DIVERSITY OPTIMIZATION OF MULTI-ECHELON PORTFOLIOS

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ABSTRACT

We present an optimization problem for a portfolio of fixed income instruments. The objective function is an extension of the *diversity score* concept put forward by Moody’s, one of the rating agencies that assess the credit quality of public bonds and other issues. The problem is formulated to take into account the portfolio’s health from several different organizational perspectives. We will examine conditions under which optimal lower-level portfolios will aggregate into near optimal higher-level portfolios.

INTRODUCTION

One of the foundations of modern finance is the concept of portfolio diversification, as pioneered by Markowitz. A measure of diversification that has been used extensively for a class of fixed income instruments, namely collateralized debt obligations (CDOs), is the *diversity score* [1] developed by Moody’s, one of the three main rating agencies. CDOs are securitizations of a portfolio of bonds, loans or other types of debt instruments.

We utilize an extension of Moody’s diversity score in the context of a multi-echelon, enterprise-wide optimization problem. The problem is ‘multi-echelon’ because the diversity score of the portfolio is calculated in several ways, each of which reflects the perspectives of a different level of the organization. The solution approach attempts to take into account the diversity scores of the different levels jointly.

The methodology discussed herein is intended to complement simulation-based credit risk tools already on the market. These are commonly referred to as ‘credit VaR (Value-at-Risk)’ tools, and they provide measurements of a portfolio’s tail risk, i.e., extreme losses under low-probability scenarios [3, 4]. A drawback of these tools is that because they are simulation-based, they are computation-intensive and thus harder to utilize in optimization, where numerous allocation strategies need to be compared.

MOODY’S DIVERSITY SCORE AND EXTENSIONS

The functional goal of Moody’s diversity score is to reduce the representation of a correlated portfolio to a smaller portfolio that is presumed to consist of uncorrelated holdings. This is accomplished by grouping the collateral assets of the CDO by industry. Moody’s may modify the classification scheme from CDO to CDO based on their view of how the assets should be grouped. The diversity score methodology implicitly allows for diversity to increase in two ways: one is to invest in different industries, and the other is to invest in different issuers within a given industry. The score table normally used for this purpose inherently builds in a preference for additional industries over additional obligors within the same industry.

The diversity score described thus far is the de facto definition. However, Moody’s has also articulated an alternative definition based on matching the statistical profile of the aggregate portfolio [2].

Furthermore, the optimization problem discussed here is not necessarily tied to a particular definition of the diversity score, although some of our proposed solution methods do assume certain functional features on the part of the score.

A MULTI-ECHELON VIEW OF THE PORTFOLIOS

A notable feature of the problem is that it attempts to address multiple echelons of a business enterprise's hierarchy. Consider the challenges facing an insurance company, for example. Insurance companies underwrite policies for which they face an uncertain stream of future liabilities. The premiums collected from these policies are invested in a mix of assets with various risk-return profiles. The objective of the company is to ensure that its enterprise-wide portfolio has assets whose yields are sufficient to cover not only the liabilities but also to provide the necessary returns to stockholders.

The company, however, is not operated as a single, monolithic portfolio. The enterprise is typically divided into several profit-and-loss centers (P&Ls), each of which may have a business objective that is distinct from other P&Ls. Each P&L, in turn, may consist of one or more separate entities for a number of reasons. One is that insurance in the United States is regulated by the states, which means that state regulators will impose different requirements on the companies underwriting policies in their states. Another reason could be that a P&L, while unified under a single business objective, was formed over time through acquisitions of smaller companies.

SOLUTION APPROACH

For simplification, we consider only two echelons of the enterprise. We refer to the higher echelon as the *parent* and the lower as the *child*, and ignore the trivial case of a single-child scenario. The parent's and children's problems cannot be solved separately since the scores of the echelons cannot vary independently. Once the asset allocations of the children are specified, the score of the parent is fixed. The converse is obviously not true because of the one-to-many relationship, though the children's asset allocations, and thus the diversity scores, would be interdependent and bound by the parent allocations.

Conceptually, if we approach the problem of the two echelons sequentially, then there are two options: either optimize the parent subject to constraints on the children, or optimize the children independently and let the parent's score be determined by the child portfolios. The difficulty with optimizing the parent first is that the onus then falls on the decision-maker to somehow assign assets to the different children. However, there would be no a priori assurance that the scores of the child portfolios are optimal. The converse problem is also challenged by the notion that optimizing for the children tells us little about the optimality of the parent.

PROBLEM FORMULATION

We formulated the problem as a non-linear mathematical model. The decision variables are the sector allocations.

Objective function: Maximize Portfolio Diversity
Subject to: Sum of sector allocations = total portfolio value.

Sector investment limitations Different variations of portfolio diversity measures were considered and will be discussed at the conference presentation.

ANALYSIS AND RESULTS

The simplest version of the problem is the case where the formulations of the parent problem and all the child problems are identical. In this case, optimization results for the parent portfolio should represent an upper bound on the diversity score achievable with the combination of child portfolios. The drawback, however, is that the nonlinearity of the score function leads to a scale issue when the parent's score is compared to the sum of the children's scores.

An alternative approach is to utilize the scale differences between the echelons as a parameter in the problem specification for one of the two echelons, along with the boundary conditions implied by the remaining, 'independent' echelon. An iterative scheme based on successive modification of the scale parameter and the specification of the 'dependent' echelon's problem yields insight into how changes in asset allocation at one echelon impact the score(s) of the other echelon.

With additional assumptions on the operational relationships of the child portfolios amongst themselves, such as allowances for inter-portfolio transfers of investment dollars, we find that the solutions of the child problems directly yield the solution to the parent problem.

CONCLUDING REMARKS

Ongoing banking regulatory reform in recent years has had an accelerating role on the management of credit risk in fixed income portfolios of not only banks but also other financial institutions. In response, vendors have brought to market sophisticated simulation tools for measuring credit risk using advanced measures like VaR and expected shortfall. One aspect in which these tools fall short is in optimization, in other words the search for alternative allocations that improve on the status quo. While the mathematics are now better understood, there are also considerable organizational impediments to making optimization a more integral part of credit risk portfolio management. This paper has brought attention to the multi-echelon or multi-layered nature of the problem that arises in practice, and has suggested an optimization approach for solving such problems.

REFERENCES

[1] Cifuentes, Arturo, and Gerard O'Connor, *The Binomial Expansion Method Applied to CBO/CLO Analysis,* Moody's Investors Service, 1996.

[2] Gluck, Jeremy, and Helen Remeza, *Moody's Approach to Rating Multisector CDOs,* Moody's Investors Service, 2000.

[3] Gupton, Greg M., Christopher C. Finger, and Mickey Bhatia, *CreditMetrics – Technical Document,* J.P. Morgan, New York, 1997.

[4] Jorion, Philippe, *Value-at-Risk,* McGraw-Hill, New York, second edition, 2001.