CONTROL LIMITS FOR P CONTROL CHARTS WITH SMALL SUBGROUP SIZES

Marilyn K. Hart, College of Business, University of Wisconsin Oshkosh, 800 Algoma Blvd., Oshkosh, WI 54901, 920-424-7195, hart@uwosh.edu

ABSTRACT

When there are small subgroups, the normal approximation to the binomial no longer applies, so the control limits on a Shewhart p chart calculated in the normal way are no longer valid. Furthermore, when there are other than the typical 24 or 25 subgroups, 3-sigma control limits may not apply. This paper discusses how to handle small subgroups on a p chart for any number of subgroups.

P CHART AND THE USE OF THE EXACT BINOMIAL PROBABILITY DISTRIBUTION

The control limits on the p chart are based upon the normal approximation to the binomial. The centerline for the p chart is: $\bar{p}$ = (total number of occurrences)/(number of units studied). With n as the subgroup size, the upper (UCL) and the lower (LCL) control limits are

$$\text{UCL(p)} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \text{ and LCL(p)} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \quad (1)$$

It is often recommend that n be at least 5/pBar for the normal approximation to the binomial to apply. For small subgroups, probability limits, i.e., the exact binomial distribution, can be used [2, 3]. The 3-sigma upper and lower control limits would be calculated by first finding the smallest value of X (X_U) and the largest value of X (X_L) satisfying the following two inequalities:

$$\sum_{X=X_U}^{n} \binom{n}{X} p^X (1-p)^{n-X} \le 0.00135 \quad (2)$$

$$\sum_{X=0}^{X_L} \binom{n}{X} p^X (1-p)^{n-X} \le 0.00135 \quad (3)$$

The upper control limit and lower control limit on the proportion are then

$$UCL(p) = \frac{X_U}{n} \text{ and } LCL(p) = \frac{X_L}{n} \quad (4)$$

However, 3-sigma limits do not maintain a satisfactory alpha-risk when the number of subgroups differs greatly from 24. Building on the work of Ott [4] with Analysis of means, studies done by Hart and Hart [1] have simplified the methodology by the use of "T-sigma" limits, which keep the total alpha-risk to between 0.05 and 0.09: Use 1.5-sigma limits (.0668 for eqs. 1 & 2) for 2 plotted points, 2-sigma (.0228) for 3-4; 2.5-sigma for 5-9 (.0062), and 3-sigma (.00135) for 10-34.

REFERENCES

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