VOLATIOLITY RISK PREMIUMS EMBEDDED IN S&P 500 INDEX RETURNS

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ABSTRACT

As evinced by the asymmetric volatility property of S&P 500 returns, this study incorporates volatility risk premiums into volatility via time-varying correlation with underlying returns. A quasi-maximum likelihood function accompanied with Kalman filter successfully estimates model parameters. The market price of volatility risk is found to be positive and increases with investment horizons. In particular, the volatility feedback is enhanced by strong asymmetries in conditional covariances for long-term returns. The existence of volatility risk premium may help solve the pricing puzzle in CAPM that empirically underprices low-beta assets but overprices high-beta assets, which reasons the importance of this study.

Introduction

The asymmetric nature of the volatility response to return shocks could simply reflect the existence of time-varying risk premiums ([7][3][2]). If volatility is priced, an anticipated increase in volatility raises the required return on equity, leading to an immediate stock price decline. The contemporaneous risk-return tradeoff appears sensitive to the use of ARCH as opposed to stochastic volatility formulations ([6]), the length of the return horizon ([4]), along with the instruments and conditioning information used in empirically estimating the relationship ([5][1]). This study proposes a stochastic volatility process allowing for time-varying correlation with underlying returns, in which the market price of volatility risk is naturally taken into account. Historical S&P 500 returns over the period January 1969–December 2004 are investigated under Kalman filtration. We successfully identify and isolate the volatility risk premium in the pricing process, and thereafter demonstrate the relative contributions of price premiums and volatility premiums to underlying returns.

The Stochastic Volatility Model

Assume the index return R_t having a stochastic volatility h_t to follow the process below,

$$\begin{aligned} R_t &= \mu_S - \mu_{h,t|t-1}/2 + \sqrt{h_t}\, x_t \\ h_t &= \varpi + \beta\, h_{t-1} + \alpha \sqrt{h_{t-1}}\, \varepsilon_t \end{aligned} \qquad (1)$$

where $x_t \sim \mathrm{ID}(\lambda,1)$, $\varepsilon_t \sim \mathrm{ID}(0,1)$, $\mathrm{Corr}(x_t,\varepsilon_t) = \rho$ and $\mu_{h,t|t-1} = \mathrm{E}(h_t | I_{t-1})$. The values of $\mu_S - \mu_{h,t|t-1}/2$

and $E(\sqrt{h_t}x_t | I_{t-1}) = \lambda\sqrt{\mu_{h,t|t-1}} + \alpha\rho\sqrt{\mu_{h,t-1|t-1}} \big/ 2\sqrt{\mu_{h,t|t-1}}$ represent, respectively, the market prices of price and volatility risks. Kalman filtration uses the linear projection to construct the conditional mean of variance, $\mu_{h,t+1|t}$, and conditional prediction error, $P_{t+1|t}$, shown as follows,

$$\mu_{h,t+1|t} = \varpi + \beta\ \mu_{h,t|t-1} + \mu^* s_{t+1} + \beta\left(\frac{P_{t|t-1}}{\mu_{h,t|t-1}} + r^* s_t\right)\left(\frac{P_{t|t-1}}{\mu^2_{h,t|t-1}} + \sigma_e^2 + \frac{2r^* s_t}{\mu_{h,t|t-1}}\right)^{-1}\left(y_t - y_{t|t-1}\right) \qquad (2)$$

$$P_{t+1|t} = \beta^2 P_{t|t-1} - \mu^{*2} + \alpha^2\mu_{h,t|t-1} + \left(\frac{P_{t|t-1}}{\mu_{h,t|t-1}} + r^* s_t\right)\left(\frac{P_{t|t-1}}{\mu^2_{h,t|t-1}} + \sigma_e^2 + \frac{2r^* s_t}{\mu_{h,t|t-1}}\right)^{-1} \times \left[\alpha^2\left(y_t - y_{t|t-1}\right) - \beta^2\left(\frac{P_{t|t-1}}{\mu_{h,t|t-1}} + r^* s_t\right) + 2\beta^2\left(\frac{\omega}{1-\beta} - \mu_{h,t|t-1}\right)^2 \mu^{-1}_{h,t|t-1}\right] \qquad (3)$$

where μ^* and r^* are adopted to recover the correlation information, or equivalently, volatility asymmetry.

The Market Price of Volatility Risk

As shown in Table I, the volatility feedback effect, along with the well-documented persistent volatility dynamics, implies an observationally equivalent negative correlation between current returns and future volatility, as a shock to the volatility will require an immediate return adjustment to compensate for the increased future risk. Empirical evidence also confirms that aggregate market volatility responds asymmetrically to negative and positive returns, and the economic magnitude is statistically significant and time-varying. Importantly, the magnitude also depends on the volatility proxy employed in the estimation, with stochastic volatilities generally exhibiting much more pronounced asymmetry. Since the S&P 500 index is generally used as a proxy of the market portfolio, the volatility shocks negatively correlated to S&P 500 returns may also be negatively correlated to aggregate consumption growth, and thus results in a negative volatility risk premium. However, the negative impact of volatility shocks on the total expected returns is offset and dominated by the volatility asymmetry. The net effect of volatility shocks on the total return rate will turn out to be positive, indicating that investors will demand a positive volatility risk premium to counter with volatility shocks.

Table I The Price and Volatility Risk Premiums

$\hat{\lambda}$ is the market price of unit volatility risk with corresponding *t*-statistics in the parentheses. The symbol of * (**) denotes that at the significance level of 5% (1%) the *t*-statistic rejects the null hypothesis of $\lambda = 0$. $v\hat{a}r\left(R_t | I_{t-1}\right)$ is the filtered conditional variance of raw returns R_t, i.e., $\hat{\mu}_{h,t|t-1} = \hat{E}\left(h_t | I_{t-1}\right)$ (stochastic volatility) or $\hat{\sigma}^2$ (constant volatility). $Corr\left(\hat{\mu}_{h,t|t-1}, \hat{\mu}_{h,t-1|t-2}\right)$ shows the magnitude of volatility persistence. $\hat{\rho} = \hat{\mu}^* \big/ \hat{\alpha}^* \sqrt{\hat{\mu}_{h,t-1|t-1}}$ illustrates the correlation between return shocks and volatility shocks. *vrp* means the "volatility risk premium" computed by $\hat{\lambda}\sqrt{\hat{\mu}_{h,t|t-1}} + \hat{\alpha}\hat{\rho}\sqrt{\hat{\mu}_{h,t-1|t-1}} \big/ 2\sqrt{\hat{\mu}_{h,t|t-1}}$ (stochastic volatility) or $\hat{\lambda}\hat{\sigma}$ (constant volatility). *prp* denotes the "price risk premium" calculated by $\hat{\mu}_S - \hat{\mu}_{h,t|t-1} \big/ 2$ (stochastic volatility) or $\hat{\mu}_S - \hat{\sigma}^2 / 2$ (constant volatility). The figures in the table are the averages of corresponding estimates in the sample period starting in January 1969 and ending in January 1990–December 2004.

Model	Constant Volatility				Stochastic Volatility			
Horizon	1 day	30 days	100 days	300 days	1 day	30 days	100 days	300 days
$\hat{\lambda}$	0.0280	0.0555*	0.0337	0.0980**	0.2572**	0.2834**	0.3048**	0.4006**
	(1.17)	(2.28)	(1.35)	(3.74)	(16.68)	(20.18)	(20.68)	(28.56)
$v\hat{a}r\left(R_t \| I_{t-1}\right)$	0.1589	0.1558	0.1501	0.1417	0.3870	0.4687	0.4310	0.4946
$Corr\left(\hat{\mu}_{h,t\|t-1}, \hat{\mu}_{h,t-1\|t-2}\right)$	—	—	—	—	0.9083	0.9935	0.9951	0.9957
$\hat{\rho}$	—	—	—	—	−0.2259	−0.5716	−0.6148	−0.5625
vrp	0.0112	0.0219	0.0131	0.0369	0.1554	0.1936	0.1997	0.2813
prp	−0.0110	−0.0153	0.0095	0.0357	−0.1552	−0.1871	−0.1773	−0.2091

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