USING TRISCANNING ALGORITHM FOR SOLVING THE TSP

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ABSTRACT

TSP is a classic combinatorial optimization problem applied in computer science, management science and operating research. The symmetric TSP consists of (n-1)!/2 combination, so it is time consuming to be solved by using the typical mathematic methods. So, many related algorithms are used to solve TSP. In this study a novel heuristic algorithm, Triscanning algorithm, is constructed which includes convex hull, geometry structure, maximum the ratio of the perimeter and triangle area to solve the TSP. Triscanning algorithm was compared with other algorithms by a 20 nodes networks. The results showed that the proposed algorithm eliminates massively unnecessary combinations and gives the best solution.
Keywords: *Convex hull; Triscanning algorithm; Euclidean plane; Traveling salesman problem*

INTRODUCTION & LITERATURE REVIEW

The traveling salesman problem (TSP) is used to find the minimum tour length that passes through every vertex exactly once. TSP is also known as an NP-hard and combinatory problem. Convex hull has been proven to be a very useful shape in dealing with many computational problems, such as automated feature recognition. Many researchers study geometrical properties such as dalaunay triangulation to solve combinatory problems [1]. The convex hull of a set of points S in a plane is the enclosing polygon with the smallest area and perimeter [2]. Flood [3] concludes that every Euclidean TSP has an optimal solution if every city lies on the boundary of the convex hull. However, for a generic TSP, the convex hull subtour needs to be combined with a heuristic search to create an insertion procedure for the points which are not on the convex hull boundary. The heuristic search may be formulated in many ways. In this paper, the novel algorithm is proposed, triangular algorithm, which uses the Graham scan algorithm [6] to initially establish a convex hull for a given set of points. For the points which are not on the convex hull boundary, the ratio of the perimeter and area of triangle and the dynamic scanning circle are applied to find the minimum path length in TSP which will be also demonstrated in this study.

METHOD

In this study a novel heuristic algorithm, Triscanning algorithm, is constructed which includes convex hull, geometry structure, maximum ratio of the perimeter and area of triangle to solve the TSP. The convex hull boundary is used for the given set of points as its initial subtour. Then, a local search heuristic is applied successively until all the given points are included in the path. Hence, the relation between each point inside the convex hull boundary and the convex edges can be established without a combinatorial search. The procedure of proposed method is introduced as follows.

1. **Graham scan algorithm:** It is used to examine three points at a time and check if the middle point can be eliminated as a possible convex hull point.

2. **Non-intersecting path:** In the Euclidean plane, the minimal TSP tour should not be intersected.
3. **Procedure for creating neighborhoods:** The proposed algorithm creates neighborhoods by clustering groups of point within regular triangle which provided by the Graham scan algorithm as an edge of the convex hull. Based on the Isoperimetric Theorem, regular triangle is chosen.
4. **Triangle family heuristic:** Some points may be located in more than one neighborhood which does not satisfy the problem definition of each point being visited only once. To solve this problem, the sweeping algorithm (Gilett, B. & Miller, L. in 1974) is applied.
5. **Neighborhood optimization heuristic:** Some points are not included in any neighborhood which also does not satisfy the problem definition. To solve this problem, a dynamic scan circle base on the shortest distance between the points which not yet included the path and the point which had included in the neighborhood is proposed in this work.

NUMERICAL EXAMPLE

Consider the n = 20 points whose coordinates are given in Table 1.

Table 1: Coordinates of points.

Point	Coordinate	Point	Coordinate
1	(10,0)	11	(0,25)
2	(25,0)	12	(0,10)
3	(45,0)	13	(10,5)
4	(55,15)	14	(45,10)
5	(55,25)	15	(45,15)
6	(55,40)	16	(25,45)
7	(55,50)	17	(25,35)
8	(45,50)	18	(30,30)
9	(20,50)	19	(35,25)
10	(10,45)	20	(40,40)

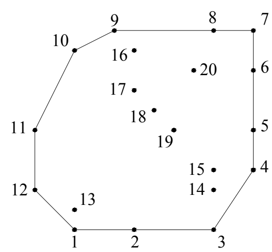

Fig. 1. Initialize a convex hull subtour

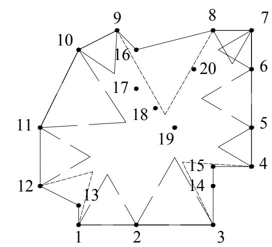

Fig. 2. Set up neighborhoods for insertion

Step 1: 8 points inside the convex hull are 13, 14, 15, 16, 17, 18, 19 and 20. The initial subtour is in Fig. 1.
Step 2: In Fig. 2, point 13 is assigned to the boundary point 12 and 1, point 14 and 15 are assign to point 3 and 4, and point 16 is assign to point 9 and 8 of the convex hull because they are the closest.
Step 3: In the Fig. 3, points 17, 18 and 20 is included into the same dynamic scan circle. Points 13 and 14 are overlapping between two dynamic scan circles. Then swap and compare them between two dynamic scan circles in order to obtain the optimal path result.
Step 4: Because point 19 is not included, the radius of dynamic scan circle is increased to reach it (Fig. 4). Since all the points are included, an optimal tour is found and the tour length is 227.93 (Fig. 5).

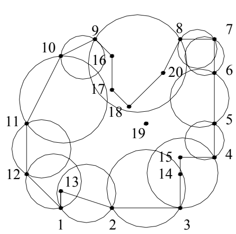

Fig. 3. Set up dynamic scan circles

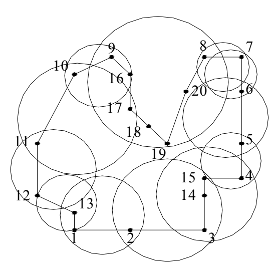

Fig. 4. Increased dynamic circle

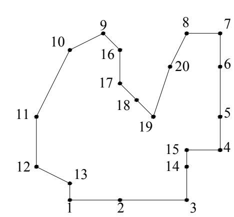

Fig. 5. Optimal solution

CONCLUSION

In this paper, Triscanning algorithm is proposed for solving the TSP to use convex hull boundary as the initial tour. Then, create a neighborhood with an equilateral triangle to be optimum inclusion strategy. Besides, the dynamic scan circle includes all the points into the convex hull and groups the neighborhood on the convex hull edge to find the global optimal solution, instead of designing a complex global linking system to deal with interdependency of the sub problems. The convex hull is accomplished by Graham scan algorithm which has a time complexity of O (n log n). The optimization phase consists of set up the convex hull neighborhood for insertion and creates a dynamic scan circle to inclusion all the other points which can be achieved in a single loop procedure. The order of the points is then optimized with nearest neighborhood method in $O(n^2)$ time. Combining the above stages results in the total cost of $O(n^2)$ which better than that of most other algorithm as compare with Table 2. Table 2 shows the order of magnitude for a few popular algorithms to solve the optimum path finding problem [9][10][11].

Table 2. Comparison of complexity of algorithm

Dynamic programming	$O(n^2 2^n)$	Nearest neighbor algorithm	$O(n^2)$	2-Opt local search heuristic	$\Theta(n^{n/2})$
Greedy algorithm	$O(n^2 logn)$	Farthest addition heuristic	$O(n^2 logn)$	Tabu search	$\Theta(n^6)$
Lin Kernighan heuristic	$O(n^{2.4})$	Christofides algorithm	$O(n^{2.5})$	Hopfield Tank neural network	$\Theta(n^4)$

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