# MANAGING SEPARATE ACCOUNTS: A PEDAGOGY

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## ABSTRACT

A successful management of individually tailored portfolios is vital to institutional equity analysts and/or fund managers. This paper discusses the importance about the analysts' ability to generate securities expected return vectors and the return variance-covariance matrices in creating and managing highly customized portfolios. A portfolio optimization process is described using the standard CAPM in varied investment environments and under differing clients' requirements. Stock returns are assumed to follow geometric Brownian motion. Various numerical solutions are provided by calibrating the market returns and the risk-free interest rate via Monte Carlo simulations. Mean reverting stochastic processes and time varying volatilities are explicitly considered.

## A. INTRODUCTION

When the capital market is *intrinsically* efficient, it is argued that the best securities valuation is the price itself, that is, the price (*P*) equals the value (*V*), i.e. $P = V$. Consequently, there are no securities with a positive net present value, i.e. $NPV = V - P = 0$. In the meantime, the capital market efficiency results in what is known as the *Value Additivity Principle*, that is, the value of a whole is equivalent to the sum of its individual pieces. We call this *relative efficiency*. We will conjecture in this paper, however, that the market may not necessarily be *relatively efficient*, as long as the holdings of individually tailored portfolios and exact portfolio weights are not known to the investing public. These portfolios are often referred to as *separately managed portfolios* or *wrap accounts*.

## B. MUTUAL FUNDS

Capital market theory in modern finance proposes that investors mix the market portfolio with the risk-free asset, according to their risk preference. It is for this reason that people are advised to buy index funds. All investors have to do is then to buy the index funds as well as holding some risk-free assets. Mutual funds are also conveniently offered in the market, so the funds may carbon copy a portfolio of index funds and risk-free assets. Although this may explain why there are a variety of mutual funds alongside every index fund, it may still be difficult to find a mutual fund, which may match the investor's investment requirement for their risk tolerance and investment horizon.

## C. EX POST VS. EX ANTE EFFICIENT PORTFOLIO FRONTIERS (EPF)

Assume that an investor currently examines an *n* number of candidate stocks from which to form an efficient portfolio. Those candidate stocks may qualify for value, growth, large or small cap stocks. As a matter of fact, one can select those stocks randomly without any specific preferences or criteria. The return on a portfolio $r_p$ with a possible $n$ number of stocks is the weighted average returns, i.e. $r_p = \sum_{i=1}^{n} x_i r_i$ , where $x_i$'s are portfolio weights and $r_i$'s are individual securities returns. The EPF depends on the expected return and the variance of returns on a portfolio, which can be computed by taking historical sample mean and sample variance-covariance or analysts' pricing model. In the former, the EPF thus obtained is *ex post*. In the latter, we will have an *ex ante* EPF.

## D. PORTFOLIO CONSTRUCTION WITH STOCHASTIC WIENER PROCESS

Assume a particular *geometric Brownian* motion $r_{pt} = \mu_{pt}\Delta t + \sigma_{pt}\varepsilon_{pt}\sqrt{\Delta t}$ for a portfolio return process where $\mu_{pt}$ and $\sigma_{pt}$ are instantaneous *annual* drift and volatility, and $\varepsilon_{pt}$ is a random disturbance with zero mean and unit variance. Define $y_{it} = r_{it} - r_{ft}$ and $X_{mt} = r_{mt} - r_{ft}$, and consider the following time series regression model, where

$$y_{it} = \alpha_i + \beta_i X_{mt} + u_{it} \qquad \text{(Equation 1)}$$

The symbol $r_{it}$ is assumed to be daily *log* price relatives for a stock *i*, i.e. $\ell n\left(\frac{P_t}{P_{t-N}}\right) \approx r_t$ and similarly, for $r_{mt}$ as well. By convention $E[u_{it}] = 0$ , $E[u_{it}^2] = \nu_{it}^2$ , and $E[u_{it}u_{ik}] = 0$ for $k \neq t,\ k = \{1, ..., T\}$ ; and $E[\varepsilon_{it} X_{mt}] = 0$. Ignoring the variance on the risk-free rate of interest $\sigma_{ft}^2$, the expected return, the standard deviation and the covariance of securities returns are $E[r_{it}] = \alpha_i + E[r_{ft}] + \beta_i E[X_{mt}]$ ; $\sigma_{it}^2 \approx \beta_{it}^2 \sigma_{Xt}^2 + \nu_{it}^2$ and $\sigma_{ijt} \approx \beta_{it}\beta_{jt}\sigma_{Xt}^2$. Deriving expressions for $E[r_p]$ and $\sigma_p$ is trivial. The optimal portfolio allocation is the solution to the problem that $\theta = \frac{E[r_p] - r_f}{\sigma_p}$ is maximized with respect to $x_i$'s subject to $\mathbf{x'1} = 1$ and $x_i \geq 0\ \forall i$ with no short sales assumption. We will pursue the following steps to calibrate the model.

First, we will assume that the securities return follows a process:

$$dS_{it} = m_{it} S_{it} dt + \nu_{it} S_{it} \varepsilon_i \sqrt{dt} \qquad \text{(Equation 2)}$$

The drift $m_{it} = \hat{E}[r_{it}] + \lambda\left(\bar{r}_{it} - \hat{E}[r_{it}]\right)$ is assumed to equal the *equilibrium* expected return under CAPM, which reverts to the mean. The annual volatility $\nu_{it}^2 = \frac{1}{T-1}\sum_{t=1}^{T} e_{it}^2$ is the forecasting error $e_{it} = y_{it} - \hat{y}_{it}$ from the regression. In calculating the *equilibrium* expected return, we assume that the risk-free rate of interest follows a *geometric Brownian* motion of the form: $dr_f = \lambda(b - r_f)dt + \sigma_f dz$ The parameter $b$ is the constant long-term mean return, $\lambda$ is the rate at which the current $r_f$ reverts to the mean and $dz$ is the *Brownian* motion, i.e. $dz = \varepsilon\sqrt{dt}$. Next, the market risk premium $X_{mt} = r_{mt} - r_{ft}$ is assumed to follow the usual return generating process: $X_{mt} = \mu_{Xt} dt + \sigma_{Xt}\varepsilon_{Xt}\sqrt{dt}$ To this end, we compute *annualized* daily *log* price relatives for S&P 500 index to estimate $\mu_{Xt}$ and $\sigma_{Xt}$. However, a time varying volatility is allowed by using an exponentially weighted moving average (EWMA) model that for a weight $0 \leq \lambda \leq 1$, $\sigma_{Xt} = \left[\lambda\sigma_{Xt-1}^2 + (1-\lambda)e_{Xt-1}^2\right]^{1/2}$. All Wiener processes are Monte Carlo simulated.

## E. THE ECONOMIC EXPLANATION OF THE MODEL

Securities are ranked by their risk premium to beta in a descending order. Graphically, this can be represented by a downward sloping curve. Next, a series of scenario portfolios will be formed from the portfolio with one best stock, the portfolio of two best stocks, the portfolio of three best stocks, and so on until adding another stock will actually lower the portfolios risk premium to risk ratio. Note that the portfolios risk is measured by their volatility. Graphically the portfolios curve will first rise but will eventually start to fall. The reason is that when the portfolio has only a few stocks, it is subject to volatility risk. Consequently, the portfolios risk premium to risk ratio would be low. As the number of securities to be included in the portfolio increases, the portfolio's standard deviation will fall, which raises the risk premium to risk ratio, but only to a certain extent. The portfolios risk premium to risk ratio will reach a peak eventually before it starts to fall again, as the majority of stocks to be added to the portfolio will have considerably low expected returns. The optimal proportion of stocks in the portfolio is then the proportion that each stock contributes to the portfolios risk premium to risk ratio. (Interested readers are referred to the full version of the author's paper.)

## F. OTHER REMARKS AND CONCLUSIONS

This paper has shown that, in principle, the best asset allocation model should be based on *ex ante* and not *ex post* efficient portfolio frontiers. The *ex ante* EPF has been generated by a series of Monte Carlo simulations under the assumption that securities would be priced in equilibrium according to the well known Capital Asset Pricing Model in varying stochastic Wiener processes. In particular, it has been shown that specific solutions to the portfolio selection problem are contingent upon the particular nature of variance-covariance matrix of securities returns. And this results from particular pricing models used in the model.

A portfolio formation process we have discussed is, however, much more general than has been presented. In reality, the following inputs are required: fund's investment horizon, either target portfolio beta or target portfolio return, various mathematical constraints to number of shares to be sold or bought, and some possible rebalancing strategies. They can be easily incorporated in the non-linear programming problems. For example, our model has an implication about certain probabilities with which to achieve the specific investment target. That is, if $z_p = \frac{r_p - E[r_p]}{\sigma_p}$; $z_p \sim N(0,1)$,

$$Pr[r_p \geq r_T] = Pr\left[z_p \geq \frac{r_T - E[r_p]}{\sigma_p}\right] = 1 - Pr\left[z_p \leq \frac{r_T - E[r_p]}{\sigma_p}\right] \qquad \text{(Equation 3)}$$

The brokerage industry has long been trying to earn fee incomes while slowly de-emphasizing its traditional commission driven businesses. It has tried the wrap account business and mutual fund allocation models to start offering their customers advisory services. But the industry was not quite able to cross the chasm. Recently, with the emergence of the Internet and its wide acceptance as a means of communication, the industry began to offer online advisory based planning and the separately managed account businesses, all of which target high net worth markets. The hard reality is that although many are enthralled by the convenience of the online advisory solutions especially in the 401(k) retirement market, the role of financial advisors is not quite clear, unless the brokerage industry simply licenses the software to end users with brokerage commissions embedded in the license fees. The only viable alternative is to aggressively pursue the separately managed market. The separately managed account not only offers good theories but also makes sound business sense.