# INVENTORY MANAGEMENT AND NEGOTIATION STRATEGIES IN A MANUFACTURER-RETAILER SUPPLY CHAIN 

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#### Abstract

This paper develops a framework that integrates inventory control with constant demand and the economic relationship between consumer demand and retail price. Within this framework, the impact of order quantity, wholesale price and retail price on the behavior of both the manufacturer and the retailer is investigated. Furthermore, this paper explores the issues and conclusions that result from coordinating the relationship between the manufacturer and the retailer.


## INTRODUCTION

This paper addresses the issues and problems of channel coordination in a manufacturer-retailer supply chain where the retailer is in a monopolistic position for the product, i.e., the ultimate consumer demand is a function of the retail price, and operating costs depend on both order quantities and retail price [1].

We start our analysis in the next section by delineating the assumed relationships and decision variables of the manufacturer and the retailer, or a group of homogenous retailers. The retailer's inventory policy is assumed to the widely used Economic Order Quantity (EOQ) model. The manufacturer's decision variable is the wholesale price to charge the retailer and the retailer's decision variables are the retail price and the order quantity.

The case where the manufacturer is the leader and the retailer is the follower is discussed immediately after assumptions. The manufacturer first declares the wholesale price, the retailer, under the Economic Order Quantity, then decides on the retail price. The unique equilibrium point is obtained [2] [4] [7].

We then address supply chain coordination. We show that if both the manufacturer and the retailer employ only the supply chain EOQ order quantity in their coordination, the order quantity, the manufacturer's annual profit and the supply chain's annual profit are higher, while the retailer's annual profit is lower than those at non-coordination. We also show that the coordinated retail price and wholesale price are lower, the coordinated manufacturer, retailer, and supply chain annual profits are higher than those at non-coordination [3] [5] [6].

## MODEL DEVELOPMENT

The manufacturer's wholesale price is $\omega$ and the retailer's retail price is $p$. It is reasonable to assume $\pi \geq \omega$. In many industries, the retail price does not exceed a certain percentage of the wholesale price. Therefore, we assume $\pi \leq k \omega$ where $k$ is a constant with $k>1$. We also assume that there exists a cap, g , for the manufacturer's wholesale price, i.e., $\omega \leq g$. The downward sloping demand function at the retail level is assumed to be $D(p)=\tilde{\alpha} p^{-\beta}$ with $\tilde{\alpha}>0$ and $\beta>0$, where $\tilde{\alpha}$ and $\beta$ are constants and $\beta$ is the price elasticity of demand $(0<\beta<1)$. Let $S_{\mathrm{r}}$ and $S_{\mathrm{m}}$ be the retailer's ordering cost per order and the
manufacturer's setup cost per setup, respectively. The retailer's annual inventory holding cost is $H_{r}$. The retailer's order size is $Q$.

The manufacturer's annual profit is equal to gross revenue minus the production setup cost. Therefore, the manufacturer's annual profit function is given by

$$
\begin{equation*}
\pi_{m}(\omega, p, Q)=\omega D(p)-S_{m} D(p) / Q \tag{1}
\end{equation*}
$$

The manufacturer's decision variable is the wholesale price, $\omega$.
Similarly, the retailer's average annual profit is equal to gross revenue minus the ordering cost and inventory holding cost. Then its functional form is given by

$$
\begin{equation*}
\pi_{r}(\omega, p, Q)=(p-\omega) D(\pi)-S_{r} D(p) Q-Q H_{r} / 2 \tag{2}
\end{equation*}
$$

The retailer's decision variables are the retail price, $p$, and the size of order quantity, $Q$.

## TWO-STAGE NON-COORDINATION GAME MODEL

The manufacturer, as the leader, first declares the wholesale price, $\omega$. The retailer then decides on the retail price, $p$. To determine the equilibrium of the two-stage game, we first solve for the reaction function in the second stage of the game.

For any given wholesale price, $\omega$, the retailer's objective is to choose the retail price, $p$, that maximizes his/her annual profit in (2) under the constraint that $k \omega \geq p \geq \omega$. Since $\partial \pi_{r}\left(\omega, p, Q_{r}\right) / \partial p>0, \pi_{r}(\omega, p$, $Q_{r}$ ) is a strictly increasing function, $p=k \omega$ is the optimal retail price for the retailer.

The optimal wholesale price, $\omega$, is determined at the first stage by maximizing the manufacturer's annual profit. Substituting $p=k \omega$ into $D(p)$ and $Q_{r}$, the manufacturer's annual profit can be rewritten as

$$
\begin{equation*}
\pi_{m}\left(\omega, Q_{r}\right)=\alpha \omega^{1-\beta}-S_{m}\left[\alpha H_{r} /\left(2 S_{r}\right)\right]^{1 / 2} \omega^{-\beta / 2} \tag{3}
\end{equation*}
$$

where $\alpha=\tilde{\alpha} k^{-\beta}$. Therefore, the manufacturer's problem is to maximize $\pi_{m}\left(\omega, Q_{r}\right)$ in (3) subject to the wholesale price cap constraint, i.e., $\omega \leq g$. Since $\mathrm{d} \pi_{m}\left(\omega, Q_{r}\right) / \mathrm{d} \omega>0, \pi_{m}\left(\omega, Q_{r}\right)$ is a strictly increasing function of $\omega$. Therefore, the manufacturer's optimal wholesale price is $\omega^{*}=g$, the retailer's optimal retail price is $p^{*}=k g$ and the retailer's optimal order quantity is $Q_{r}^{*}=\left[2 \alpha S_{r} g^{-\beta /} / H_{r}\right]^{1 / 2}$.

## COORDINATION GAME MODEL

In this section, we consider the situation in which both the manufacturer and the retailer are willing to coordinate to maximize their supply chain profit. The supply chain profit function is defined as the sum of the manufacturer's and the retailer's profits:

$$
\begin{equation*}
\pi_{s}(\omega, p, Q)=\pi_{m}(\omega, p, Q)+\pi_{r}(\omega, p, Q), \tag{4}
\end{equation*}
$$

i.e.,

$$
\begin{equation*}
\pi_{s}(p, Q)=p D(p)-\left(S_{r}+S_{m}\right) D(p) / Q-Q H_{r} / 2 \tag{5}
\end{equation*}
$$

For any given retail price, p, it is easy to show that the supply chain ordering, setup and inventory holding cost is minimized by the following supply chain EOQ formula:

$$
\begin{equation*}
Q_{s}=\left[2\left(S_{r}+S_{m}\right) D(p) / H_{r}\right]^{1 / 2} . \tag{6}
\end{equation*}
$$

Theorem 1. For the given retailer's non-coordinated retail price, $p^{*}=k g$, and manufacturer's noncoordinated wholesale price, $\omega^{*}=g$, the relationships between $Q_{s}\left(p^{*}\right)$ and $Q_{r}\left(p^{*}\right)$, between $\pi_{m}\left(\omega^{*}, p^{*}\right.$, $\left.Q_{s}\left(p^{*}\right)\right)$ and $\pi_{m}\left(\omega^{*}, p^{*}, Q_{r}\left(p^{*}\right)\right.$, between $\pi_{r}\left(\omega^{*}, p^{*}, Q_{s}\left(p^{*}\right)\right)$ and $\pi_{r}\left(\omega^{*}, p^{*}, Q_{r}\left(p^{*}\right)\right)$, and between $\pi_{s}\left(p^{*}\right.$, $\left.Q_{s}\left(p^{*}\right)\right)$ and $\pi_{s}\left(p^{*}, Q_{r}\left(p^{*}\right)\right)$ are as follows:

$$
\begin{align*}
& \quad Q_{s}\left(p^{*}\right)>Q_{r}\left(p^{*}\right)  \tag{7}\\
& \pi_{m}\left(\omega^{*}, p^{*}, Q_{s}\left(p^{*}\right)\right)>\pi_{m}\left(\omega^{*}, p^{*}, Q_{r}\left(p^{*}\right)\right)  \tag{8}\\
& \pi_{r}\left(\omega^{*}, p^{*}, Q_{s}\left(p^{*}\right)\right)<\pi_{r}\left(\omega^{*}, p^{*}, Q_{r}\left(p^{*}\right)\right)  \tag{9}\\
& \pi_{s}\left(p^{*}, Q_{s}\left(p^{*}\right)\right)>\pi_{s}\left(p^{*}, Q_{r}\left(p^{*}\right)\right) \tag{10}
\end{align*}
$$

Theorem 2. For any $\omega$ satisfying $\omega_{\max } \geq \omega \geq \omega_{\min }$ and $p=k \omega,(\omega, p)$ is a feasible solution and

$$
\begin{equation*}
\pi_{s}\left(p, Q_{s}(p)\right)>\pi_{m}^{*}+\pi_{r}^{*} \tag{11}
\end{equation*}
$$

Furthermore, we have

$$
\begin{equation*}
\pi_{s}^{* *}>\pi_{s}^{*}, \pi_{m}^{* *}>\pi_{m}^{*}, \pi_{r}^{* *}=\pi_{r}^{*} . \tag{12}
\end{equation*}
$$

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