# OPTIMAL WARRANTY POLICIES WITH PREVENTIVE MAINTENANCE FOR REPAIRABLE PRODUCT

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## ABSTRACT

In this study, we deal with a model to study the effect of PM carried out by both parties on a repairable product sold under a non-renewing, free-repaired warranty policy. During the warranty period, the seller will repair the deteriorated but repairable product and charges no fee to the buyer. After the warranty period, the deteriorated product is also repaired by the seller but the buyer will be charged the cost of repair. Under such a framework, cost analysis of four PM options is analyzed from both the seller's perspective and the buyer's.

#### INTRODUCTION

Warranty is a key promotional tool for the seller since it has become an essentially competitive strategy employed by sellers to boost their market share, profitability and corporate image. To the seller, offering a warranty implies additional costs including preventive maintenance (PM), repair and replacement that depend on the transaction contract and product characteristics. From the buyer's perspective, however, PM can either lengthen the lifetime of product or effectively reduce its failure rate. Well performing PM during the warranty period may provide the consumer better product service in the post-warranty period and reduce the cost of repair the deteriorated product. Several papers integrate warranty and PM [3]. Yeh and Lo [4] investigated PM warranty policies for repairable products. A more realistic approach can refer to Djamaludin et al. [1] and Kim et al. [2]. Here we deal with a framework focusing on an repairable product sold with warranty and then study the PM policies in this paper.

### MODEL FRAMEWORK AND FORMULATION

All malfunctions of a product in the warranty period [0,W) are repairable by the seller with a free warranty policy and the charges no fee to the buyer. The lifetime or life expectancy of each product is *L*. In the post-warranty period [W,L), the cost of repairing a deteriorated product is borne by the buyer. In the post-warranty period [W,L), the cost of PM is borne by the buyer according to a similar reason. Let  $r_m(t)$  be the failure rate function of the product under level *m* of PM. Let *m*1 be the level of PM in the warranty period [0,W) and *m*2 be the level of PM in the post-warranty period [W,L).

Option A: In this case, there is no PM; then m1 = m2 = 0. The failure rate function  $r_m(t) = r_0(t)$  is over the whole period of the product life,  $0 \le t < L$ , and the repair cost of the product to the seller is given by

$$C_{AM} = C_R \int_0^w r_0(t) dt .$$
<sup>(1)</sup>

The repair cost of product of the buyer can be also expressed as

$$C_{AB} = C_R \int_W^L r_0(t) dt \,. \tag{2}$$

Option B: In this case, the failure rate function is

$$r_{m}(t) = \begin{cases} r_{m1}(t) &, \quad 0 \le t < W \\ r_{m2}(t) - r_{m2}(W) + r_{m1}(W) &, \quad W \le t < L \end{cases}$$
(3)

The expected total cost of seller  $C_{BM}$  is

$$C_{BM} = C_R \int_0^W r_{m1}(t) dt + C_{m1} \cdot W , \qquad (4)$$

where  $C_{m1}$  denotes cost of PM per unit time under fixed level *m*1.From the buyer's perspective, the expected total cost of buyer  $C_{BB}$  also includes the cost of repair and PM

$$C_{BB} = C_R \int_W^L (r_{m2}(t) - r_{m2}(W) + r_{m1}(W)) dt + C_{m2} \cdot (L - W), \qquad (5)$$

where  $C_{m^2}$  denotes the cost of PM per unit time under fixed level  $m^2$ .

Option C: In this case, the failure rate function is

$$r_{m}(t) = \begin{cases} r_{0}(t) & , \ 0 \le t < W \\ r_{m2}(t) - r_{m2}(W) + r_{0}(W) & , \ W \le t < L \end{cases}$$
(6)

The expected total cost of seller is

$$C_{CM} = C_{AM} = C_R \int_0^W r_0(t) dt \,.$$
<sup>(7)</sup>

The expected total cost to the buyer is

$$C_{CB} = C_R \int_{W}^{L} (r_0(W) - r_{m2}(W) + r_{m2}(t))dt + C_{m2} \cdot (L - W) .$$
(8)

Option D: In this case, the failure rate function is

$$r_{m}(t) = \begin{cases} r_{m1}(t) &, \ 0 \le t < W \\ r_{0}(t) - r_{0}(W) + r_{m1}(W) &, \ W \le t < L \end{cases}$$
(9)

The expected total cost of seller  $C_{DM}$  is

$$C_{DM} = C_{BM} = C_R \int_0^W r_{m1}(t) dt + C_{m1} \cdot W .$$
(10)

The expected total cost to the buyer is

$$C_{DB} = C_R \int_W^L (r_0(t) - r_0(W) + r_{m1}(W)) dt .$$
(11)

## DISCUSSION

From a cost analysis under the seller's perspective, option A is equivalent to option C and option B is

equivalent to option D. Let ENR  $\frac{r^2}{d}(r)$  be the expected number of repair per unit time, with failure rate

$$r = r(t)$$
 over period  $(t1, t2]$ . We have  $ENR_{t1}^{t2}(r) = \frac{\int_{1}^{2} r(t)dt}{(t2-t1)}$ .

Lemma 1: Options B and D are better than Options A and C during the warranty period if and only if  $ENR_{0}^{W}(r_{0}) - ENR_{0}^{W}(r_{m1}) > \frac{C_{m1}}{C_{R}}$ . The term  $ENR_{0}^{W}(r_{0}) - ENR_{0}^{W}(r_{m1})$  can be explained as the difference between

the expected number of repairs with and without PM during warranty period. The right side of lemma 1's condition is the ratio of cost between performing PM and repair.

Lemma 2: Option A is better than option C and option D is better than option B if

 $ENR_{W}^{L}(r_{0}) - ENR_{W}^{L}(r_{m2}) < \frac{C_{m2}}{C_{R}}$ . The term  $ENR_{W}^{L}(r_{0}) - ENR_{W}^{L}(r_{m2})$  can be explained as the effect of performing

PM or not, under the post-warranty period. The right side of lemma 2's condition can also be explained as the cost ratio performing PM and repair.

Lemma 3: Option B is always better than option C from the buyer's perspective. It is reasonable for the buyer that one would not pay any charge for performing PM during the warranty period.

## CONCLUSION

In this paper, we construct a framework of four possible options with PM. The goal of this research is to compare the options and reconfirm the importance of carrying out PM. We formulate various scenarios for theoretical analysis and obtain decision-making criteria. The expected number of repair per unit time (*ENR*) and the ratio between the cost of PM rates ( $C_{m1}$  and  $C_{m2}$ ) and the cost of unit repair are the essential factors for decision-making. Besides, PM is also shown to reduce the expected total cost.

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