PURCHASE PLANNING UNDER BUSINESS VOLUME PRICE BREAKS: A MULTIOBJECTIVE APPROACH

Nasr-Eddine Dahel, Fisher Graduate School of International Business, Monterey Institute of International Studies, 460 Pierce Street, Monterey, CA 93940, 831-647-4602, edahel@miis.edu

ABSTRACT

This paper presents an analytical approach for simultaneous optimization of the number of vendors to employ and the order quantities to allocate to these vendors in a multi-product sourcing environment. The proposed approach can be used to support purchasing decisions in sourcing environments where vendors with varying prices quality and delivery performance levels offer volume discounts based on the total value of multi-product orders they receive from the buyer. The paper discusses the multicriterion nature of the purchasing decision, presents a multiobjective mathematical model, and proposes a solution methodology. Results of an extensive experiment show that the computational efficiency of the proposed model to be quite satisfactory.

INTRODUCTION

Selecting vendors with various capabilities and performance levels from a large supplier base is a difficult and time-consuming task. In his seminal work on vendor selection criteria, Dickson (1966) identified 23 different criteria by which purchasing managers have selected vendors in various procurement environments. In practice, procurement cost, product quality, delivery performance, and supply capacity have been found to be the most frequently used vendor evaluation criteria. A review of 74 supplier selection articles, by Weber et. al. (1991) found that these four criteria received the greatest amount of attention in the recent literature.

The joint consideration of procurement cost, product quality, delivery performance, and supply capacity criteria complicates the selection decision because competing vendors have different levels of achievement under these criteria. For example, the vendor with the least expensive price in a given industry may not have the best delivery performance or product quality. Vendor selection is therefore an inherently multiobjective decision that seeks to reduce procurement cost, maximize quality, and maximize delivery performance concurrently.

The presence of volume discounts further complicates the selection problem since the buying decision is no longer based on a single product that can be purchased from one or more vendors, but on the collection of items that can be sourced from a single vendor. In this case discounts are based on the aggregate value of multi-product orders placed with the vendor over a given period of time, regardless of the magnitude or value of each order quantity. As such, vendor selection is a multicriterion decision that affects the number and types of vendors to employ, as well as the order quantities to place with these vendors.

This article introduces a multiobjective mixed integer programming model to support vendor selection decisions. The mathematical model is formulated in such a way to simultaneously determine the optimal number of vendors to employ and the order quantities they must supply to each facility or plant in the supply chain so as to concurrently minimize total purchase cost, maximize product quality, and maximize on-time deliveries, while satisfying capacity and demand requirement constraints.

MODEL DEVELOPMENT

Consider a procurement situation in which $i = 1, 2,\ldots, I$ items are to be purchased for $k = 1, 2,\ldots, K$ plants from $j = 1, 2,\ldots, J$ vendors, that provide different levels of item price, product quality, delivery performance, and supply capacity for each item they sell. Also, depending on the buyer's total purchases value vendor j offers a business volume discount having $r = 1, 2,\ldots, R_j$ discount brackets. For example a three-discount bracket schedule may be such that purchases worth less than \$100,000 get 0% discount, purchases worth \$100,000, but not exceeding \$500,000 get an across the board 5% discount applicable to all purchases, not just those above the \$100,000 cutoff point; and purchases worth \$500,000 are discounted 10% to \$450,000.

Let J_i be the set of vendors offering item i; I_j be the set of items offered by vendor j; K_j be the set of plants that can be supplied by vendor j; and K_i be the set of plants demanding item i. Also, let D_{ik} = units of item i demanded by plant k; c_{ijk} = unit price of item i quoted by vendor j for delivery to plant k; q_{ijk} = percentage of rejected item i units from vendor j at plant k; t_{ijk} = percentage of item i units from vendor j missing their scheduled delivery time window at plant k; S_{ij} = maximum quantity of item i that may be purchased from vendor j due to capacity constraints or other considerations; u_{jr} = upper cutoff point of discount bracket r for vendor j; and d_{jr} = discount coefficient associated with bracket r of vendor j's cost function.

Define decision variables as follows. x_{ijk} = units of item i purchased from vendor j for delivery to plant k. v_{jr} = volume of business awarded to vendor j in discount bracket r; observe that v_{jr} is greater than zero only if the dollar amount of purchases made from vendor j falls within bracket r of its cost function; otherwise it is zero.

$$y_{jr} = \begin{cases} 1, \text{if the volume of business awarded to vendor } j \text{ falls on segment } r \text{ of its cost function;} \\ 0, \text{otherwise.} \end{cases}$$

Mathematical Formulation

$$\min Z = [Z_1, Z_2, Z_3] \tag{1}$$

$$Z_1 = \sum_{j\in J}\sum_{r\in R_j}(1-d_{jr})v_{jr} \tag{1a}$$

$$Z_2 = \sum_{i\in I}\sum_{j\in J_i}\sum_{k\in K_j} q_{ijk}x_{ijk} \tag{1b}$$

$$Z_3 = \sum_{i\in I}\sum_{j\in J_i}\sum_{k\in K_j} t_{ijk}x_{ijk} \tag{1c}$$

subject to:

$$\sum_{j\in J_i} x_{ijk} = D_{ik}, \qquad i\in I, \quad k\in K_i; \tag{2}$$

$$\sum_{k\in K} x_{ijk} \le S_{ij}, \qquad i\in I, \quad j\in J_i; \tag{3}$$

$$\sum_{i\in I_j}\sum_{k\in K_i} c_{ijk}x_{ijk} = \sum_{r\in R_j} v_{jr}, \qquad j\in J; \tag{4}$$

$$v_{jr} \le u_{jr}y_{jr}, \qquad j\in J, \quad r\in R_j; \tag{5}$$

$$v_{j,r+1} \ge u_{jr}y_{j,r+1}, \qquad j\in J, \quad r=1,\ldots,R_j-1; \tag{6}$$

$$\sum_{r \in R_j} y_{jr} = 1, \qquad j \in J; \qquad (7)$$

$$y_{jr} = \{0,1\}, \quad v_{jr} \geq 0, \qquad j \in J, \quad r \in R_j; \qquad (8)$$

$$x_{ijk} \geq 0, \qquad i \in I, \quad j \in J_i, \quad k \in K_j. \qquad (9)$$

Constraint (2) ensures that the total demand of each item at each plant will be satisfied. Constraint (3) ensures that the total number of items procured by each supplier to all plants is within the production and shipping capacity of that supplier. Constraint (4) determines the dollar amount of business awarded to vendor *j*. Constraints (5)-(6) link the purchase of the item with the business volume discount to the appropriate segment of the discount pricing schedule for each vendor. Constraint (7) ensures that only one discount bracket for each vendor's volume of business will apply. Constraints (8) and (9) ensure integrality and nonnegativity on the decision variables. Equation (1) specifies the multiobjective function whose components are given by equations (1*a*), (1*b*), and (1*c*). Equation (1*a*) minimizes the total purchase cost. Equation (1*b*) minimizes the number of defective items, and Equation (1*c*) minimizes the number of items missing their scheduled delivery time window.

A number of optional constraints may be added to the above formulation to account for additional requirements of the procurement decision. These constraints may be applied uniformly across all items and vendors or selectively to specific products or suppliers.

Market Share Constraint. This constraint specifies that the buyer is willing to purchase no more than a given percentage P_i of item *i* total demand ϕ_i from a given supplier. With $\phi_i = \sum_k D_{ik}$, this market share constraint may be expressed as: $\sum_{k \in K} x_{ijk} \leq P_i \phi_i, \quad i \in I, \quad j \in J_i$. Whenever this option is selected, constraint (3) should be rewritten as: $\sum_{k \in K} x_{ijk} \leq \min(S_{ij}, P_i \phi_i), \quad i \in I, \quad j \in J_i. \quad (3')$. This way, constraint (3') enforces the dual requirement of supplier's capacity and supplier's market share without increasing problem size.

Business Volume Constraint. This constraint limits the buyer's volume of business with supplier *j* to a maximum dollar value U_j. Often, larger buyers would like to limit the amount of business they award to a single vendor to achieve their own supplier diversification goal, and also prevent small suppliers from becoming too dependent on them. This constraint is expressed as follows: $\sum_{r \in R_j} v_{jr} \leq U_j, \quad j \in J$.

Maximum Number of Supplier Constraint. This constraint limits the number of vendors the buyer is willing to do business with to a maximum of *M* suppliers. Often, decreasing the number of suppliers helps the buying organization reduce administrative cost due to individual transactions, and facilitate the development of long-term supplier partnerships. This constraint requires replacing constraint (7) by the following set of equations:

$$\sum_{r \in R_j} y_{jr} \leq 1, \qquad j \in J;$$

$$\sum_{j \in J} \sum_{r \in R_j} y_{jr} \leq M.$$

These optional constraints ultimately affect the type and number of vendors selected, their respective order quantities, as well as the total cost, quality and delivery outcomes of the procurement process.

SOLUTION METHODOLOGY

Two basic approaches may be used to solve multiobjective programming problems. These are the preference-oriented approach and the generating approach. The preference-oriented approach consists of techniques that rely on a formal characterization of preferences among the objectives prior to solving the problem. Generating techniques are suitable to situations where the articulation of preferences among the objectives is postponed until a range of alternative *noninferior* solutions is examined (see Cohon, 1978, for a comprehensive discussion). These solutions help the decision maker to better understand the tradeoffs between the objectives before selecting a best-compromise solution. Tradeoffs between the objectives are however relatively difficult to understand when more than two objectives are at hand. For this reason, practitioners often prefer the preference-oriented approach to generating techniques. An application of the preference-oriented approach to our problem is discussed next.

Preference Oriented Approach. Assume that our procurement manager is in a position to articulate a value judgment between the objectives of high product quality and on-time delivery in the form of some dollar value attached to such objectives. Let p_{ik} be the dollar penalty caused by one defective unit of item *i* at plant *k* to the purchasing organization. Also, let l_{ik} be the dollar penalty the organization suffers as a result of one unit of item *i* missing its scheduled delivery time window at plant *k*. The Multiobjective function (1) can now be rewritten as:

$$\min Z = \sum_{j\in J}\sum_{r\in R_j}(1-d_{jr})v_{jr} + \sum_{i\in I}\sum_{j\in J_i}\sum_{k\in K_j} p_{ik}q_{ijk}x_{ijk} + \sum_{i\in I}\sum_{j\in J_i}\sum_{k\in K_j} l_{ik}t_{ijk}x_{ijk}$$

or

$$\min Z = \sum_{j\in J}\sum_{r\in R_j}(1-d_{jr})v_{jr} + \sum_{i\in I}\sum_{j\in J_i}\sum_{k\in K_j}(p_{ik}q_{ijk} + l_{ik}t_{ijk})x_{ijk} \qquad (1')$$

Equation (1') is a single dimension (dollars) objective function, and our model can be now solved as a single-objective optimization problem. The optimal solution to equation (1') subject to constraints (2)-(9) represents the *best-compromise solution* in respect to the articulated p_{ik} and l_{ik} values.

COMPUTATIONAL EXPERIENCE

An extensive computational experiment consisting of 192 different procurement environments obtained through the process of combining various values of the number of items, vendors, discount brackets, and plants was designed to test the computational efficiency of problem (1')-(9). The values of input factors were fixed as follows: items = 100, 200, and 300; vendors = 15, 20, 25, and 30; discount brackets = 3, 4, 5, and 6; plants = 1, 2, 3, and 4. For each such procurement configuration, 10 randomly generated problems were run on a personal computer using the LINGO optimization package, with the clock time elapsing between the beginning of the run and the reporting of the optimal solution recorded.

Analysis of the optimization results reveals two observations. First, the proposed model is computationally efficient. The largest problem of 4 plants, 300 items, 30 vendors, and 6 discount brackets was solved to optimality in about 2 CPU minutes. Second, solution times appear to grow exponentially in the number of plants, but are relatively more sensitive to the number of vendors and their respective discount brackets than to the number of items.

References and the full version of this paper available from the author upon request.