OPTIMAL POLICY FOR PRODUCTION, STOCK RATIONING AND DYNAMIC PRICING IN A MULTI-CHANNELS SUPPLY CHAIN

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ABSTRACT

In this study, we consider a capacitated supply system with single item and several demand channels. Based on the different lost sales cost and stochastic demand, the problem of optimizing the production planning, stock rationing and dynamic pricing is modeled as an M/M/1 make-to-stock queue. With the objective of minimizing total expected cost, we use discounted dynamic programming to construct the problem and adopt value iteration approach to analyze the optimal policy with an infinite horizon.
Keywords *：Stock Rationing; Dynamic Pricing; Dynamic Programming; Yield Management*

INTRODUCTION

In recent years, innovative pricing strategy has become a common strategy to effectively manage inventory. In some industries, “yield management” is concerned to adjust the perishable resource dynamically according to the inventory level or time left in the selling season. And for retailing, the strategy is popularly used for market cleaning. With the advent of the Internet and the high connectivity, differentiated and dynamic pricing through the supply chain has also become prevalent in this e-business era. Based on the inventory level and production capacity, firms could change their pricing policy for different channels at any moment. Especially for a supply chain with limited capacity, the prices charged to different demand channels and the possible inventory allocation becomes the key decisions for managers. We could implement “inventory rationing” to reserve the limited resource for channels with higher economic values, and use dynamic pricing skill to stimulate or restrain downstream demand. Many research results for stock rationing can be found in [7] [8] [9] [10] [11] [12], and many papers addressing pricing and inventory control can be found in [1] [2] [3] [4] [5] [6]. But as the best of our knowledge, no papers try to explore the interaction among production, inventory rationing and dynamic pricing. Hence, our objective is thus to analyze a combined problem where pricing decisions are considered jointly with inventory rationing and taking into account production capacity. Using dynamic programming approach, we hope to obtain some structural results or managerial insights for both academic researchers and practitioners.

PROBLEM FORMULATION AND MATHEMATICAL MODEL

To obtain the analytical insight and to determine the optimal policy, we construct the research problem as follows. Suppose a production facility produces one type of product and stock them in a warehouse. Demand from N different channels would independently arrive at any moment. When a downstream request arrives, the facility should satisfy the request immediately. Otherwise, a lost sales cost will be incurred due to insufficient inventory levels or stock rationing. Production and dynamic pricing could be used to mitigate the shortage. Especially for raising the price, it is an effective strategy to hold downstream demand back. Once the shortage has been replenished or the inventory levels is in an over condition, price would be reduced to stimulate the request. Furthermore, we formulate the above problem as an M/M/1 queueing model and with the following parameters:

μ : for a production facility, the production time is an Exponential distribution with mean equals $1/\mu$.

λ_i : for the downstream channel i, the request rate follows a Poisson distribution with mean λ_i.

c_i : for different downstream channel i, the unit lost sales cost is c_i, $c_1 > c_2 > c_3 > ... > c_N$.

$C(X)$:system total cost under inventory level X.

$H(X)$: holding cost under inventory level X.

$P_i(X)$: the price of channel i under inventory level X, $i = 1,2,...,N$.

To construct the model with an one-dimension Markov Chain, we set $X(t)$ as the inventory level at time t. Furthermore, to reduce the complexity of a continuous time Markov model, the uniformization method proposed by Lippman [13] in 1975 is used. Without loss of generality, we assume $\alpha + \gamma = 1$ and $\gamma = \mu + \sum_{i=1}^{N} \lambda_i$, $\alpha \in R$ is the discount factor, which satisfies $0 < \alpha < 1$. After this step, the time scale is redefined as a discrete time type, and the optimal discounted cost function could describe our decision problem as

$$C(X) = \frac{1}{\alpha+\gamma}\Big\{H(X) + \mu \underset{a_1}{MIN}\left[a_1 C(X) + (1-a_1)C(X+K)\right]$$

$$+ \sum_{i=1}^{N} \lambda_i \underset{a_2}{MIN}\left[a_2(\ C(X) + k_i c_i \ ) + R_i C(X)\right] - \underset{a}{MAX} \sum_{i=1}^{N} P_i\left(a_1 X + (1-a_1)K - (1-a_2)k_i\right)\Big\} \tag{1}$$

where (a) K is the amount of product manufactured by the facility, k_i is the demand of channel i

(b) R_i is an operator and $R_i C(X)$ equals $a_2(C(X) + k_i c_i$ for $X = 0$ or $(1-a_2)C(X-k_i)$ otherwise.

(c) $A = (a_1, a_2) \to \{0,1\}^2$, $a_1 = 1$ if and only if not to produce, $a_2 = 1$ if and only if reject the request.

The above function has four components. Besides of the holding cost $H(X)$, the first minimization is respect to the decision of manufacturing. If $C(X+K) < C(X)$ holds, it means that to manufacture K unit products would have a lower total cost. Thus the facility should produce the goods. The second minimization is for the reservation policy. Under $X > 0$, $C(X) + c_i k_i > C(X-k_i)$ shows the system cost due to offer k unit products are less than the lost sales cost. Hence, releasing the holding inventory to downstream channel would be the best policy. The third part of objective function is about the pricing policy. Price in each time period varies with inventory level. After observing the inventory level, the best price of channels could be decided. As soon as the system's inventory level is low, any request would be asked for a higher price.

MODEL ANALYSIS AND PROPOSITIONS

We will use value iteration to analyze the objective function. On the basis of this method, optimal policy is chosen in each sate to minimize the total expected discounted cost. Based on the value iteration approach, it is to select an initial value function at first, and then choose actions which minimize future expected cost. After the n-step function is defined, this function will uniformly converge to the real optimal value by letting $n \to \infty$. Hence we could get the decision policy. We characterize the structures of optimal combined strategies in the following propositions：

Proposition 1: The optimal cost function is convex and bounded.

Proposition 2: The optimal policy for production is a base-stock policy.

Proposition 3: The optimal policy for inventory rationing is a reservation policy

Proposition 4: The optimal price does not always decrease with increasing inventory.

Proposition 2 means that it should be a critical inventory level S such that it should start to produce if the inventory level is lower than S. Otherwise, the production should be stopped. The Proposition 3 shows that there exists a series of rationing levels $R_1, R_2, ..., R_N$ and the demand of class k should be satisfied with holding inventory if the stock level is upper than R_k. Otherwise, it should be rejected.

CONCLUSION

In this paper, production planning, inventory rationing and dynamic pricing are considered simultaneously in the face of demand uncertainty. For researchers, this study is a new starting point for yield management. For practitioners, recognizing this integration and trying to build the best insight into the real world is needed in the near future.

REFERENCE

Due to the page limitation, the references are omitted. Readers please contact with the authors if the complete reference list is needed.