THE TWO-PHASE REVENUE-SHARING CONTRACT COORDINATING SUPPLY CHAIN WITH CONSTRAINT

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ABSTRACT

We consider the supply chain management with two risk-averse participants including one supplier and one retailer and the retailer is directly confronted with random demand over one selling period. Under the condition of satisfying his own risk constraint the participant maximizes his expected profit. The usual contracts, i.e. the revenue–sharing, cannot coordinate the supply chain with risk constraint. So we propose and analyze a Two-phase Revenue-Sharing Contract to coordinate such supply chain under the condition of the information sharing.

INTRODUCTION

Studies on supply chain management have undergone a rapid development in theory and practice today. One of the most important directions of research is the development of coordinating supply chain under the uncertain demand. In order to reach supply chain coordination, we need the execution of a precise set of actions. Unfortunately, those actions are not always in the best interests of both participants in the supply chain, i.e. the both participants are primarily concerned with optimizing their own objectives, and self-serving focus often results in poor performance. Currently, there are a few widely used contracts such as the buy-back contract, the revenue-sharing contract, and so on. Especially, the revenue-sharing contract has received more attention for it is easy to implement. Paper [3] has studied these contracts in the contexts of a perfectly competitive retail market. Usually, researchers assume that participants are all risk-neural. In fact, in the field of economics and finance, participants are often assumed to be risk-averse and they maximize their profits. For recent study, see [2].

Our primary objective is to investigate the coordination of supply chain with risk constraint in hedging against demand uncertainty. Assume that supplier and retailer have agreed on the revenue-sharing contract. And we develop a serial of supplier-retailer models on this contract. Results are derived for expected system profit with coordination on revenue-sharing contract. Here, coordination means that retailer and supplier share information regarding their relevant costs, prices and market demand, and then they jointly select the amount to be ordered by the retailer and produced by the supplier, at the same time, the participant's risk constraint can be satisfied. Here, we define the participant's risk constraint as the set of actions adopted by the participant that results in probability of his actual profit below his target profit level not more than a designated value.

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We show that delivery order quantity from supplier to the retailer has a minimal value. If the order quantity is below this value, the risk probability reaches to the maximum. Otherwise, the risk probability is a monotone increasing function of the order quantity. At the same time, the supply chain cannot be coordinated at all time by the general contracts mentioned before. So we develop a two-phase revenue-sharing contract to coordinate supply chain with risk constraint, i.e. the revenue-sharing mechanism of the first phase is similar to that used in [1] and the revenue-sharing mechanism of the second phase in this paper is aimed to ensure a limited portion of the excessive inventories to be refundable. And at the end of the paper, we illustrate the importance of the information

sharing: if the information is not sharing, the retailer will get more profit by concealing the true demand. However, the profit of the supplier and the system will both decrease, and the supply chain cannot be coordinated, which exposes the limitation of our study and put the new further directions for research.

FORMULATION

Assume that there is one selling season with uncertain demand, and let X be random demand. Let F be the distribution function of demand, and f the corresponding density function, where F is differentiable and strictly increasing. The retailer's price is p , and the supplier marginal cost is c and the retailer's magical cost is c_r , which is not including any payment to the supplier. Let $c = c_r + c_s$ and $c < p$ is reasonable. At the same time, the retailer earns v unit unsold in the second market at the end of season, i.e. v is the unit salvage expense and $v<c$.
Let $s(q)$ be the expected sales of the retailer who order the quantity q from the supplier at the start of the selling season, where $s(q) = q - \int_0^q F(x)dx$. So the revenue is $ps(q)$ and salvage revenue at the end of selling season is $v(q - s(q))$. The total revenue is $(p-v)s(q) + vq$. And then the total expected profit of supply chain can be described as $E[\pi_0(q, X)] = (p-v)s(q) - (c-v)q$, where the actual profit of supply chain $\pi_0(q, X) =$ $p\min\{q, X\} + v(q - X)_+ - cq.$ The sign $x_+ : x$ if x>0; 0, otherwise. Obviously the function $E[\pi_0(q, X)]$ is strictly concave. The optimal order quantity is

$$q^0 = F^{-1}\left(\frac{p-c}{p-v}\right). \tag{1}$$

Here, we just introduce the concept of risk constraint. Let α be the target profit, and parameter $\beta \in [0,1]$; The risk constraint can be described as the probability that the participant's actual profit less than the target profit α is no more than β , i.e. $\Pr\{\pi(q, X) < \alpha\} \le \beta$. And we call $\Pr\{\pi(q, X) < \alpha\}$ as the risk probability (RP). Obviously, as to risk-constraint pairs (α_1, β_1) and (α_2, β_2) , if $\alpha_1 \le \alpha_2$ and $\beta_2 \le \beta_1$, and then the second pair means a higher aversion to risk than does the first.

Now consider an ideal supply chain management that has only one decision-maker, i.e. the retailer and the supplier unite as one. And we just only consider the whole supply chain's problem with assuming system's risk-constraint pair (α_0, β_0). It can be written as the following uncertain programming (UP):

$$\text{(UP)} \quad \max E[\pi_0(q, X)] = (p-v)s(q) - (c-v)q \tag{2}$$

$$\text{s.t.} \quad \Pr\{p\min\{q, X\} + v(q - X)_+ - cq < \alpha_0\} \le \beta_0 \tag{3}$$

Proposition 1 In the optimal system, there is a minimal order quantity $q_{\min}^0 = \alpha_0/(p-c)$, and the risk probability of the channel is $RP = 1$ if $q < q_{\min}^0$; otherwise, $RP = F((\alpha + (c-v)q)/(p-v))$. And the system can be coordinated if the delivery order quantity q^* satisfies the following condition.

$$q^* = \begin{cases} q^0 & if \quad F(\dfrac{\alpha_0 + (c-v)q^0}{p-v}) \le \beta_0 \\ \dfrac{F^{-1}(\beta_0)(p-v) - \alpha_0}{c-v} & if \quad F(q_{\min}^0) \le \beta_0 < F(\dfrac{\alpha_0 + (c-v)q^0}{p-v}) \end{cases} \tag{4}$$

All proofs are omitted for space, and the corresponding proofs can be acquired from the authors.

RP IN THE REVENUE-SHARING CONTRACT

Now we consider the special case that the participant's target profit level is defined as his expected profit. We know The revenue-sharing contract generally includes two parameters, the first is the wholesale-price the retailer pays per unit, ω, the second, ϕ, is the retailer's share of revenue generate from each unit, the rest $1-\phi$ is the supplier's share, that's, the supplier charges ω per unit purchased plus the retailer gives the supplier a percentage of her revenue. And ϕ depends on the member's bargaining power. And the result of bargaining will be $E[\pi_r(q,X,\phi)] = \phi E[\pi_0(q,X)]$, $E[\pi_s(q,X,\phi)] = (1-\phi)E[\pi_0(q,X)]$. And the problem of the retailer with the risk-constraint pair $(E[\pi_r(q,X,\phi)], \beta_r)$ can be expressed as the following uncertain programming.

$$\text{(UP)} \quad \max E[\pi_r(q,X,\phi)] = \phi(p-v)s(q) - (\omega + c_r - \phi v)q \tag{5}$$

$$\text{s.t.} \Pr\{\phi(p-v)\min\{q,X\} - (\omega + c_r - \phi v)q < E[\pi_r(q,X,\phi)]\} \le \beta_r \tag{6}$$

And then the risk probability of the retailer is $RP = \Pr\{\min\{q,X\} < s(q)\} = \Pr\{X < s(q)\}$.
As to the problem of the supplier with the risk-constraint pair $(E[\pi_s(q,X,\phi)], \beta_s)$, we have

$$\text{(UP)} \quad \max E[\pi_s(q,X,\phi)] = (1-\phi)(p-v)s(q) - [c_s - \omega - (1-\phi)v]q \tag{7}$$

$$\text{s.t.} \quad \Pr\{(1-\phi)[p\min\{q,X\} + v(q-X)_+] - (c_s - \omega)q < E[\pi_s(q,X,\phi)]\} \le \beta_s. \tag{8}$$

Similarly, as to the problem of the system with the risk-constraint $(E[\pi_0(q,X)], \beta_0)$, we have

$$\text{(UP)} \quad \max E[\pi_0(q,X)] = (p-v)s(q) - (c-v)q \tag{9}$$

$$\text{s.t.} \Pr\{p\min\{q,X\} + v(q-X)_+ - cq < E[\pi_0(q,X)]\} \le \beta_0 \tag{10}$$

Considering the RPs of the supplier and the system, we can have the following conclusion by dealing with the same way in solving the risk constraint of the retailer.

$$\Pr\{\pi_s(q,X,\phi) < E[\pi_s(q,X,\phi)]\} = \Pr\{\pi_r(q,X,\phi) < E[\pi_r(q,X,\phi)]\}$$

$$= \Pr\{\pi_0(q,X) < E[\pi_0(q,X)]\} = \Pr\{X < s(q)\} = F[s(q)].$$

That's, the retailer, the supplier and the system have same RP, which is equivalent to the probability that

the demand is less or equal to the expected sales. Now we will decide on the wholesale-price, ω, in the revenue-sharing contract. Obviously, when $\omega = \phi c - c_r$, the retailer gets the expected profit $E[\pi_r(q, X, \phi)] = \phi E[\pi_0(q, X)]$, and the supplier gets the expected profit $E[\pi_s(q, X, \phi)] = (1-\phi)E[\pi_0(q, X)]$.
Similar to proposition1, we can get the solution of the retailer's order quantity is

$$q_0^* = \begin{cases} q^0 & \text{if} \quad F(s(q^0)) \le \beta_r \\ F^{-1}(\beta_r) + \int_0^{q^0} F(x)dx & \text{if} \quad F(s(q^0)) > \beta_r \end{cases} \tag{11}$$

Clearly if $F(s(q^0)) > \beta_r$, the retailer orders $q_0^* < q^0$, in which the system will not be coordinated.
From the section, we know that the usual revenue-sharing contract cannot coordinate the supply chain with risk constraint in some cases. In fact, other contracts studied in [1] cannot do it, too.

Assume the supply chain is composed of the retailer with risk-constraint pair (α_r, β_r) and the supplier with risk-constraint pair (0,1). The supplier's risk-constraint pair (0,1) means the supplier has no risk constraint to some certain for if his lost profit is from this retailer, he can salvage his loss from other retailers. We can measure the supply chain's risk-constraint pair (α_0, β_0). We cannot assume the supply chain has the same risk-constraint pair as (α_r, β_r) for the supplier has no risk constraint; similarly, we also cannot suppose that supply chain has no risk constraint as the supplier for existing the risk-constraint retailer. So in this section, the assume that the supply chain's risk-constraint pair (α_0, β_0) should satisfy $0 \le \alpha_0 \le \alpha_r$ and $\beta_r \le \beta_0 \le 1$. And the system with risk constraint is coordinated if actions taken by participants to maximize the profit of system, and at the same time satisfy participants' (including system) risk constraints. Assuming the parameter ϕ of the revenue-sharing contract is pre-negotiated. Then the problem of the retailer is.

$$\text{(UP)} \quad \max E[\pi_r(q, X, \phi)] = \phi[(p-v)s(q) - (c-v)q] \tag{12}$$

$$\text{s.t.} \Pr\{\phi[(p-v)\min\{q, X\} - (c-v)q] < \alpha_r\} \le \beta_r \tag{13}$$

Proposition 2. In the revenue-sharing contract, there is a minimal order quantity of the retailer, i.e. $q_{\min}^r = \alpha_r / \phi(p-c)$. And RP=1 if $q < q_{\min}^r$; otherwise, $RP = F((\alpha_r + \phi(c-v)q)/\phi(p-v))$. Furthermore, the delivery order q_r^* shall satisfy the following two cases.

$$q_r^* = \begin{cases} q^* & \text{if} \quad F(\dfrac{\alpha_r + \phi(c-v)q^*}{\phi(p-v)}) \le \beta_r \\ \dfrac{\phi F^{-1}(\beta_r)(p-v) - \alpha_r}{\phi(c-v)} & \text{if} \quad F(q_{\min}^r) \le \beta_r < F(\dfrac{\alpha_r + \phi(c-v)q^*}{\phi(p-v)}) \end{cases} \tag{14}$$

Obviously, the supply chain is coordinated by ordering q^* if $F[(\alpha_r + \phi(c-v)q^*)/(p-v)] \le \beta_r$ and the risk constraint of system is also satisfied. Else, the retailer will order $[\phi F^{-1}(\beta_r)(p-v) - \alpha_r]/\phi(c-v)$, and it's obvious that the channel's expected profit cannot be coordinated if there is no motivating mechanism. In order to induce the retailer to order q^*, a two-phase revenue-sharing contract carried out by the supplier is constructed, which can be described as the following strategies:
Case1: When the order quantity q satisfies that $q < q_r^*$, the revenue-sharing contract with parameter ϕ is executed, and no products are returned.
Case2: When the order quantity satisfies $q_r^* < q \le q^*$, a revenue-sharing contract with parameter ϕ is

executed for q_r^*. If the rest quantity $q-q_r^*$ is more than $x-q_r^*$, the surplus quantity $q-x$ returns to the supplier for full refund. Else if the demand is larger, then the retailer sells out the rest $q-q_r^*$. So whatever conditions the demand maybe, the retailer's expected profit doesn't decrease for the supplier's providing full refundable policy. Then in order to compensate the supplier's such policy, the rest potential profit, i.e. $(p-c)\min\{q-q_r^*, x-q_r^*\}$ will be shared further with parameter θ, the retailer's share, and the rest $1-\theta$ is the supplier's share. So the actual profit of the retailer is

$$\pi_r(q,X,\phi,\theta)=\begin{cases}\phi[(p-v)X-(c-v)q_r^*] & if \quad X\le q_r^*\\ \phi(p-v)q_r^*+\theta(p-c)\min\{q-q_r^*, X-q_r^*\} & if \quad X>q_r^*\end{cases} \tag{15}$$

Where the parameter θ should satisfy the following equation

$$\int_0^{q_r^*}\phi[(p-v)x-(c-v)q_r^*]f(x)dx+\int_{q_r^*}^{\infty}[\phi(p-v)q_r^*+\theta(p-c)\min\{q-q_r^*, X-q_r^*\}]f(x)dx$$

$$=\phi[(p-v)s(q)-(c-v)q]. \tag{16}$$

Case3: When the order quantity satisfies $q>q^*$, the system's profit will not increase any more, so the return policy will not be provided for the products exceeding over q^* - q_r^*.

Proposition 3. The two-phase revenue-sharing contract can coordinate the channel with the risk-constraint pair (α_0,β_0), and the retailer's risk-constraint pair (α_r,β_r) where $\alpha_0\le\alpha_r$ and $\beta_0\ge\beta_r$.

However, the two-phase revenue-sharing contract is not perfect when the information such as the market demand and costs cannot be shared. In the next section, we illustrate the contract exposes its shortage when the market demand report is disguised by the retailer.

DISCUSSION

In order to get a better understanding of the underlying coordination problem and illustrate the flexibility of profit allocation using the described coordination mechanism, we consider a numerical example.

We assume that the information (parameters) is known. The parameters are specified as follows: the external demand of the final products *X: U*[10,20], and the retailer's price *p*=12, and the marginal cost *c*=6, and each unit salvage profit *v*=2. Let ϕ=0.5. The risk-constraint pairs of the retailer, the supplier and the supply chain are (α_r,β_r)=(40,0.4), (α_s,β_s)=(α_s,1), and (α_0,β_0)=(0,1), respectively. Then the retailer's order quantity is q_r^*=15, and the channel's order quantity is q^*=16. Using the coordination mechanism for $q_r^*\le q\le q^*$, we get the parameter θ=0.093. So the profits of the retailer and the supplier are $E[(\pi_r)]=E[(\pi_s)]$=39, and the expected profit of supply chain is 78.

Here we consider the special case: the retailer cheats the supplier by announcing demand of products *X: U*[12,22], and giving a strong risk-constraint pair (α_r,β_r)=(40,0.2) (other parameters are same as before), and then q_r^*=15, q^*=18, and θ=0.228. But under the condition of the true demand, the expected profits of the retailer, the supplier and the supply chain will be $E[(\pi_r)]$=40.19, $E[(\pi_s)]$=35.81, and $E[(\pi_0)]$=76. So the expected profit of the retailer will increase, inversely, the supplier's expected profit will decrease. At the same time, the channel's expected profit will not reach the maximum.