# ESTIMATION OF REMAINING PAVEMENT LIFE WITH AN ABSORBING MARKOV CHAIN

*Peter M. Ellis, Dept. of Business Administration, Utah State University, Logan, UT, 84322-3510, 435-797-2372, peter.ellis@usu.edu*
*W. Spencer Guthrie, Dept. of Civil and Environmental Engineering, Brigham Young University, Provo, UT, 801-422-3864, Guthrie@byu.edu*

## ABSTRACT

The interesting work by Tack and Chou utilizes Markov chain analysis in the process of pavement deterioration. They developed a Markov chain of successive states of road deterioration using data from a road system in Ohio. This work references that same Markov transition probability matrix **P** to demonstrate the utility of an absorbing Markov chain in estimating remaining pavement life. **P** is modified in response to the realistic state transition expectation that pavement condition will either remain in the same state or deteriorate to states of poorer condition over successive time periods.

## INTRODUCTION

Pavement condition is often categorized into discrete condition states, and pavement deterioration models are used to predict changes in condition over time [*2*]. Transition probabilities can be calculated to quantify the likelihood that the condition of a pavement will change from one state to another over a given time period. This probabilistic prediction approach ideally accounts for uncertainties arising from unobserved explanatory variables, the presence of measurement errors, and the stochasticity of the deterioration process itself [*2*]. The state transition probabilities presuppose that the road receives regular maintenance but has not been restored with repairs that would either extend the time for remaining in the given state or move the condition back to a higher state.

### The Absorbing Markov Chain

This work references the Markov transition probability matrix **P** developed by Tack and Chou [6] to demonstrate the utility of an absorbing Markov chain in estimating remaining pavement life. In particular, the requirement that pavement condition monotonically decrease with increasing pavement age is emphasized [*7, 8, 9*]. An overview of Markov chain analysis is provided first, followed by a comparison of remaining life estimations based on original and modified versions of the above-mentioned stochastic matrix **P**. In the modified version, the condition of the pavement is prohibited from spontaneously improving over time.

The Markov chain is represented with $\mathbf{P} = (p_{ij})$. The one of interest here was obtained by Tack and Chou (6) and is shown in Table 1. They obtained this matrix from a set of consecutive-year pavement condition ratings (PCRs) in the state of Ohio, which were determined from visual inspections of pavement condition by trained raters. The pavement condition states in increasing order from 1 to 10 correspond to PCR ranges with endpoints of 100 to 95, 95 to 90, 90 to 85, 85 to 80, 80 to 75, 75 to 70, 70 to 60, 60 to 50, 50 to 40, and 40 to 0. A PCR value of 100 represents a perfect pavement, while a failed pavement tends to have PCR values near 40 [*6*]. The general relationship between PCR and pavement condition is displayed in Figure 1 [*10*].

The type of Markov chain illustrated in Table 2 is called an absorbing Markov chain. Such a Markov process has states that, once entered, cannot be escaped. The transition probability matrix **P** will be ordered so that it has the following form:

$$\mathbf{P} = \begin{matrix} \{Q: & R\} \\ \{0: & I\} \end{matrix}$$

Q is the submatrix of transitions among the non-absorbing states. If the system is initially in a non-absorbing state, it will eventually fall into an absorbing state, but before that happens it might experience any number of transitions among the several nonabsorbing states. R is the submatrix of direct transitions to any of the absorbing states from any of the non-absorbing states over a single time period. The 0 submatrix is needed because the system has zero probability of making a transition back to a non-absorbing state after having fallen into an absorbing state. The I submatrix is an identity matrix because once an absorbing state has been reached, the system will forever after remain in that same state (*11*). The absorbing Markov chain analysis calculates the expected number of time periods during which the system will continue to exist in nonabsorbing states prior to eventually falling into some absorbing state. This is most helpful in road studies because it permits estimating the lifetime of a road before it falls into a state of sufficient disrepair that it is no longer useable.

The probability that the system will fall from nonabsorbing state i to absorbing state j at transition n is given as $Q^{n-1}R$. Therefore, the probability of eventual transition from any nonabsorbing state i to absorbing state j is $\mathbf{N} = R + QR + Q^2R + \ldots = (I - Q)^{-1}R = FR$, where $F = (I - Q)^{-1}$.

The probability matrix of eventual transition from any non-absorbing state to any absorbing state is $\mathbf{N} = \mathbf{FR}$. In the case of road deterioration, there is just one absorbing state, that being state 10. Therefore, the probabilities of eventual transition to an absorbing state yields $\mathbf{N} = (1,1, \ldots ,1)^T$.

The **P** matrix used here is from the Ohio road study by Tack and Chou [6]. They developed the transition probabilities from annual condition observations of a road section in Ohio. It is presented in table 1.

The absorbing Markov chain analysis was carried out with the modified **P** matrix of table 2. The fundamental matrix **F** with its row totals is shown in table 3. The amounts shown in the fundamental matrix **F** are the expected number of time periods that the system will be in nonabsorbing state j, given that the system was initially in nonabsorbing state i. Thus, the row totals of **F** are the expected number of time periods until absorption, given that the system was initially in nonabsorbing state i.

Table 3 shows that the expected lifetime of a road in new condition is 34.93 years. This expected road life is quite compatible with standard design expectations. The table also shows that the expected lifetimes until absorption do not necessarily change linearly with initial pavement condition. These expected values are set according to the original $p_{ij}$ values, so no particular road life pattern necessarily emerges. For roads in nearly new condition, the expected remaining life persists at about 30 years, while for roads in state 9 the expected remaining life plummets to just 6.25 years.

State 10 is a road condition rating that describes a road that has become totally impassable. This state is clearly unacceptable on modern roads. In fact, it is thought that road conditions of states 5, 6 or 7 might be the lowest ones tolerated by the general driving public. Therefore, the fundamental matrix **F** was obtained from the revised **P** matrix by successively fixing the terminal state to be either 5, 6 or 7. The results are displayed in table 4. If a road is initially in state 1 (new condition) and the terminal condition is declared to be state 5, then the expected total road life is 6.0 years. If the terminal state is declared to be state 6 then the expected road life is 8.0 years. Finally, if the road is permitted to fall to state 7 before being declared unusable, and the initial state is 1, then the expected road life is 9.7 years.

**Conclusion**

The results of the absorbing Markov chain analysis show the expected remaining lifetime of a pavement that is treated only with usual regular maintenance and not repaired to the extent that road condition improves to a higher state. Interested readers may request the complete paper, which contains all the references and tables.

## REFERENCES


[1] Brecher, A. Infrastructure: A National Priority. *SWE*, Vol. 41, No. 6, 1995, pp. 14-15.

[2] Madanat, S. and W. H. W. Ibrahim. Poisson Regression Models of Infrastructure Transition Probabilities. *Journal of Transportation Engineering*, Vol. 121, No. 3, May/June 1995, pp. 267-272.

[3] Vepa, T. S., K. P. George, and A. R. Shekharan. Prediction of Pavement Remaining Life. In *Transportation Research Record 1524,* TRB, National Research Council, Washington, D.C., 1996, pp. 137-144.

[4] Li, N, W.-C. Xie, and R. Haas. Reliability-Based Processing of Markov Chains for Modeling Pavement Network Deterioration. In *Transportation Research Record 1524,* TRB, National Research Council, Washington, D.C., 1996, pp. 203-213.

[5] Butt, A. A., M. Y. Shahin, K. J. Feighan, and S. H. Carpenter. Pavement Performance Prediction Model Using the Markov Process. In *Transportation Research Record 1123,* TRB, National Research Council, Washington, D.C., 1987, pp. 12-19.

[6] Tack, J. N. and Y. J. Chou. Pavement Performance Analysis Applying Probabilistic Deterioration Methods. In *Transportation Research Record 1769,* TRB, National Research Council, Washington, D.C., 2001, pp. 20-27.

[7] Hawk, H. BRIDGIT Deterioration Models. In *Transportation Research Record 1490*, TRB, National Research Council, Washington, D.C., 1995, pp. 19-22.

[8] Jiang, Y., M. Saito, and K. C. Sinha. Bridge Performance Prediction Model Using the Markov Chain. In *Transportation Research Record 1180,* TRB, National Research Council, Washington, D.C., 1988, pp. 25-32.

[9] Shahin, M. Y., M. M. Nunez, M. R. Broten, S. H. Carpenter, and A. Sameh. New Techniques for Modeling Pavement Deterioration. In *Transportation Research Record 1123*, TRB, National Research Council, Washington, D.C., 1987, pp. 40-46.

[10] Pavement Condition Rating Procedures. Ohio Department of Transportation. http://www.dot.state.oh.us/pavement/Pubs/PCR%20Procedure.pdf. Accessed July 31, 2003.