# **TEACHING UNDERGRADUATES ABOUT DYNAMIC SYSTEMS**

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# ABSTRACT

This study expands the study of undergraduate students' learning about the behavior of dynamic systems. In a lower-division social sciences calculus class, a computer simulation was tested as a vehicle for improving the effectiveness of the typical applied calculus course introducing dynamic systems. Pre- and post-assessments were compared to assess the knowledge of the students. Results suggest that the simulation approach did improve student knowledge of dynamic systems.

### INTRODUCTION

Ample research has indicated accelerating paces of change in business, both at the industry and firm levels. With innovation rates averaging nearly 10 percent annually and even faster in technological sectors [1], business graduates who are capable of assessing, interpreting, and making effective decisions in dynamics situations are more likely to succeed than those who cannot recognize or understand the implications of dynamic rates of change. Consequently, teachers of business mathematics and other business courses are seeking improved methods of instruction and assessment regarding dynamic systems.

In a prior study [2], the first author reviewed the literature describing intelligence, knowledge, and learning in order to provide a theoretical basis for developing more effective methods for teaching students about dynamic systems. This review suggested the usefulness of computer simulation as a teaching tool to link the academic conceptual development of mathematical abstraction skills to personal (prior) latent knowledge from common experiences of dynamic systems. Specifically, simulation could provide an iterative, low-cost [3] experience of the consequences of multiple interrelationships in a system that includes delays between cause and effect. In the current study, the authors developed a simulation tool to link the concepts of system dynamics to the common experience of filling a bathtub. Pre- and post-tests were conducted to evaluate the effectiveness of the simulation.

### THE SIMULATION EXERCISE

The computer simulation represented a typical bathtub (simulation available from the first author), with a single inflow (faucet) and outflow (drain) and allowed the user to alter the rates of the flows. Created in Visual Basic, the simulation was provided on personal computers to students in concert with a self-paced exercise in which the student conducted three activities with the simulation, each of which elicited the student's manual calculation of flows and resulting accumulation at various points in time, as well as the student's graphical estimate of the accumulation of water in the bathtub over the time. This approach was based on the notion that, by "seeing" the simulated concepts, the students could gain deeper understanding of the mathematical concepts of rates of flow and integral (accumulation) calculus.

# **EXPERIMENTAL METHOD**

To test the hypothesis that the simulation with the structured self-paced exercise would improve student understanding of dynamic systems, a pre-test and post-test were conducted using a set of assessment instruments developed by Sweeney and Sterman [4]. The four assessment problems (available from the authors), tested student mastery of fundamental concepts of dynamics systems, the accumulation of stocks based inflow and outflow rates. These two ideas form the conceptual heart of integral (accumulation) and differential (rates of change) calculus.

In each problem students were provided with the initial accumulation of a resource (either cash in an account or water in a bathtub), along with a graph depicting the rates at which resource was flowing into and out of the accumulation. The students were asked to create a graph of the accumulation over time based upon the net flow. Two of the four problems (CF1 and BT1) provided a square wave pattern of inflow with a constant outflow, while the other two (CF2 and BT2) provided a sawtooth inflow with a constant outflow. All of the problems assumed knowledge of basic arithmetic, Cartesian graphing, and an intuitive understanding of rates/flows and accumulations/stocks.

Each student received only a single problem (CF1, BT1, CF2, or BT2) in each of the pre-test and a post-test. Distribution of problems was random among students in eleven sections of a calculus survey course for social sciences majors. The pre-test took place early in the semester and the post-test took place several weeks later, following completion of the structured learning simulation activity. No effort was made to give individual students the same type of problems in both the pre- and post-tests. After discarding responses due to scoring and response irregularities, the authors had 254 coded assessments from the pre-test and 187 from the post-test. Demographics including age, gender, class standing, and major were collected to check the randomness of the assignment of the four problems.

The null hypothesis stated that the students' performance on the four assessment problems on the pre-test would equal performance on the post-test.

 $H_o: p_{PRE} = p_{POST}, H_a: p_{PRE} \neq p_{POST}$ 

Performance was measured as the fraction, p, of students answering the problem correctly. It was assumed that an increase in performance on the post-test could be explained by the experience of the structured simulation exercise. This assumption was verified by comparing pre-test and post-test performance with the performance of an equivalent group of students from the same course in a prior semester who completed the assessment problems without doing the structured simulation exercise.

# RESULTS

Analysis of age, gender, class standing, and major revealed no bias in assignment of the four problems to students based on these demographic variables. Self-reported ethnicity suggested that the students were predominantly (89%) Caucasian. The random method of distributing the four problems to students attending class on a particular day created two concerns: Not only was it not possible to ensure the administration of the same problem to a particular student on both the pre-test or post-test, but also the identical population of students on both the pre- and post-test was also not assured. Furthermore, not all students completed the simulation exercise.

In light of these concerns, the following table lists the summary success rates for only the students who completed the online simulation exercise and who took both the pre-test and the post-test.

| TABLE 1. | <b>Results for</b> | students | who | took | both | the | pre-test | and | post-tes | t |
|----------|--------------------|----------|-----|------|------|-----|----------|-----|----------|---|
|          | Niuma              | Correct  |     | 0/   |      |     |          |     |          |   |

|          | Num | Correct | %      |
|----------|-----|---------|--------|
| Pretest  | 101 | 4       | 3.96%  |
| Posttest | 101 | 14      | 13.86% |

H<sub>o</sub>:  $p_{PRE} = p_{POST}$ , H<sub>a</sub>:  $p_{PRE} \neq p_{POST}$  $X^2 = 6.10$ , significant at  $\alpha = .05$ 

These results suggest the performance difference between the pre- and post-test for students who completed the simulation exercise was significant at the 0.05 level. To ascertain that the performance difference was due to completing the simulation exercise rather than to other factors, the post-test results were also compared to the performance results from a sample of 165 students in the same course in a prior semester who did not complete the simulation exercise. While the post-test performance in the current assessment was higher than in the prior semester, the difference was not significant at the 0.05 level.

# TABLE 2. Comparison of post-test performance to prior students

|                | Num | Correct | %      |
|----------------|-----|---------|--------|
| Prior Semester | 165 | 14      | 8.5%   |
| Posttest       | 101 | 14      | 13.86% |

H<sub>o</sub>:  $p_{Prior} = p_{POST}$ , H<sub>a</sub>:  $p_{PRE} \neq p_{POST}$  $X^2 = 1.92$ , not significant at  $\alpha = .05$ 

# DISCUSSION

This study provided support for the contention that the targeted simulation exercise can improve student understanding of accumulations and rates in dynamic systems. The results, however, were tempered by concerns regarding the student sampling procedures. Moreover, even though the study suggested that the simulation exercise improved performance on the post-test, the percentage of students able to accurately represent the behavior of an accumulation, given the inflow and outflow, over time was surprisingly low—particularly for students who have completed a significant part of a calculus course. Significant additional research is needed to refine simulation exercises and experimental methods to strengthen research on how to teach undergraduates effectively to interpret and assess dynamic systems.

### REFERENCES

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