ILLUSTRATING DYNAMIC PROGRAMMING: AN EXCEL IMPLEMENTATION FOR THE TSP

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ABSTRACT

This paper presents an Excel model of a dynamic programming solution to the (asymmetric) traveling salesperson problem for small problems with up to ten cities. The primary aim of the model is to illustrate the dynamic programming solution process in a classroom setting. Modeling and extensibility issues presented by the NP-hard nature of the problem are discussed.

The model

The purpose of the model described in this paper is to introduce and illustrate the complexity of the traveling salesperson problem (TSP) and its solution using dynamic programming (DP) in a classroom setting. While the benefits of a dynamic programming approach are that it will guarantee an optimal solution to the TSP it is generally agreed that it may not be appropriate, i.e., too cumbersome, for large TSP problems due to the NP-completeness. This is dramatically illustrated in the model presented. It is, of course, possible to code up a DP algorithm to solve the problem but the intention here was to show the workings of the method and the development of a solution. Excel is chosen as the modeling medium, partly because it is now the default modeling medium and students are familiar and comfortable with the interface and partly because the target audience are business students for whom coding, in the sense of programming, was beyond their skill level.

As can be seen from its many diverse applications, the TSP does not always have to refer to the nodes as being cities. However, for simplicity, we shall stay with the classical terminology of cities. Given n cities numbered 1 through n, since the eventual route has to be continuous we can say that we arbitrarily start in city 1. The DP approach, successively builds up partial optimal routes, starting at city 1 and visiting all remaining pairs of cities, then triplets, then sets of four, five….. n-1 cities. In DP terminology each of these is called a stage. At each stage we keep track of the optimal route, which is then referred back to in the next stage. This approach was implemented in Excel using text processing and lookup functions. Student exercises are, for example, to extend the model from a 4 to 5, or 5 to 6 city problem.

Discussion and Conclusion

The mathematical formulation will be provided and the Excel implementation demonstrated during the presentation. Issues to be discussed include: how are ties handled; extensibility issues (including use of the DOS program COMBI); and why the approach is not practical for >10 cities. Models developed for TSPs up to 10 cities and teaching implications will be discussed. In conclusion, this is a useful approach for small TSPs to demonstrate DP and the NP nature of the TSP, but not for solution of large problems. Benefits are that numerous problems can be run once the model of a particular dimension has been setup – it is just a matter of entering the data; and due to the way the routes are developed it is possible to immediately obtain results for partial optimal routes for fewer than n cities.