COST ESTIMATION AND CURVILINEAR DATA ANALYSIS: A LEARNING CURVE EXAMPLE

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ABSTRACT

Most managerial/cost accounting textbooks perform a logarithmic transformation of the curvilinear learning function so that students can perform linear regression when estimating costs. With the enhanced statistical capabilities of spreadsheets, learning curves with their underlying power functions are now available as another trend type for data analysis. Students can determine whether learning curves better fit the historical data, approximate the learning rate for the best model, and estimate costs when similar learning will take place.

INTRODUCTION

Most management/cost accounting textbooks discuss learning curves by first describing the log-linear model and contrasting the cumulative average-time and the individual unit-time variations of it. Then both models are used with an assumed learning rate to predict labor hours for increasing levels of production. All of this is presented without performing curvilinear data analysis to determine which model to select and its unique learning rate. When faced with learning curve data for estimating labor costs, most accounting textbooks prefer to perform a logarithmic transformation of the data so that linear regression techniques may be used.

LEARNING CURVES

The learning curve relationship is commonly modeled with a power function described as the log-linear or constant percentage model. The log-linear model below recognizes that labor hours decrease systematically by a constant percentage each time the volume of production increases geometrically (usually a doubling of units).

$$(A, \text{ or } I_n) = aX^b$$

The choice of a dependent variable depends on whether the cumulative average-time learning model (*A*) or the individual unit-time learning model (I_n) is selected. The dependent variable and independent variables are defined as *A* = the average cumulative labor hours for *X* number of units, I_n = the number of labor hours required to produce the last *n*th unit, *a* = the number of labor hours required to produce the first unit, *X* = cumulative number of units produced, and *b* = learning exponent, which is always negative. The negative learning exponent b is equal to (log r)/(log f), where r is the rate of learning represented by the constant percentage decrease in hours, and f is the factor increase in output (usually in terms of 2).

ALPHA COMPANY: CURVILINEAR ANALYSIS OF LEARNING

Alpha Company is preparing for the government a bid to build seven LUV lunar vehicles. Because of its experience building similar space equipment, the government requested Alpha Company to build the prototype LUV lunar vehicle. Upon completion of the LUV prototype, the government released to other approved contractors the lunar vehicle manufacturing specifications and its $1,500,000 cost. Listed below are the direct costs for materials, labor, and manufacturing overhead. Indirect costs were applied at 20% of total direct manufacturing costs. The $300,000 equipment purchased by the government will be made available to the selected contractor.

Direct materials	$ 640,000
Direct labor (2,000 hours @ $150)	300,000
Direct manufacturing overhead (2,000 hours @ $30)	60,000
Indirect costs ($1,000,000 @ 20%)	200,000
Purchase of reusable equipment	300,000
Total	$ 1,500,000

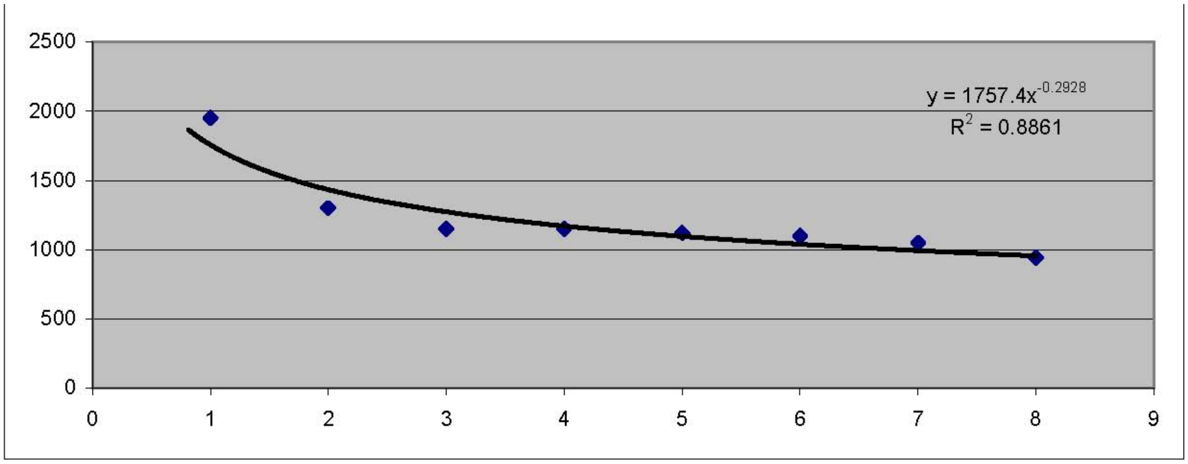
The government and Alpha Company recognize that the 2,000 direct labor hours incurred for the LUV prototype should not be extended to the next seven lunar vehicles because of anticipated learning effects. Alpha projects that, with their highly skilled and stable labor force, the next seven LUV lunar vehicles could be built with the same amount of learning experienced with a similar space vehicle project. Six year ago, Alpha Company built eight DEF lunar vehicles for the government, with the first being a prototype. From their job-order costing records, the direct labor hours incurred for each of the eight DEF vehicles are below:

1. 1,950	2. 1,300	3. 1,150	4. 1,150
5. 1,120	6. 1,100	7. 1,050	8. 940

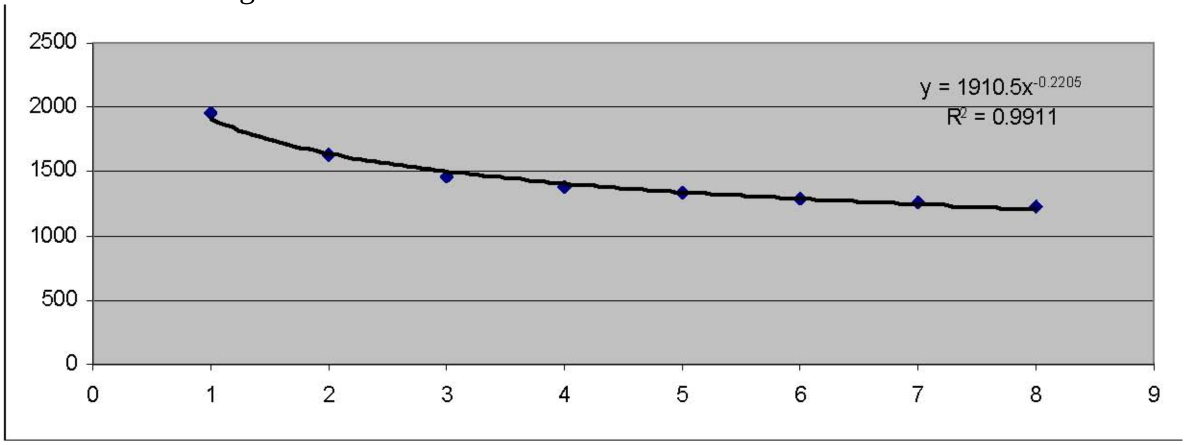
Curvilinear Data Analysis of DEF Learning

Alpha Company relies on its historical labor data for the DEF vehicles in preparing its bid. Two related analyses are performed on the DEF data: what learning curve model to adopt, and its percentage of learning. The analysis performed for both the individual unit-time and the cumulative average-time models prepares a scatterplot of direct labor hours incurred for the eight DEF vehicles, and then adds a power function curve, equation, and r-squared value. Learning curve for the individual unit-time model: Y = 1,757.4 X -0.2928, with 0.8861 r-squared value, and for the cumulative average-time model: Y = 1,910.5 X -0.2205, with 0.9911 r-squared value.

Individual Unit-Time Model



Cumulative Average-Time Model



Unit	Individual	Total	Average	Unit	Individual	Total	Average
1	1,950	1,950	1,950	5	1,120	6,670	1,334
2	1,300	3,250	1,625	6	1,100	7,770	1,295
3	1,150	4,400	1,467	7	1,050	8,820	1,260
4	1,150	5,550	1,388	8	940	9,760	1,220

Approximate Direct Labor Hours for LUV Cost Estimate

The data analysis supports the use of the cumulative average-time model because of its larger r-squared value. Its learning rate is 86% or 10^(-0.2205*Log(2)). Alpha Company estimates the direct labor hours for the next seven LUV lunar vehicles to be 1,272 direct labor hours {2000*8^(Log(0.86)/Log(2))}, given 2,000 hours for the prototype. Therefore, the total number of direct labor hours estimated to complete the next seven units is 8,176 ((1,272*8) – 2,000). A cost estimate of $7,142,016 is calculated below for the next seven LUV lunar vehicles.

Direct materials ($640,000*7)	$ 4,480,000
Direct labor (8,176 hours @ $150)	1,226,400
Variable manufacturing overhead (8,176 hours @ $30)	245,280
Other manufacturing overhead ($5,951,680 @ 20%)	1,190,336
Total	$ 7,142,016

The log of the cumulative average-time model yields the same result if linear regression is performed on the equivalent logarithmic DEF data. Log (Y) = Log $(1{,}910.5\ X^{-0.2205})$ = Log (1,910.5) + Log $(X^{-0.2205})$ = 3.2811 - 0.2205X. If no learning is modeled, the overstated cost estimate will be $8,400,000 based on 14,000 hours. If the incorrect individual unit-time model is used with the 86% learning rate, the cost estimate will be $7,561,704 based on 10,119 hours.

SUMMARY

The analysis of curvilinear learning data is facilitated by enhanced statistical spreadsheet capabilities. Textbooks and instructors should utilize this added capability to teach students to perform curvilinear data analysis. By not transforming learning curves into linear relationships, students will better recognize the importance of learning when preparing cost estimates.