CAN WE ALWAYS LEARN ABOUT THE MARKET?

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ABSTRACT

An *n*-firm market game is introduced with a deterministic learning process leading to price beliefs in which the firms cannot detect the errors in their price estimations, even if all price believes are still wrong. That is, the estimation errors of the different firms compensate for each other making estimation errors undetectable and therefore learning impossible.

INTRODUCTION

In the literature of mathematical economy a large number of researchers have developed models and procedures of learning about market conditions and about the behavior of the competitors. Most deterministic models are based on comparing expected and actual outcomes and based on their discrepancy, certain adjustments are made in the model parameters. Stochastic models use Bayesian statistics, and the distribution functions of the uncertain model parameters are repeatedly updated after new observations become available. The works of [1] [2] [3] [4] are considered classical articles in this field.

In this paper we will introduce a model, in which no learning is guaranteed, since the misbeliefs of the different participants have a certain compensating effect resulting in no discrepancies between expected and actual outcomes even in cases when all participants' beliefs are wrong.

THE MATHEMATICAL MODEL

Assume *n* firms produce the same product or offer the same service to a homogeneous market. Let x_k be the output of firm *k*, and let $Q=\sum_{k=1}^{n} x_k$ be the total output of the industry. Assuming linear price and cost functions we will use $p(Q)=B-AQ$ as the market price and $C_k(x_k)=\alpha_k x_k+\beta_k$ as the cost function of firm *k*. Therefore its profit is

$$\Pi_k = x_k\left(B-AQ\right)-\left(\alpha_k x_k+\beta_k\right). \tag{1}$$

However we assume that the firms do not know the price function, firm *k* has only an estimate $p_k(Q)=B-A_kQ$ of it, where A_k might be different than *A*.

Firm k thinks as follows. An n-person noncooperative game describes this situation where the firms are the players, the set of feasible strategies for any player is $[0,\infty)$, and the payoff function of any player l (including itself) is

$$x_l\left(B-A_kQ\right)-\left(\alpha_l x_l+\beta_l\right). \tag{2}$$

With fixed $x_1,\dots,x_{l-1},x_{l+1},\dots x_n$ output levels let $Q_l=\sum_{i\neq l} x_i$, then with any given Q_l the best output choice of firm l is given by differentiating (2),

$$B-2A_kx_l-A_kQ_l-\alpha_l=0$$

implying that the best reply of player l is as follows:

$$x_l=\frac{B-\alpha_l-A_kQ_l}{2A_k}$$

showing that firm k believes that the equilibrium is

$$x_l^{(k)}=\frac{B-(n+1)\alpha_l+\sum_{i=1}^{n}\alpha_i}{(n+1)A_k}\qquad (1\le i\le n) \tag{3}$$

with total believed output of the industry

$$Q^{(k)}=\frac{nB-\sum_{i=1}^{n}\alpha_i}{(n+1)A_k}$$

and equilibrium price

$$p^{(k)}=B-A_kQ^{(k)}=\frac{B+\sum_{i=1}^{n}\alpha_i}{n+1}. \tag{4}$$

However in reality each firm simultaneously thinks in the same way, so they independently determine their believed equilibrium outputs (given by (3) with $l = k$), so the actual output of the industry becomes

$$Q=\sum_{k=1}^{n}x_k^{(k)}=\frac{1}{n+1}\left[\left(B+\sum_{i=1}^{n}\alpha_i\right)\sum_{k=1}^{n}\frac{1}{A_k}-(n+1)\sum_{k=1}^{n}\frac{\alpha_k}{A_k}\right]$$

with actual equilibrium price

$$p=B-AQ=B-\frac{A}{n+1}\left[\left(B+\sum_{i=1}^{n}\alpha_i\right)\sum_{k=1}^{n}\frac{1}{A_k}-(n+1)\sum_{k=1}^{n}\frac{\alpha_k}{A_k}\right]. \tag{5}$$

The discrepancy between the actual and believed prices for firm k is therefore

$$D^{(k)}=\frac{1}{n+1}\left(nB-A\left(B+\sum_{i=1}^{n}\alpha_i\right)\sum_{k=1}^{n}\frac{1}{A_k}+A(n+1)\sum_{k=1}^{n}\frac{\alpha_k}{A_k}-\sum_{i=1}^{n}\alpha_i\right). \tag{6}$$

Notice that $D^{(k)}$ is the same for all firms, so if there is a discrepancy between the actual and believed prices, then all firms have the same signal about the error they made in price estimation.

LEARNING POSSIBILITIES

Assume first that $D^{(k)} > 0$ meaning that the actual price is higher than the believed price for all firms. Then they want to increase their price beliefs by decreasing the value of A_k. Notice that

$$\frac{\partial D^{(k)}}{\partial A_k} = \frac{1}{n+1}\left[A\left(B + \sum_{i=1}^{n} \alpha_i \right)\frac{1}{A_k^2} - A(n+1)\frac{\alpha_k}{A_k^2}\right] = \frac{A}{(n+1)A_k^2}\left[B + \sum_{i=1}^{n} \alpha_i - (n+1)\alpha_k \right] = \frac{x_k^{(k)}}{A_k} > 0, \tag{7}$$

so $D^{(k)}$ also decreases. Similar is the situation if $D^{(k)} < 0$, then all firms want to decrease their price beliefs by increasing the value of A_k. In both cases $D^{(k)}$ moves to the right direction: positive $D^{(k)}$ value decreases and negative $D^{(k)}$ value increases. By using small increments, by the repeated application of this adjustment process the value of $D^{(k)}$ might become zero for all practical purposes.

Assume next that $D^{(k)} = 0$, that is the right hand side of (6) becomes zero. Since $D^{(k)}$ is the same for all *k*, it occurs if and only if the single equation for the *n* unknown A_k values is satisfied. Clearly the true value of *A* for all A_k variables satisfies the equation, which is linear in the $1/A_k$ values. So there are infinitely many different solutions. All of these solutions represent cases when all firms believe in wrong price functions, but for all of them the actual and believed prices coincide. Therefore none of the firms recognizes any discrepancy between the prices therefore all believe that their price estimations were correct, so none of them is willing to change it. Hence no learning is possible. Even when $D^{(k)} \neq 0$ at the beginning, the adjustment process usually converges to another wrong price estimations that cannot be detected anymore, so all firms will believe that their estimate are correct even if they are not.

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