TOWARD A THEORY OF SECURITIES TRADING

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ABSTRACT

This is a paper on intraday security price forecasting and trading technologies for best exection. The author presents an intraday stock price prediction model by simulating stochastic processes of bid and asked prices with fractal volatility. The model introduces a unique adaptive learning process, which improves the accuracy of forecasts. Then, the model is taken to the trading environment where the trader's objective is to minimize the market impact. The situation analyzed is similar to that found in game theory. Intraday price predictions are used as potential price limit strategies. The solution to the problem is one of mixed strategies, which is found via the simplex method in the standard LP problems.

INTRODUCTION

The best securities trading strategy must give the best execution. Such strategy involves two aspects of trading. One is the trader's ability to predict the price over the entire trading time horizon. And the other aspect is to minimize the market impact, when orders are placed.

In this paper, we begin with Professor Stigler's securities trading model where security prices are determined in the *flow* market to buy (or sell) and not in the *stock* market for securities to own (or hold). Therefore, even when the *stock* demand for securities remained unchanged, it is possible to see changes in security prices in the *flow* market and in fact, in multiple times. It is this frequency of trades, which causes the price volatility. In this context, Professor Stigler presented a trading rule to explain how the prices are determined. Professor Stigler's model overlooks the following.

First, frequency of trading would differ and hence, the price volatility would differ at different trading times. A correct analysis must examine the difference between the trading times and the calendar times. In this way, the fractality in volatility observed on the calendar time scale can be effectively accounted for. Second, an implicit assumption that every trader is able to buy and sell however many shares they wish at any given price, i.e. the assumption of perfect competition, is unrealistic. In reality, any reasonably large order impacts the market. Third, the equilibrium structure changes frequently. We ought to explain how the bid and asked prices change as a consequence. Fourth, the stochastic process of security prices and of order arrivals as assumed in the Stigler's model is one of uniform distribution. This is contrary to some latest findings that the stock prices follow the lognormal distribution. We offer an alternative securities trading model.

SECURITIES TRADING MODEL

The securities trading model we propose follows several procedures. First, we will generate *daily* log bid and ask price relatives, i.e. $\ell n\left(\frac{B_t}{B_{t-390}}\right) \approx b_t$, and $\ell n\left(\frac{A_t}{A_{t-390}}\right) \approx a_t$, given that there are 390 trading minutes in a day. We only look at the market inside bid and ask to learn the *marginal* behavior of traders. Second, we then compute the mean and standard deviations for b_t, i.e. $\bar{b}_t$ and σ_{bt}, and a_t, i.e. $\bar{a}_t$ and σ_{at}. The time interval is $\Delta t = \frac{1}{390}$. Then, the minute average price drift will be given by $\bar{b}_t \cdot \Delta t$ and $\bar{a}_t \cdot \Delta t$. However, the minute average volatility will be computed somewhat differently. If the total number of trading in each minute is f_{td} in a given day, *d*, then, the relative trading frequency at time *t* is $v_t \equiv (1\ minute) \cdot \frac{f_t}{\sum_{t=1}^{390} f_t}$. To convert from the daily to minute volatility, obtain the fractal power as $p_t = \frac{1}{2} \cdot \frac{\ell n(v_t)}{\ell n(\Delta t)}$.

Third, we will model a semi-random walk stochastic process similar to the Wiener process for the bid and ask prices as $\frac{\Delta B}{B} = \bar{b}\Delta t + \sigma_b \varepsilon (\Delta t)^p$ and $\frac{\Delta A}{A} = \bar{a}\Delta t + \sigma_a \varepsilon (\Delta t)^p$. It should be noted that if $v_t = \bar{v} = \frac{1}{390}$, then $p = \frac{1}{2}$, as is with the standard *Geometric Brownian* motion. The simulated *willing* bid and asking prices are, for $t = 1, 2, ..., 390$, $B_{t+1} = B_t \cdot \left[1 + \left(\frac{\Delta B}{B}\right)_{t+1}\right]$ and $A_{t+1} = A_t \cdot \left[1 + \left(\frac{\Delta A}{A}\right)_{t+1}\right]$.

Fourth, apply the Stigler's trading rule that if a buy order arrives at the market before a sell order, and if B_{t+1} is greater than A_{t+1}, the resulting equilibrium security price is given by $P_{t+1} = B_{t+1}$. Otherwise, no trade will take place. Similarly, if a sell order comes before a buy order, A_{t+1} must be lower than B_{t+1} in order for a trade to occur. This completes the entire process of the price determination in the Stigler's context. In summary, the price is determined when the bid "crosses" the asking price.

ADAPTIVE LEARNING

Any forecasting model must carry with it its own prediction errors. A successful trading technology must be able to quickly recognize the errors and adapt to the forecasting values. The conventional adaptive expectation model, however, is not complete, as it essentially deals with a simple two-period paradigm. When extended into multiperiods, any revision of expectation applies uniformly to all other future periods without affecting trends. That is, the traditional adaptive expectation model will never capture errors made about trend reversal. We propose the following model.

Let G_{th} be initial forecasting values at time *t* for a future time *h*, as described above, where $G_{th} = \phi_{th} G_{th-1}$. If α_{th} is an error made in a forecast done for *h* at time *t*, and the actual realized price at *t* + *h* is P_{t+h}, then $P_{t+h} = G_{th} - \alpha_{th}$. Therefore, if $\alpha_{th} > 0$, $G_{th} > P_{t+h}$ meaning that the prediction was overshot, and as such, the market should be predicted as bearish. If $\alpha_{th} < 0$, the opposite is true. The adaptive learning model that we are proposing here modifies G_{th}'s based on these α_{th} errors. We will call these modified predictions as F_{th}'s, as opposed to G_{th}.

If χ_{th} is the adjustment coefficient, our adaptive expectation takes the form of $F_{th} = G_{th} - \chi_{th}\alpha$. The symbol α is the latest error. If $0 \le \chi_h < 1$, the process is partial, while if $\chi_h > 1$, it is lumpy or abrupt. Note that if $P_{t+h} = F_{th}$, then $\alpha_{th} = \chi_{th}\alpha$. That is, the *ex post* error equals the adjustment factor times the predicted (or estimated) error.

TRADING STRATEGIES: AN APPLICATION

Simulated intraday security prices provides useful information to professional buy side traders in obtaining the most "desirable" execution price. Generally, intraday forecasting figures can be used as a series of *limit order* either to buy or to sell, as stated earlier. In order to put this into the game theory framework, we consider the trader's trading horizon, *z*. If $z = 5$ minutes, there are 78 trading intervals, i.e. $\frac{390}{5} = 78$, and hence 78 strategies.

Estimate the expected number of shares to be traded (*q*) as a regression function of bid and asks, i.e. $q_t = \beta_0 + \beta_1 b_t + \beta_2 a_t + \beta_3 a_t b_t + \varepsilon_t$. Now one can construct a payoff matrix in a trading game based on the regression. Using the results from regression, each cell in the payoff matrix can be represented by $\{a_{ij}\}$; $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$. The maximum expected number of shares that can be sold can be achieved by sellers' placing mixed orders at different prices, but subject to satisfying the condition that $\sum_i a_{ij} p_i \ge V$, where *V* is the value of the game and p_i is the probability with which an event $\{a_{ij}\}$ may occur for any given *j*. Similarly, the buyer can also place mixed orders at different prices to ensure that $\sum_j a_{ij} q_j \le W$, where *W* is the value of game and q_j is the probability with which an event $\{a_{ij}\}$ may occur for any given *i*. The solution values for p_i and q_j will then entail trading plans for each time interval.

SUMMARY AND CONCLUSIONS

A securities trading model as presented in this paper is rich in contents. The standard error of forecasting can be used to compute the probability that a stock's price may rise or fall by so much in some specified time. It can also compute the Value at Risk or for that matter, the Value to Gain, all of which can be used as a trading tool. In a longer term context, if we are concerned with monthly or yearly returns, we could easily incorporate some market equilibrium models into the analysis such as Capital Asset Pricing Model.

The best execution often means the trader's ability to beat what is known as Value Weighted Average Price (VWAP). The model presented in this paper will certainly help to achieve that trading goal through the adaptive prediction model, on the one hand, and through the game theoretic mixed strategies, on the other, to minimize the market impact cost.