CAPITAL BUDGETING IN LARGE-SCALE INTERDEPENDENT PROJECTS USING CERTAINTY EQUIVALENCE

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ABSTRACT

Project selection is a major problem in managerial decision making. In this study, a deterministic model that schedules project starts is formulated as a binary integer program. This model is applicable in various settings such as selection of engineering projects in corporate planning, or in other planning environments in which the candidate projects are interdependent. Previous studies using similar methodologies which handled risk adjustments required estimation of parameter values that may not be realistic. This paper introduces certainty equivalence approach in handling risk in capital budgeting using deterministic models.

INTRODUCTION

Project selection is a major problem in managerial decision making. For instance, it is central to the portfolio selection process in investment planning and evaluation of engineering projects in engineering economic analysis. Because effective and efficient management of scarce resources is of paramount importance in every organization, this area has received considerable attention in the literature. Essentially, it is a resource allocation problem: determining the distribution of limited budgetary resources among competing alternative projects. This allocation of scarce resources under capital rationing must be done in order maximize value of the firm. In this study, a deterministic model that schedules project starts is formulated as a binary integer program. It is argued that the best way to utilize this model is in the context of a decision support system. Since such project selection problems usually have long planning horizons and far-reaching strategic impacts on the enterprise, the decision makers should use judgment and insight in addition to the scientific decision making tools. Furthermore, the results obtained must be analyzed by the decision makers based on their experiences of similar situations in the past and their intuition about the future.

Not all projects involved in these analyses are independent. In this study, two main types of interdependencies among the candidate projects are considered. *Mutually exclusive projects:* A set of projects may have the same objective, and therefore, at most one can be selected. *Dependent projects:* It is possible for a major, or primary, project to have a number of secondary, dependent projects. The dependent projects can be selected only if the primary project is selected. Furthermore, there may be a certain level of timing dependency between the primary and the dependent projects. For example, a dependent project can start at the earliest so many time periods prior to (or after) the start of the primary project. There may also be similar restrictions on the completion of the projects.

PREVIOUS WORK

The description given in the previous section is basically a multi-period capital budgeting problem with side conditions. The capital budgeting problem determines which projects to fund given a constraint on available capital. The net present value (NPV) of each project is calculated. and the objective is to maximize the NPV of the sum of the chosen projects subject to funding constraints. The capital budgeting problem is also referred to as the multidimensional knapsack problem [4]. A comprehensive review of knapsack problems is given by Pisinger and Toth [5]. Benli and Yavuz [2] formulated the interdependencies among candidate projects as a 0-1 programming problem and reported very favorable computational results. Value maximization requires that both NPV of the projects and risk of the projects must be accounted for simultaneously. Previous mathematical programming approaches to capital budgeting problems concentrated on net present value maximization ignoring impact of capital budgeting on risk of the firm. Benli and Bilici [3] introduced risk of the firm as a constraint so that risk of the firm will not creep upwards as project selection in the future tends to do if risk is not accounted for. Since introduction of total risk must be handled in a nonlinear manner systematic risk or beta of the projects was used in that study.

MODEL

Assume there are N projects to schedule. If project j $(j = 1, \dots, N)$ is selected, it can start at any year k $(k = 1, \dots, T)$ and continue for the duration, d_j , without preemption. In order to allow any project j to start at period T, at the latest, and be completed at the end of period $T + d_j$, the planning horizon is taken as $T' = T + (D - 1)$, where $D = max_j \{d_j\}$. The associated indicator variable is defined as: x_{jk} is equal to 1, if project j starts at year $k = 1, \dots, T'$, and equal to 0, otherwise. Let ρ_{jl} be the resource requirement of project j during the year l $(l = 1, \dots, d_j)$ of its inception, and define

$$r_{jkt} = \begin{cases} 0, t < k, \\ \rho_{j(t-k+1)}, k \le t < k + d_j, \\ 0, t \ge k + d_j. \end{cases} \tag{1}$$

Then r_{jkt} is the resource requirement of project j during year t if it is started in year k. Let R_t denote the total amount of resource available in year t $(t = 1, \dots, T')$. Clearly, it is not unrealistic to assume that the returns from projects are random variables and that it is possible to assign subjective probabilities for its possible outcomes. Let G_{jl} be the random variable, with a known probability distribution, denoting return of project j during the year l $(l = 1, \dots, d_j)$ of its inception, and define

$$A_{jkt} = \begin{cases} 0, t < k, \\ G_{j(t-k+1)}, k \le t < k + d_j, \\ 0, t \ge k + d_j. \end{cases} \tag{2}$$

Then A_{jkt} is the random variable denoting return of project j during year t if it is started in year k. In order to introduce certainty equivalence approach in handling risk, we need to define certainty equivalent of a random variable. Very simply stated, the certainty equivalent is “[t]he amount of cash someone would require with certainty at a point in time to make the individual indifferent between that certain amount and an amount expected to be received with risk at the same point in time." [29] More precisely, suppose that the decision maker has a utility function $U(.)$ Suppose that A_{jkt} is the random variable denoting return of project j during year t if it is started in year k. Then the expected utility of this return is $E[U(A_{jkt})]$. The certainty equivalent is the (nonrandom) amount $CE(A_{jkt})$ such that $U[CE(A_{jkt})] = E[U(A_{jkt})]$. [See [19] for a detailed discussion.]

As a measure of decision maker's risk aversion of the return of project j during year t if it is started in year k, define $\gamma_{jkt} = CE(A_{jkt}) / E[A_{jkt}]$; or $CE(A_{jkt}) = \gamma_{jkt} E[A_{jkt}]$. Let S denote an ordered set of pairs $(i, j) \in [1, \ldots, N] \times [1, \ldots, N]$ where $i \neq j$. Define $q_{ij} > 0$, as the maximum allowable time lag for project j to start before project i is started, and define $q_{ij} < 0$, as the minimum allowable time lag for project j to start after project i is started. Similarly, $r_{ij} > 0$ is defined as the maximum allowable time lag for project j to be completed after project i is completed and let $r_{ij} < 0$ denote the minimum allowable time lag for project j to be completed before project i is completed. Note that the following strict inequality must hold for all pairs $(i, j) \in S$, $q_{ij} + d_i + r_{ij} > d_j$. If $q_{ij} + d_i + r_{ij} = d_j$, then the start and the completion time for project i is fixed with respect to project j, and therefore the projects i and j can be treated as a single project. Clearly, if $q_{ij} + d_i + r_{ij} < d_j$, then there is no feasible way to schedule project i with respect to project j. Finally, for project i to be able to start in year k $(k = 1, \ldots, T)$, project j must have started at the earliest,

$$\nu(i,j,k) = \min\{\max\{1, k - q_{ij}\}, T'\} \tag{3}$$

and at the latest,

$$\mu(i,j,k) = \min\{\max\{1, k + d_i + r_{ij} - d_j\}, T'\} \tag{4}$$

Let G_h denote the sets of mutually exclusive, disjunctive projects $(h = 1, \ldots, H)$. At most one project can be selected from each set. The objective is to maximize the total discounted certainty equivalents of returns. Letting α be the risk free interest rate and recalling that $CE(A_{jkt}) = \gamma_{jkt} E[A_{jkt}]$, define net present worth of project j as,

$$p_{jk} = \sum_{t=1}^{T} (1+\alpha)^{-(t-1)} \gamma_{jkt} E[A_{jkt}],\ j = 1, \cdots, N; k = 1, \cdots, T. \tag{5}$$

Then the problem can be stated as,

$$\max \sum_{j=1}^{N} \sum_{k=1}^{T} p_{jk} x_{jk} \tag{6}$$

subject to

$$\sum_{k=1}^{T'} x_{jk} \leq 1,\ j = 1, \cdots, N \tag{7}$$

$$\sum_{j=1}^{N} \sum_{k=1}^{T'} r_{jkt} x_{jk} \leq R_t,\ t = 1, \cdots, T' \tag{8}$$

$$\sum_{k=1}^{T'} x_{ik} = \sum_{k=1}^{T'} x_{jk},\ \forall (i,j) \in S \tag{9}$$

$$x_{ik} \leq \sum_{m=\nu(i,j,k)}^{\mu(i,j,k)} x_{jm},\ k = 1, \cdots, T; \forall (i,j) \in S \tag{10}$$

$$\sum_{j \in G_h} \sum_{k=1}^{T} x_{jk} \le 1, h = 1, \cdots, H \tag{11}$$

$$x_{jk} \in \{0,1\}, j = 1, \cdots, N; t = 1, \cdots, T'. \tag{12}$$

Constraint (7) ensures that any project can start only once. Constraint (8) states that the total resource requirement of the selected projects must be less than or equal to the total amount of resource available for that year. Constraint (9) ensures that, for all pairs $(i, j) \in S$, if project i is chosen then project j must also be chosen, and vice versa. Relaxing this constraint, allows the possibility for choosing project j without necessarily choosing project i. Constraint (10) enforces the feasible start times of projects i and j with respect to each other, where $v(i, j, k)$ and $\mu(i, j, k)$ are defined in (3) and (4), respectively. Constraint (11) ensures the selection of only one project in each group G_h $(h = 1, \ldots, H)$.

CONCLUSIONS

Effective decision making is vital for any enterprise in coping with the rapid technological, social, and economical changes. Scientific decision making tools are essential for effective decision making. However, in many enterprises these tools are still not in extensive use and the decisions are generally made based on judgment and intuition. This managerial shortcoming results in an inadequate decision making process, thus reducing the competitiveness of these enterprises. The problem investigated in this study is the modeling of selecting interdependent projects over a planning horizon in order to satisfy the strategic goals of an enterprise and achieve value maximization. The binary programming model developed is essentially a multi-period capital budgeting problem with side conditions. Experimental runs of models of similar size and structure resulted in very promising computational times in the study by Benli and Yavuz [4], suggesting that the problems with actual data can be run well under few minutes. This is especially important because “what if” analysis is an indispensable part of analysis of strategic problems of the type discussed in this paper.

REFERENCES

[1] Benli, Ö. S. & Bilici, H. Capital budgeting in large-scale interdependent projects. Presented at *Thirty-Fourth Annual Meeting of the Western Decision Sciences Institute*, Vancouver, British Columbia, Canada, 2005.

[2] Benli, Ö. S. & Yavuz, S. Making project selection decisions: A multiperiod capital budgeting problem. *International Journal of Industrial Engineering,* 2002, *9*(3):301–310.

[3] Luenberger, D. G. *Investment Science.* Oxford University Press, New York, 1998.

[4] Murty, K. G. *Operations Research: Deterministic Optimization Models*. Prentice Hall, N. J., 1995.

[5] Pisinger, D. & Toth, P. Knapsack problems. In D.-Z. Du and P.M. Pardalos, editors, *Handbook of Combinatorial Optimization* (Vol. 1). Kluwer Academic Publishers, Boston/Dordrecht/London, 1998.

[6] VanHorne, J. C. and Wachowicz, Jr., J. M. *Fundamentals of Financial Management*. Prentice Hall, N. J., 2001.