt}\right)\right]+\left[\sum_{t=1}^{\|\mathcal{T}\|-\tau_{jkm}}\tau_{jkm}z_{jkmt}\right]+\\ &\left[\sum_{t=\|\mathcal{T}\|-\tau_{jkm}+1}^{\|\mathcal{T}\|-1}\left(\|\mathcal{T}\|-t\right)z_{jkmt}\right]\Bigg)\leq\|\mathcal{T}\|\left(C_{mh}+r_{mh}\right)\sigma_m \qquad \forall m,h\end{aligned} \tag{16}$$

$$\sum_{i\in\mathcal{AMM}(\mathcal{I})} x_{ijkmt} \leq \gamma_m u_{jkmt} \qquad \forall j,k;\ m\notin\mathcal{SEA}(\mathcal{M});\ \zeta_k \leq t \leq \eta_k \tag{17}$$

$$\sum_{i\in\mathcal{NAM}(\mathcal{I})} x_{ijkmt} \leq \gamma_m y_{jkmt} \qquad \forall j,k;\ m\notin\mathcal{SEA}(\mathcal{M});\ \zeta_k \leq t \leq \eta_k \tag{18}$$

$$\sum_{i\in\mathcal{AMM}(\mathcal{I})\bigcup\mathcal{NAM}(\mathcal{I})} x_{ijkmt} \leq \gamma_m y_{jkmt} \qquad \forall j,k;\ m\in\mathcal{SEA}(\mathcal{M});\ \zeta_k \leq t \leq \eta_k \tag{19}$$

$$x_{"PAX"jkmt} \leq \lambda_m y_{jkmt} \qquad \forall j,k;\ m\in\mathcal{SEA}(\mathcal{M});\ \zeta_k \leq t \leq \eta_k \tag{20}$$

$$x_{"PAX"jkmt} \leq \lambda_m p_{jkmt} \qquad \forall j,k;\ m\notin\mathcal{SEA}(\mathcal{M});\ \zeta_k \leq t \leq \eta_k \tag{21}$$

$$\sum_{j} z_{jkmt} = \sum_{j}\left[p_{jkm(t-\tau_{jkm}-\alpha_m-\beta_m)} + u_{jkm(t-\tau_{jkm}-\alpha_m-\beta_m)} + y_{jkm(t-\tau_{jkm}-\alpha_m-\beta_m)}\right] \quad \forall k,m;\ \zeta_k \leq t \leq \eta_k \tag{22}$$

$$v_{jmth} = v_{jm(t-1)h} + \sum_{k\ni\mu_k=h}\left[z_{jkm(t-\tau_{jkm})} - p_{jkmt} - u_{jkmt} - y_{jkmt}\right] \qquad \forall j,m,h;t\geq 2 \tag{23}$$

$$v_{jm"1"h} = q_{jmh} + \sum_{k\ni\mu_k=h}\left[-p_{jkm"1"} - u_{jkm"1"} - y_{jkm"1"}\right] \qquad \forall j,m,h \tag{24}$$

$$x_{ijkmt} = 0 \qquad t<\zeta_k;\ t>\eta_k \tag{25}$$

$$n_{"1"j}, n_{"2"j}, p_{jkmt}, q_{jmh}, r_{mh}, s_{ikt}, u_{jkmt}, v_{jmth}, x_{ijkmt}, y_{jkmt}, z_{jkmt} \geq 0 \tag{26}$$

$$p_{jkmt}, u_{jkmt}, y_{jkmt}, z_{jkmt} \qquad \text{integer} \tag{27}$$

$$w_j \in \{0,1\} \tag{28}$$

The objective function (1) minimizes the total cost, equal to the sum of the FSL opening costs, the transport cost, the cost of procuring new vehicles and the shortfall cost for not satisfying demand requirements. Constraint (2) limits the total number of available vehicles systemwide. Constraint (3) limits the total vehicles available at each FSL. Note that $v_{jm"1"} \geq 0$, together with constraint (24), eliminate the need for a version of constraint (3) at $t = 1$. Constraints (4), (5), (6) limit the loading space available at each FSL for each of three "classes" of vehicles: air, land and sea. Within a class of vehicles, different "modes" are assumed to consume differing fractions of this loading space. Each of these differing modes of transport is also assumed to consume the loading space for a different length of time (for example, compared to a C-130 cargo aircraft, loading the much larger C-5A aircraft requires four times the ground space for three times the length of time). The FOL unloading space constraints (7), (8), (9) impose similar restrictions at the FOLs.

Demand constraint (10) compares the cumulative arrivals by time t against the cumulative demand by time t, with unmet demand recorded in the shortfall variable s. FSL storage is limited through constraints (11) and (12) for munitions, and through constraints (13) and (14) for nonmunitions. These constraints, along with constraint (15), also govern the opening of FSLs and their sizing. Note the implicit assumption that the FSL configuration will be fixed in place at the beginning of phase 1. Constraint (16) limits the average fleetwide utilization of the transport vehicles over each phase.

The remaining constraints are necessary for mathematical "bookkeeping". Constraints (17) and (18) translate tons of commodities transported via non-sea transport modes into transport vehicles, for munitions and nonmunitions, respectively. Constraint (19) similarly translates tons of all non-personnel commodities transported into sea vehicles. Constraints (20) and (21) translate personnel transported into transport vehicles, for sea and non-sea modes, respectively. Note the assumption that for non-sea modes, vehicles may not simultaneously transport more than one of munitions, nonmunitions or personnel. Once vehicles p, u and y finish unloading at FOL k, constraint (22) reassigns those vehicles to return trips to FSLs. Constraints (23) and (24) are flow balance equations that track the number of available vehicles at each time period.

3. COMPUTATIONAL RESULTS

The mathematical program of Section 2 was coded using GAMS version 20.7, and solved using the GAMS/CPLEX solver. A set of 18 potential FSLs was identified, along with a set of 10 potential scenarios, with each scenario comprising a separate phase h (note that no location names will be given for the illustrative results presented in this report). The order of occurrence for these scenarios was randomized. The transport costs for future years were discounted at a 10% rate between phases, allowing the objective function to minimize the net present value over the 10 phases in the study. A single type of transport vehicle, the C-17 cargo airplane, was assumed available. The maximum load and unload space values A_{1j} and B_{1k} were set to 4.0 at all FSLs and to 2.0 for all FOLs (i.e., at most 4 C-17 aircraft can be loaded at one FSL at a time). Demands were defined according to the characteristics of each scenario. Data pertaining to costs, facility characteristics, vehicle characteristics and commodities were obtained from USAF sources. The vehicle inventory C_{mh} was set equal to 60, and r_{mh} set equal to 0 for all phases. We varied the number of days allowed for transport (closure date) to examine the associated changes in performance, allowing no shortfall below demand (i.e., $s_{ikt} = 0$).

We first solved the associated set covering problem, allowing 5 FSLs to be opened. We then examined the performance of this set covering solution on the transport-constrained problem. This solution was found to be infeasible for a 3-day closure. Subsequent examination of this case determined that both "throughput" and "total number of vehicles" constraints were violated.

We next solved the transport-constrained problem, allowing the mathematical programming model to select the utilized FSLs. Figure 1 presents the minimal total cost achieved for each of the three closure dates studied, with the cost separated into its construction and transportation components.

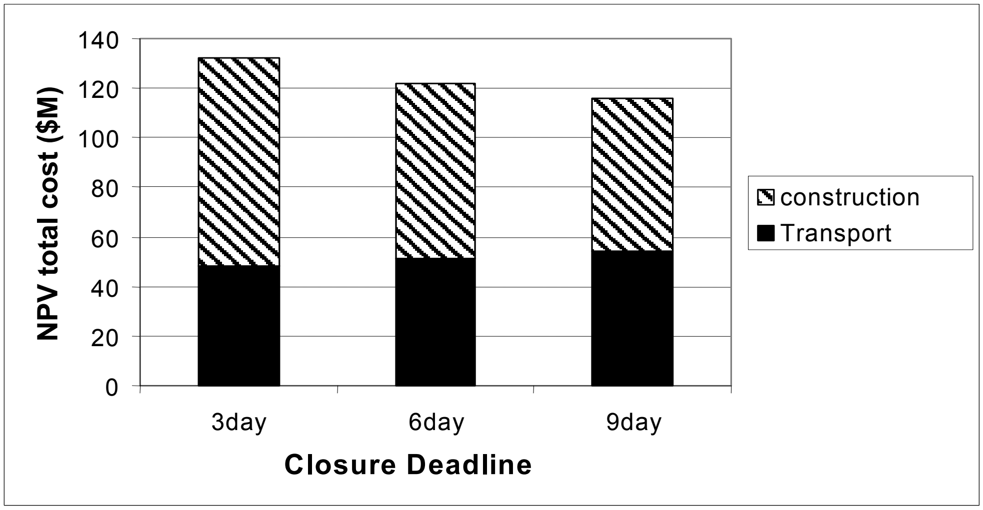

FIGURE 1. Minimal Total Cost for each Closure Date

An important tradeoff occurs between the total cost and the closure date. A less stressing closure date makes the FSL throughput constraints less binding, allowing for potentially fewer FSLs to be opened, and for less load balancing across FSLs. Notice that the transport costs increase as the closure date increases, while the construction costs decrease with respect to the closure date. This is intuitive, since relaxing the closure date also allows FOLs to be served by more distant FSLs, which may require fewer FSLs to be opened, but more expensive routes to be utilized.

To further examine these effects, consider Figure 2, which presents the number of FSLs opened for each closure date, along with the storage space allocation at each FSL. Note that each FSL label (e.g., FSL2) denotes the same location across different closure dates.

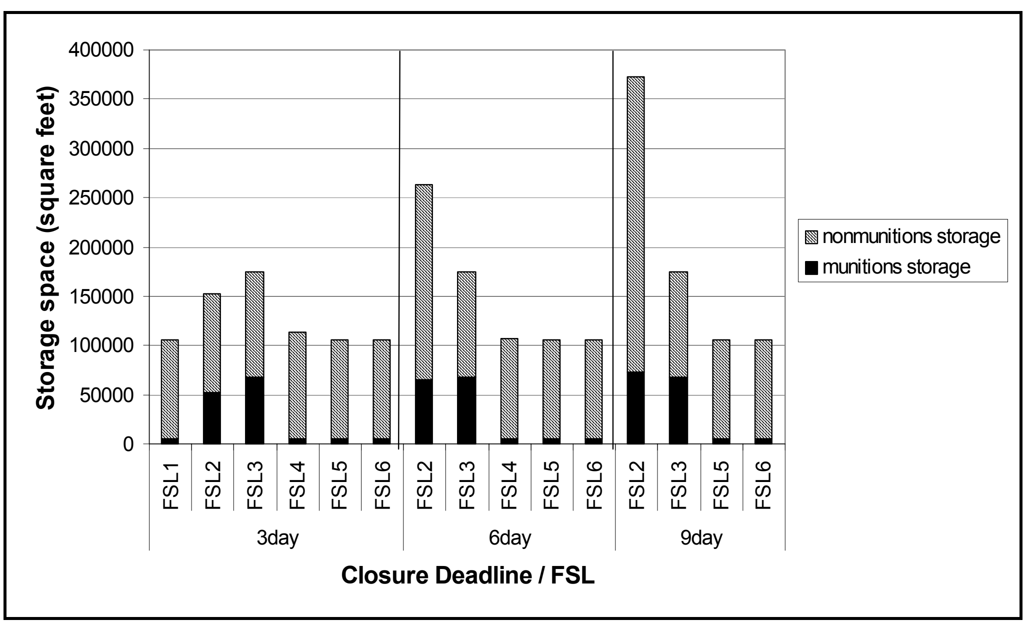

FIGURE 2. Required Storage Space at the FSLs

Notice that six FSLs are utilized for the 3-day solution. The set covering solution, which did not take into account throughput constraints, underestimated the number of FSLs needed in this case. Those FSLs whose bar resembles that of FSL1 are those that were opened at the minimum allowed size. It is not surprising to note that as fewer FSLs are opened, the storage space required at individual FSLs increases. This may be of special interest if FSLs are to be located in foreign countries. While such a posture may have cost benefits, it increases the reliance on individual FSLs, which in turn increases the risk associated with denial of access at each of these locations.

4. CONCLUSIONS AND RECOMMENDATIONS FOR IMPLEMENTATION

This analysis has demonstrated that FSL postures that are proposed without accounting for transport constraints may prove inferior once these transport considerations are included in the analysis. While this study utilized a cost minimization objective, cost is only one consideration in the larger tradeoff between resource allocation and operational capability. The results of this analysis yield global "portfolio options" of FSL structures and WRM allocations that should include tables of metrics such as locations, technologies and costs that quantify the capability sacrificed or afforded by differing policies.

Before the final portfolio of FSL options is produced, a refinement of the model solutions should be performed from a political perspective. This may alter the FSL list and thereby affect the results of the optimization process. Some FSLs suggested by the model may be deemed impossible due to politics, practicality, or risk. Other FSLs not suggested by the model may merit consideration due to these same factors. These considerations may influence the inputs to the next iteration of model runs. This post-optimality analysis can then iterate until an acceptable set of portfolios is determined. Ultimately, policy makers will be able to consider various mixes of FSL postures with their respective capabilities and effectiveness.

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