LAV AND LS ESTIMATION AND INFERENCE PROCEDURES IN REGRESSION MODELS WITH ASYMMETRIC ERROR DISTRIBUTIONS

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ABSTRACT

A Monte Carlo simulation is used to compare estimation and inference procedures in least absolute value (LAV) and least squares (LS) regression models with asymmetric error distributions. This paper compares mean square errors (MSE) of coefficient estimates to assess the relative efficiency of the estimators. Hypothesis tests for coefficients are compared on the basis of empirical level of significance and power. For the LAV regression, the likelihood ratio (LR) test, Lagrange multiplier test, and the bootstrap test are examined. Several versions of the LR and bootstrap tests are considered. The usual t-test is used for LS regression. Factors considered that might influence estimation and test performance include the disturbance distribution and the sample size.

LEAST ABSOLUTE VALUE ESTIMATION AND TESTING

In most previous studies comparing the performance of LAV and LS estimation, the distributions examined have been symmetric. "Fat-tailed" distributions that introduce outliers have been used, but these have typically been symmetric fat-tailed distributions (Laplace, Cauchy, etc). This paper examines the performance of LAV and LS coefficient estimators when the regression disturbances come from asymmetric distributions. Also, hypothesis tests for coefficient significance are examined. For the LAV regression, the tests compared include the likelihood ratio (LR) test and the Lagrange multiplier (LM) test suggested by Koenker and Bassett [12] as well as a bootstrap test. The tests are compared in terms of both observed significance level and empirical power. Several versions of the LR and bootstrap tests are considered. The LAV tests are also compared with the traditional t-test for LS regression.

The model considered in this paper is the simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{1}$$

where y_i is the i^{th} observation on the dependent variable, x_i is the i^{th} observation on the explanatory variable, and ε_i is a random disturbance for the i^{th} observation. The distribution of the disturbances may not be normal or even symmetric in this examination. The parameters β_0 and β_1 are unknown and must be estimated. For a discussion of algorithms to produce LAV coefficient estimates, see Dielman [3] [4].

Bassett and Koenker [1] showed that the LAV coefficient estimator has an asymptotic distribution that converges to a normal distribution with nuisance parameter λ, where λ^2 / n is the asymptotic variance of the sample median for a sample of size n from the disturbance distribution. The test we will consider is the basic test for coefficient significance, i.e., H_0: $\beta_1 = 0$. Koenker and Bassett [12] proposed three procedures for conducting hypothesis tests on the coefficients of LAV regression models. The three tests are based on Wald, likelihood ratio (LR), and Lagrange multiplier (LM) test statistics, each of which has the same limiting chi-square distribution. The LR and LM statistics will be examined in the Monte Carlo simulation. In previous studies, the Wald test has been shown to be inferior to the LR and LM statistics in small samples, so it is not included in this study (See, for example, Dielman and Pfaffenberger [5] [6] [7] and Dielman [2]). The LR test statistic requires the estimation of the scale parameter λ, whereas the LM test statistic does not. One often-suggested estimator for λ can be computed as follows:

$$\hat{\lambda} = \frac{\sqrt{n'}\,[e_{(n'-m-1)} - e_{(m)}]}{z_{\alpha/2}} \quad \text{where} \quad m = \frac{n'+1}{2} - z_{\alpha/2}\sqrt{\frac{n'}{4}} \tag{2}$$

The $e_{(.)}$ are ordered residuals from the LAV-fitted model. A value of α = 0.05 is usually suggested. This estimator will be referred to as the SECI estimator. See McKean and Schrader [14], Sheather [15], Dielman and Pfaffenberger [5] and Dielman and Rose [9] [10] for discussion and use of this estimator. Also, see Dielman and Pfaffenberger [6] for a discussion of additional research relating to the problem of estimating λ.

When computing the variance of the slope coefficient in a LAV regression, the estimator in equation (2) will be used. However, four different options in constructing this estimator will be considered. These options are as follows:

SECI1: $\hat{\lambda}_1$ uses $z = 1.96$ (the α = 0.05 value) and n' = total number of observations (n).

SECI2: $\hat{\lambda}_2$ uses $t_{0.025}$ with n degrees of freedom rather than the z value and n' = total number of observations (n).

SECI3: $\hat{\lambda}_3$ uses $z = 1.96$ (the α = 0.05 value) and $n' = n - r$ where r is the number of zero residuals.

SECI4: $\hat{\lambda}_4$ uses $t_{0.025}$ with $n - r$ degrees of freedom rather than the z value and $n' = n - r$ where r is the number of zero residuals.

The notation L1, L2, L3 and L4 will be used to indicate the LR test using variance estimator 1, 2, 3, or 4. Much of the literature in this area recommends using the estimator SECI3.

The bootstrapping methodology provides an alternative to the LR and LM tests. In a LAV simple regression, for example, a bootstrap test statistic for H_0: $\beta_1 = 0$ can be computed in several ways, as discussed in Li and Maddala [13]. The following procedure will be used in this study: The model shown as equation (1) is estimated using LAV estimation procedures, and residuals are obtained. The test statistic $T = \left|\hat{\beta}_1 - 0\right| / se(\hat{\beta}_1)$, is computed from the regression on the original data, where $se(\hat{\beta}_1)$ represents the standard error of the coefficient estimate. The residuals, e_i ($i = 1,2,\ldots n$), from this regression are saved, centered, and resampled (with replacement, excluding zero residuals), to obtain a new sample of "disturbances", e_i^*. The e_i^* values are used to create pseudo-data as follows:

$$y_i^* = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i^* \tag{3}$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are the initial LAV estimates of the intercept and slope. The coefficients in equation (3) are then re-estimated to obtain new parameter estimates, $\hat{\beta}_1^*$ and $\hat{\beta}_0^*$, and the following test statistic is computed: $T^* = \left|\hat{\beta}_1^* - \hat{\beta}_1\right| / se(\hat{\beta}_1^*)$. The test statistic value, T^*, is computed and saved, and the process is repeated a large number of times. For a test to be performed at a particular level of significance, α, the critical value is the $(1 - \alpha)^{th}$ percentile from the ordered test statistic values. If the original test statistic, T, is larger than this critical value, then the null hypothesis H_0: $\beta_1 = 0$ is rejected. As with the LR test, four versions of the bootstrap, (B1, B2, B3, B4) will be examined depending on which estimator of λ is used.

DESCRIPTION OF THE SIMULATION EXPERIMENT

The simulation is based on the model in equation (1). The sample sizes used are *n* = 14, 30 and 100. The disturbances are generated using stable distributions with the following combinations of characteristic exponent (alpha) and skewness parameter (beta): Beta = 0.0, 0.4 and 0.8 with Alpha = 1.2; Beta = 0.0, 0.4 and 0.8 with Alpha = 1.8; Beta = 0.0 and Alpha = 2.0 (normal). Stable distributions are infinite variance distributions when the characteristic exponent is less than 2.0, so the LAV estimator would be expected to outperform LS in these cases. When the characteristic exponent equals 2.0 (and beta is zero), the distribution is normal and LS will be optimal.

The independent variable is generated as a standard normal random variable, independent of the disturbances. Bootstrap tests used 199 bootstrap replications. The value of β_0 is set equal to zero (without loss of generality). The value of β_1 is set equal to 0.0 to assess the level of significance and is set equal to 0.2, 0.4, 0.6, 0.8, 1.0 and 2.0 to examine power. For each factor combination in the experimental design, 5000 Monte Carlo simulations are used, and the number of rejections of the null hypothesis H_0: $\beta_1 = 0$ is counted for each setting. All testing is done using a nominal 5% level of significance.

SUMMARY OF RESULTS

Estimation

Table 1 contains ratios of mean square errors for estimates of the slope coefficient for sample size *n* = 14 in Panel A, *n* = 30 in Panel B and *n* = 100 in Panel C. Results suggest that the LAV estimator is preferred over LS for alpha of 1.8 or smaller, although the advantage decreases as alpha approaches two (normal distribution). The advantage of LAV over LS also decreases as the skewness (beta) of the distribution increases with Alpha = 1.2. The LAV estimator does not perform as well in this case, relatively speaking, when the disturbance distribution is skewed. It still outperforms the LS estimator in terms of estimator efficiency, however.

TABLE 1: RATIOS OF MEAN SQUARE ERROR OF ESTIMATES OF SLOPE (LS MSE / LAV MSE)

	Panel A: n = 14			Panel B: n = 30			Panel C: n = 100		
		Beta			Beta			Beta	
	0.0	0.4	0.8	0.0	0.4	0.8	0.0	0.4	0.8
Alpha									
1.2	71.4	60.5	38.6	71.4	83.4	56.1	57.3	50.7	38.5
1.8	1.2	1.2	1.1	1.3	1.4	1.5	1.2	1.3	1.3
2.0	0.8			0.8			0.8		

Hypothesis Tests

Tables 2 – 4 (for *n* = 14, 30 and 100 respectively) contain the median percentage of trials in which H_0: $\beta_1 = 0$ is rejected for various combinations of test and coefficient value. The medians are taken over the disturbance distributions. Thus the results for the symmetric distributions include Stable distributions with alpha = 1.2, 1.8 and 2.0 when beta = 0.0. The asymmetric distributions include Stable with alpha = 1.2 and 1.8 when beta is either 0.4 or 0.8. When the coefficient value is zero, the empirical significance levels can be assessed; when it is non-zero, we can compare the power for the tests.

Among the LR tests, LR2 consistently has median significance level closer to nominal when *n* = 14 and 30. There is little difference when *n* = 100. Performance is similar for skewed and symmetric distributions. Among the bootstrap tests, there is little difference in performance for any of the experimental settings. The LM and LS t-test both have empirical level of significance close to nominal. The LAV and LS tests have relatively similar median power for the symmetric distributions. This is not too surprising because one of the three symmetric distributions is normal and another is Stable with alpha of 1.8 which is not too different from a normal distribution. The differences in power between the LAV and LS tests can be much greater when the distributions are asymmetric. The LAV tests perform better than the LS t-test when disturbances are not symmetric.

Making a choice among the LAV tests is somewhat difficult because the differences in power are relatively small. It does appear that the LR2 test maintains relatively high power, even when the level of significance is lower compared to the other tests. Also, the LM test seems to be consistently lower in power. This negates some of the advantage this test might have due to the fact that it does not need an estimate of the nuisance parameter. The LR3 test tends to have higher power than the other alternatives, but at the cost of considerably higher levels of significance. The LR2 test seems to maintain a relatively good balance.

As noted, the bootstrap tests have levels of significance that tend to be close to the nominal level. They tend to have somewhat lower power, however. Power for the bootstrap tests can be slightly lower than that for LR2, even when the level of significance is the same or lower for LR2. Increasing the number of bootstrap iterations might improve the power of these tests.

When sample size is large (*n* = 100), there is little difference among any of the LAV based tests. These tests still improve on the LS t-test even in large samples when disturbances are asymmetric.

All Tabled results and References are available upon request.