INTEGRATING CUSTOMER PREFERENCES INTO TOLERANCE SPECIFICATION PROCEDURES USING A MULTICRITERIA DECISION FRAMEWORK

Srikrishna M. Govindaluri, School of Business, Ramapo College of New Jersey, 505 Ramapo Valley Rd., Mahwah, NJ 07430, USA, (201)684-7383, sgovinda@ramapo.edu
Thierry Rakotobe-Joel, School of Business, Ramapo College of New Jersey, 505 Ramapo Valley Rd., Mahwah, NJ 07430, USA, (201)-684-7096, trakotob@ramapo.edu

ABSTRACT

We propose a multicriteria optimization framework to integrate customer preferences into tolerances specification procedures. The usual approach to specification of tolerances is to use a quality loss function approach for trading off costs and quality associated with tolerance specification decision. Despite its popularity, the loss function approach in specifying tolerances often fails to incorporate the qualitative preferences of the customer. As a consequence, resources expended on capturing the customer preferences such as customer attributes and product characteristics in the preliminary phases of design are barely utilized in the majority of the current tolerance determination models. The proposed model assigns customer-perceived relative importance weights to individual quality characteristics and then incorporates these weights to synthesize Pareto-optimal solutions, which will then represent optimal tolerances for the product.

INTRODUCTION

In a typical screening inspection, a product is either reworked or scrapped when a quality characteristic (*QC*) of interest falls outside the interval between upper and lower limits used for screening inspection. These limits are referred to as upper and lower screening limits (*USL* and *LSL*) and the interval between the limits is referred to as the tolerance. The choice of screening limits or tolerance is a primary determinant of the effectiveness of a screening inspection. Implementing tight tolerance may provide higher outgoing quality; however, the rejection costs can be excessive. On the other hand, a loose tolerance results in a lower outgoing quality and a lower rejection cost since lesser products are reworked or scrapped. A number of research works in the literature have considered the tradeoff between rejection costs and quality for specifying an optimal tolerance. A simple method to perform this tradeoff is quantifying quality in monetary terms using a Taguchi or an empirical loss function so that quality and rejection costs can be summed to yield a single objective optimization model [1-6]. The tolerance specified can also have an effect on the manufacturing cost in addition to quality and rejection costs. Along these lines, Shin and Govindaluri [7] propose a closed-form solution for optimal tolerances that accounts for manufacturing cost as well.

Despite the extensive work conducted in the area of tolerance optimization, the incorporation of customer preferences in this problem has often been overlooked. In this paper, we propose a multicriteria optimization framework to integrate customer preferences into tolerance optimization. Unlike the majority of earlier models in the literature, the model proposed within this framework not only considers quality and costs but also incorporates customer satisfaction as an optimization criterion. An overview of the multicriteria framework is presented in the third section.

RESEARCH MOTIVATION

As discussed in introduction, the majority of models consider a tradeoff between quality and costs by using a quality loss function to quantify effects of good or poor quality in monetary terms. Quality loss is a function of standard deviation and the costs incurred in repairing or replacing a defective product. Thus the tightness of a tolerance according to quality loss function based models in the literature is based on minimization of monetary measures: quality loss, rejection costs, and manufacturing cost. However, a decision based merely on a monetary criterion may not be the best decision. For example, the policy of minimizing quality loss and costs does not eliminate the possibility where a *QC* considered highly important by the customer is assigned a loose tolerance. Hence, it becomes necessary to find a systematic method to integrate customer preferences into tolerance optimization. The integration will ensure that QCs having a higher value of customer-perceived relative importance have a relatively tighter tolerance. Moreover, it also allows utilization of the customer preference information captured by expending much time and effort in the preliminary phases of design.

OVERVIEW OF THE PROPOSED MULTICRITERIA FRAMEWORK

The overall procedure involves developing mathematical models to establish relationships between tolerance or screening limits and expected total cost, $E[T]$ (i.e., sum of expected rejection and manufacturing costs and expected quality loss) and between tolerance and customer satisfaction due to individual *QC*s. Customer satisfaction due to each *QC* is measured in terms of the expected deviation $E[D_i]$ from target value y_{0i} for QC_i discussed in the fourth section. Next, separate single objective optimization models are employed to determine the individual minima for each of the objectives $E[T]$ and $E[D_1]$, $E[D_2]$, …, and $E[D_n]$ in a nonsimultaneous fashion. Finally, a multicriteria optimization model is employed to determine the Pareto-optimal solution using a weighted-Tchebycheff norm that simultaneously minimizes expected total cost and maximizes customer satisfaction for all individual *QC*s. This method is superior to the simplistic weighted sum and goal programming approaches that may fail to explore the full set of Pareto points for a multiobjective optimization problem. Achieving the best compromise between the multiple criteria $E[T]$, $E[D_1]$, $E[D_1]$, …, $E[D_n]$ when determining Pareto-optimal solutions requires tradeoff decisions among multiple objectives. For this reason, the determination of customer-perceived relative importance of QC_1, QC_2, …, QC_n, and $E[T]$ is included in the framework.

MODELING OF COSTS AND QUALITY LOSS IN TERMS OF TOLERANCE

In this section, the expressions for rejection costs, manufacturing cost, quality loss, and expected deviation in terms of screening limits are developed.

Empirical Quality Loss Function

When the historical data concerning costs associated with quality losses are available, an empirical relationship between the quality loss and the quality characteristic QC_i represented by random variable y_i can be developed using regression analysis. The linear regression form of the empirical loss function $L(y_i)$ for the QC_i is given by [6][8][9]

$$L(y_i) = b_{0i} + b_{1i} y_i + b_{2i} y_i^2 + \cdots, \tag{1}$$

where the b_i's are unknown parameters estimated using sample data. If $f(y_i)$ is the probability density function (p.d.f) for the random variable Y_i, the expected loss under a 100% inspection scheme can be evaluated as [9]

$$E[L(y_i)] = \int_{LSL_i}^{USL_i} L(y_i)\, f(y_i) dy_i \ . \tag{2}$$

Rejection Costs

Suppose C_{Si} is the scrap cost incurred when a product falls below the *LSL*, and C_{Ri} the rework cost when the product falls above the *USL,* then expected rejection cost $E[R]$ incurred due to products that do not meet the screening limits is defined by

$$E[R] = \sum_{i=1}^{n} C_{Si} \int_{0}^{\mu_i - \delta_{1i}\sigma_i} f(y_i) dy_i + C_{Ri} \int_{\mu_i + \delta_{2i}\sigma_i}^{\infty} f(y_i) dy_i \ . \tag{3}$$

If Y is expressed in terms of μ_i and σ_i such that $y_i = \mu_i + z_i\sigma_i$, then

$$E[R] = \sum_{i=1}^{n} C_{Si} \int_{0}^{-\delta_{1i}} \phi(z_i) dz_i + C_{Ri} \int_{\delta_{2i}}^{\infty} \phi(z_i) dz_i$$

$$= \sum_{i=1}^{n} C_{Si} \Phi(-\delta_{1i}) + C_{Ri}[1 - \Phi(\delta_{2i})] \, , \tag{4}$$

where $\phi(\bullet)$, $\Phi(\bullet)$, and z denote the standard normal p.d.f, cumulative normal distribution, and standard normal random variate, respectively [9].

Manufacturing Cost

Using the model suggested by Kim and Cho [8], the expected manufacturing cost $E[M]$ is described by a polynomial model as follows:

$$E[M] = a_{0i} + a_{1i}t_i + a_{2i}t_i^2 + ..., \tag{5}$$

where t_i are tolerances and a_j's are coefficients of the polynomial function determined using regression analysis. The term t_i can be defined in terms of δ_{1i}, δ_{2i}, μ_i and σ_i as

$$t_i = USL_i - LSL_i = (\mu_i + \delta_{2i}\sigma_i) - (\mu_i - \delta_{1i}\sigma_i) = \delta_{1i}\sigma_i + \delta_{2i}\sigma_i \tag{6}$$

Substituting $t_i = \delta_{1i}\sigma_i + \delta_{2i}\sigma_i$, in Equation (6), we get

$$E[M] = \sum_{i=1}^{n} a_{0i} + a_{1i}(\delta_{1i} + \delta_{2i})\sigma_i + a_{2i}(\delta_{1i} + \delta_{2i})^2 \sigma_i^2 + ... \tag{7}$$

Equation (7) defines the manufacturing cost in terms of δ_{1i} and δ_{2i} [9].

Customer Satisfaction

Customer satisfaction is high when lower expected deviation from target values is achieved for *QC*s having higher values of customer-perceived relative importance. According to Taguchi [2], the reduction in customer satisfaction is squarely proportional to the deviation from the target value of a *QC*. Letting y_{0i} be the target value of QC_i, the expected deviation can be given by

$$E[D_i] = \int_{LSL_i}^{USL_i} (y_i - y_{0_i})^2 f(y_i) dy_i = \int_{\mu_i - \delta_{1i}\sigma_i}^{\mu_i + \delta_{2i}\sigma_i} (y_i - y_{0_i})^2 f(y_i) dy_i \tag{8}$$

Substituting $y = \mu + z\sigma$ and letting $\phi(\cdot)$ and $\Phi(\cdot)$ denote standard and cumulative standard normal random variable

$$E[D_i] = \int_{-\delta_{i1}}^{\delta_{2i}} \left[(\mu_i + z_i\sigma_i)^2 - 2y_{0i}(\mu_i + z_i\sigma_i) + y_{0i}^2\right]\phi(z_i)\,dz_i$$
$$= \left[\begin{array}{l} \mu_i^2\Phi(z_i) + z_i\sigma_i(-\phi(z_i)) + \sigma_i^2(z_i\phi(z_i) - \Phi(z_i)) \\ -2y_{0i}\mu_i\Phi(z_i) - 2y_{0i}\sigma_i(-\phi(z_i)) + y_{0i}^2\Phi(z_i) \end{array}\right]_{-\delta_{i1}}^{\delta_{2i}} \tag{9}$$
$$= \left[\Phi(z_i)\left((\mu_i - y_{0i})^2 - \sigma_i^2\right) - 2\sigma_i\,\phi(z_i)\,(1 - \frac{z_i^2}{2} - y_{0i})\right]_{-\delta_{i1}}^{\delta_{2i}}$$

TOLERANCE OPTIMIZATION

A tolerance optimization problem can be structured using the following general multiobjective optimization model:

$$\begin{array}{l}\text{Minimize } E[T], E[D_1], E[D_1], \ldots, \text{and } E[D_n] \\ \text{subject to } \mathbf{g} \geq 0,\end{array} \tag{10}$$

where $E[T]$ is determined as follows:

$$E[T] = E[L(y)] + E[R] + E[M]$$
$$= \sum_{i=1}^{n} \left[\begin{array}{l} b_{0i}[\Phi(\delta_{2i}) - \Phi(-\delta_{1i})] + b_{1i}\mu_i[\Phi(\delta_{2i}) - \Phi(-\delta_{1i})] - b_{1i}\sigma_i[\phi(\delta_{2i}) - \phi(-\delta_{1i})] \\ + b_{2i}\mu_i^2[\Phi(\delta_{2i}) - \Phi(-\delta_{1i})] - b_{2i}^2\mu_i\sigma_i[\phi(\delta_{2i}) - \phi(-\delta_{1i})] \\ + b_{2i}\sigma_i^2[\delta_{2i}\phi(\delta_{2i}) + \delta_{1i}\phi(\delta_{1i}) - (\Phi(\delta_{2i}) - \Phi(-\delta_{1i}))] \\ + C_{Si}\Phi(-\delta_{1i}) + C_{Ri}[1 - \Phi(\delta_{2i})] + a_{0i} + a_{1i}(\delta_{1i} + \delta_{2i})\sigma_i + a_{2i}(\delta_{1i} + \delta_{2i})^2\sigma_i^2 \end{array}\right].$$

The feasible decision space Ω is defined in terms of the screening limit vector $\{\boldsymbol{\delta} | \boldsymbol{\delta} = \delta_{11}, \delta_{12}, \cdots \delta_{i1}, \delta_{i2}, \cdots \delta_{n1}, \delta_{n2}\}$ in the Euclidean space as $\Omega = \{\delta \mid \delta \in R^u,\ \mathbf{g} \geq 0\}$, where **g** are real-valued functions defined in Ω. In addition, **E**= [$E[T]$, $E[D_1]$, $E[D_1]$, …, $E[D_n]$] represents the image of δ in the objective space. In order to define utopian point consider the following n+1 single objective optimization models

$$\begin{array}{ll}\text{Minimize } \mathrm{E}[D_i] \text{ for } i = 1, 2, \ldots, n & \\ \text{subject to} \quad a_i > \delta_i > b_i \text{ and } E[T] \geq 0 & \end{array} \tag{11}$$

$$\begin{array}{ll}\text{Minimize } \mathrm{E}[T] & \\ \text{subject to} \quad a_i > \delta_i > b_i \text{ and } E[D_i] \geq 0 & \end{array} \tag{12}$$

Let $\boldsymbol{\delta}_j^I$ be the solutions for the n models given by (10) and $\boldsymbol{\delta}_T^I$ be the solution for (12). Also, $E[D_i]^I$ be the minimum value obtained for $E[D_i]_j$ on solving Equation (11) for each i=1, 2, … n and $E[T]^I$ be the minimum value obtained on solving Equation (12). The utopian point in the objective space is then given by $\mathbf{E}^I$ = [$E[D_1]^I$, $E[D_2]^I$, …, $E[D_n]^I$, $E[T]^I$]. The pre-image of the utopian point, δ^I is called utopian solution and can often be infeasible because of the conflict between multiple objectives.

MULTICRITERIA TOLERANCE OPTIMIZATION MODEL

The optimization model given by Equation (10) seeks to determine δ within Ω that can attain closer proximity to the utopian point $\mathbf{MSE}^I$. Let δ^* and $\mathbf{E}^*$ denote the feasible solution and its image in the objective space, respectively. In order to determine δ^* where $\delta^* \in \Omega$, the principle of the Edgeworth Pareto-optimality can be employed [11]. That is, δ in Ω is considered a Pareto solution if and only if there does not exist some other $\delta \in \Omega$ such that $\mathbf{E} < \mathbf{E}^*$. A popular method to determine the Pareto set is the minimization of the distance of $\mathbf{E}$ from the utopian point $\mathbf{E}^I$, where the distance is expressed using weighted θ–norm defined in Equation (13). A Pareto solution δ^* can then be determined by minimizing the distance $d_\theta(\mathbf{w}^T)$ from the $\mathbf{MSE}^I$ given by

$$d_\theta(\mathbf{w}) = \mathbf{w}(\mathbf{E}-\mathbf{E}^I)\Big|_\theta = \left[\begin{array}{l} \displaystyle\sum_{i=1}^{n} w_i \left(\frac{E[D_i]-E[D_i]^I}{E[D_i]^{\max}-E[D_i]^I}\right)^\theta \\ \displaystyle + w_T\left(\frac{E[T]-E[T]^I}{E[T]^{\max}-E[T]^I}\right)^\theta \end{array}\right]^{1/\theta}, \qquad (13)$$

where $E[D_i]^{\max}$ (i =1, 2, …, n) is defined as the maximum value in the j^{th} row of Table 1.

TABLE 1. PAY-OFF MATRIX

	$E[D_1]$	…	$E[D_n]$	$E[D_T]$
$\delta_1{}^I$	$E[D_1]_1 = E[D_1]^I$	…	$E[D_n]_1$	$E[D_T]_1$
⋮	⋮	⋮	⋮	⋮
$\boldsymbol{\delta}_n^I$	$E[D_1]_n$	…	$E[D_n]_n = E[D_n]^I$	$E[D_T]_n$
$\boldsymbol{\delta}_T^I$	$E[D_1]_T$	…	$E[D_n]_T$	$E[D_T]_T = E[D_T]^I$

Using the weighted θ–norm, multicriteria model can be written as

$$\begin{aligned} &\text{Minimize } d_\theta(\mathbf{w}) \\ &\text{subject to } \mathbf{g} \geq 0. \end{aligned} \qquad (14)$$

Here, the vector $\mathbf{w} = [w_1, w_2, …, w_n, w_T]$ represents the customer-perceived relative importance of n+1 objectives or criteria $E[D_i]$ and $E[T]$. The vector $\mathbf{w}$ can be determined by using a combination of entropy method and house of quality [10-12].Commonly-used weighted norms are weighted unity norm or $d_1(\mathbf{w})$, weighted quadratic norm or $d_2(\mathbf{w})$, and weighted Tchebycheff metric or $d_\infty(\mathbf{w})$, which is also referred to as weighted ∞–norm depending on whether θ is 1, 2 or ∞. The compromise programming method based on the weighted ∞–norm is recommended by many researchers [13-16], since it reduces the computational burden and achieves lower deviations from $\mathbf{MSE}^I$. Further, the Tchebycheff metric based compromise programming method is therefore capable of exploring all non-dominated solutions to multicriteria problems irrespective of the convexity of a decision space. The denominator in Equation (12), $MSE_j^{\max}(\mathbf{x}) - MSE_j^I(\mathbf{x})$, is used to normalize the variability measures for the m QCs so that the measures take the values between 0 and 1. This normalization converts incommensurable measures into dimensionless indices so that the summation in the right-hand side of Equation (10) can be performed on a consistent basis. When $\theta = \infty$, Equation (14) can be represented by the equivalent β–problem given by

$$\begin{aligned} &\text{Minimize } \beta \\ &\text{subject to} \\ &w_i\left(\frac{E[D_i]-E[D_i]^I}{E[D_i]^{\max}-E[D_i]^I}\right) \leq \beta \end{aligned}$$

$$w_T\left(\frac{E[T]-E[T]^I}{E[T]^{\max}-E[T]^I}\right)\leq\beta$$

$$\mathbf{g}\geq 0,\text{ and }\beta\geq 0. \tag{15}$$

The well-known β–formulation given in Equation (15) is computationally less tedious than the formulation given in Equation (14).

NUMERICAL EXAMPLE

Consider a product with two QCs, QC_1 and QC_2. A market survey of customers using entropy method [10] determined the relative importance of the QC_1, QC_2, and $E[T]$ as 02.5, 0.35, and 0.4 respectively. The empirical and manufacturing equations given by equations (1) and (5) were determined using least-squares regression. Tables 2 and 3 show the solutions obtained for single objective optimization models and the corresponding values of the objective functions $E[T]$, $E[D_1]$, and $E[D_2]$. The maximum and utopian values of $E[T]$, $E[D_1]$, and $E[D_2]$ are shown in Table 4. As seen in Table 4, the utopian point is $[E[T]^I, E[D_1]^I, E[D_2]^I]$ = [123.28, 0.1, 0.1]. When determining the Pareto solution the optimization minimizes the distance from the utopian point, where the distance is expressed by the weighted ∞-norm or the Tchebycheff weighted norm. The weights incorporated into the weighted norm relate to the customer perceived relative importance weights, $\mathbf{w}$ = [02.5, 0.35, 0.4]. The Pareto solution for weight vector [0.25, 0.35, 0.4] was found to be (0.53, 0.42, 0.89, 0.32).

TABLE 2. UTOPIAN SOLUTIONS

δ_j^I	$(\delta_{11},\delta_{12},\delta_{21},\delta_{22})$
δ_1^I	(0.962, 0.1, 1.133, 0.1)
δ_2^I	(0.292, 0.632, 1.133, 0.1)
δ_T^I	(0.962, 0.1, 0.346, 0.675)

TABLE 3. PAY-OFF MATRIX

	$E[T]$	$E[D_1]$	$E[D_2]$
δ_1^I = (0.962, 0.1, 1.133, 0.1)	123.28	4.99	5.6
δ_2^I = (0.292, 0.632, 1.133, 0.1)	157.95	0.1	5.6
δ_T^I = (0.962, 0.1, 1.133, 0.1)	190.16	4.99	0.1

TABLE 4. MAXIMUM AND UTOPIAN VALUES

$E[T]^{\max}$	190.16	$E[T]^I$	123.28
$E[D_1]^{\max}$	4.99	$E[D_1]^I$	0.1
$E[D_2]^{\max}$	5.6	$E[D_2]^I$	0.1

CONCLUSIONS

The paper investigates the effect of customer preferences other than just monetary measures such as quality loss and rejection and manufacturing costs, on screening limit or tolerance decisions. The model presented in this paper is one of the most comprehensive models presented and the direction of this research is in harmony with concurrent engineering that attempts to incorporate the voice of customer in all phases of design and manufacture.

REFERENCES

[1] Tang, K., 1988, Economic Design of Product Specifications for a Complete Inspection Plan, *International Journal of Production Research*, 26(2), 203-217.

[2] Phadke, M.S., 1989, *Quality Engineering Using Robust Design*, Prentice Hall, New Jersey.

[3] Tang, K. and Tang, J., 1989, Design of Product Specifications for Multi-Characteristic Inspection, *Management Science*, 35(6), 743-756.

[4] Fathi, Y., 1990, Producer-Consumer Tolerances, *Journal of Quality Technology*, 22(2), 138-145.

[5] Kapur, K.C. and Cho, B.R., 1996, Economic Design of the Specification Region for Multiple Quality Characteristics, *IIE Transactions*, 28(3), 237-248.

[6] Phillips, M.D. and Cho, B.R., 2000, An Empirical Approach to Designing Product Specifications: A Case Study, *Quality Engineering*, 11(1), 91-100.

[7] Shin, S.S., Govindaluri, S.M., and Cho, B. R., 2005, Integrating the Lambert W function to a Tolerance Optimization Problem, *Quality Reliability Engineering International*, 21(8)

[8] Kim,Y.J. and Cho, B.R., 2000, The Use of Response Surface Designs in the Selection of Optimum Tolerance Allocation, *Quality Engineering*, 13(1), 35-42.

[9] Govindaluri, S.M. and Cho, B.R., 2002, Designing cost-effective limits: An empirical approach, *Proceedings of Industrial Engineering Research Conference*, Orlando, Florida.

[10] Hwang, C.L. and Yoon, K., 1981, *Multiple Attribute Decision Making – Methods and Applications*: *A State-of-the Art Survey*. Springer-Verlag, New York.

[11] Wasserman, G., 1993, On How to Prioritize Design Requirements during the QFD Process *IIE Transactions* 25(3), 59-65.

[12] Govindaluri, S.M. and Cho, B.R., 2005, Integration of customer and designer preferences in Robust Design, *International Journal of Six Sigma and Competitive Advantage*, 1(3), 276-294.

[13] Das, I. and Dennis, J.E., 1997, A Closer Look at the Drawbacks of Minimizing Weighted Sums of Objectives for Pareto Set Generation in Multicriteria Optimization Problems. *Structural Optimization*, 14, 63-69.

[14] Chen, W., Wiecek, M., and Zhang, J., 1999, Quality Utility – A Compromise Programming Approach to Robust Design, *ASME Journal of Mechanical Design* 121(2): 179-187.

[15] Dai, Z., Scott, M.J., and Mourelatos, Z.P., 2003, Robust Design Using Preference Aggregation Methods, *Proceedings of ASME Design Engineering Technical Conference*, ASME paper no. DETC/DAC-48715.

[16] Sen P, Yang B-J (1998) *Multiple Criteria Decision Support in Engineering Design*. Springer, London.