ON LIMITATION OF PERT METHODS FOR LARGE SCALE PROJECTS

Khosrow Moshirvaziri, Information Systems Department, California State University, Long Beach, Long Beach, CA, 90840, 562-985-7965, moshir@csulb.edu
Ömer S. Benli, Information Systems Department, California State University, Long Beach, Long Beach, CA, 90840,562-985-5918, obenli@csulb.edu

ABSTRACT[1]

The PERT based Project Management tools do not take an adequate account of risk [1]. Monte Carlo Simulation provides an improved methodology to quantify schedule risk. This paper investigates the use of this methodology and its shortcomings in an Engineering environment for better understanding, designing, and managing a project such that the detrimental effects of these risks can be minimized.

INTRODUCTION

Critical Path Method (CPM) detects interdependencies among activities and determines those activities requiring extra attention and scare resources in the precedence network. The Program Evaluation Review Technique (PERT) developed by the U.S. Navy in the 1950s merely for managing the Polaris missile project. It was perceived as an effective tool to model and account for uncertainty associated with the planning of large industrial projects.

The first step in any CPM or PERT analysis is to identify the activities required to complete the project, and then to list their predecessor activities. Identifying activities and their precedence relationships is valuable since it clarifies the scope of the project and puts structure on what at the outset may seem an overwhelming task. In both CPM and PERT, the time to complete each activity is estimated. To complete a CPM analysis, a point estimate of the completion time is sufficient. These time estimates are used to compute the earliest possible project completion time, as well as the earliest and latest starting time for each activity, the slack, or amount of time by which the start and/or end time can be adjusted without affecting the completion time of the overall project. The critical path is the sequence of activities with zero slack times; in other words they must each start at their earliest possible start time and be completed in no more than the estimated time for the overall project to be completed on time.

In the PERT method estimates of the optimistic, most likely, and pessimistic time required to complete each activity are needed. Such estimates are used to obtain probability distribution of the completion time on the critical path. This is done under the following assumptions: (1) The critical path does not change; (2) The project completion time is a normally distributed random variable; (3) The activity completion times are statistically independent and identically distributed random variables (iid), and (4) Activity's completion time is a "PERT-beta" distributed random variable.

This paper discuses the value of using simulation to relax some of the restrictive assumptions of PERT [3] and to generate more reliable, though not necessarily definitive, probability distributions for the completion time and budget for an entire project. Large-scale engineering projects are usually original, one-of-a-kind projects. At its current state, PERT is not fully capable of handling all the features of such projects. PERT is designed to handle <u>only</u> the so-called "PERT-distribution" for random activity durations with the three estimates for activity durations. However, in many projects there is a need for other probability distributions, as well as "activities with selection probabilities." PERT assumes that there exists "a single dominant critical path" in the project network, which most

[1] This is an abbreviated version of the paper. The full paper is available from the authors upon request.

likely not true in most very-large scale engineering projects. Additionally, PERT assumes that random variables representing the task times on the critical path are "independently and identically distributed" (IID) when making its conclusions as to the completion probabilities of the project, which most likely not true in most very-large scale engineering projects. This paper emphasized these issues and proposes some suggestions.

TECHNICAL CONSIDERATION

The risk of failing to complete activities and entire projects on time and the resulting cost overruns are critical elements of all operations in project management. The PERT method introduces uncertainty into the network by treating each activity's completion time as a random variable. The probability distribution of the activity time random variable is almost universally taken to be "Beta Distribution". Its distribution function called the "Beta Function**":**

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

A random variable *X* has the beta distribution, if its density function is given by the following

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1.$$

The two parameters α and β are shape parameters and hence produce a family of distributions. The triangular distribution as well as the uniform distribution, as assumed in CPM (Critical Path Method) networks, are special cases of beta distribution, and normally used to avoid dealing with the mathematical difficulties arising in the analysis under the Beta distribution... The expected value and standard deviation of beta distribution is used in PERT systems as a standard practice. Special properties of beta distribution will be used as discussed to arrive at analysis that is more reliable.

ANALYSIS AND MODELING

One needs to focus on individual task completion times, examining the probability distribution generally used to model the uncertainty in activity completion times in simulation and in PERT setting. One must also examine the common assumptions used to derive estimates of the parameters required to describe the beta distribution and to derive mean and standard deviation of project completion times in the underlying PERT representation. It is also worth mentioning that PERT approximations of project duration and the associated probability statements derived from those approximations may sometimes be misleading. The most widely recognized source of bias is near critical paths that emerge as critical in a realization of the project. However, simulation can overcome the bias in the PERT and in particular in standard deviation of completion times by bypassing the PERT formulas. This requires more careful choices parameters of the beta distribution so that a wider variety of shapes can be used to model activity completion times in a network simulation. The formal beta distribution used in many simulation software packages such as EXTEND, CORE, and in MS EXCEL is a continuous distribution that has the following distinct properties: (1) beta has finite limits, (2) it can be asymmetrical, and (3) finally beta distribution is flexible. These properties lay a foundation for Variance Reduction Analysis of the project's completion time.

BUDGETING ISSURES

All projects have to be supported financially. The budget allocated to the project, however, is subject to uncertainty due to various financial, market, and political risks. Thus, an important piece of the analysis is to incorporate budget uncertainty into project time-cost tradeoff. In a realistic model, one formulates financial feasibility as a stochastic constraint, transforms it into a deterministic equivalent in the case of normal, beta, or triangular distribution, and solves the equivalent model accordingly. Typical result is a minimum time-cost curve, which relates the shortest project duration to different levels of budget and available resources. An important assessment is to show the degree to which budget uncertainty relates to financial constraint and, thus, provide extra contingency duration when necessary. For example, if the financial constraint has to be met at a higher probability level, extra contingency costs are necessary to ensure an on-time completion.

At the activity level, a more elaborate budget analysis may be incorporated. This is commonly known as “Project Crashing” or “Crashing and Time/Cost Tradeoffs” in which budget and generally resource allocation alternatives are analyzed in terms of their incremental effects on the completion time of individual activities and on the overall project’s completion time. There are three basic cost categories involved in most projects - direct costs, indirect costs, and penalty costs. Shortening the project time may decrease these costs, the shorter the project, the lower the indirect costs. A project may also involve penalty costs; these are incurred if the project extends beyond a specific date. Generally, because of complication in solving underlying models under crashing, heuristic approaches are used. The minimum project duration under crashing phenomenon is not part of the simulation tools and must be incorporated separately. Typically, optimization modeling tools are used to make more reliable estimates of the minimum project’s completion time under crashing. Software tools dealing with PERT systems or those capable of carrying out crashing analysis, would normally assume a linear, or convex, activity cost function. This implies, for example, the cost of performing an activity is a linear or convex function of its duration. Quite the contrary, in a realistic setting, the cost is a concave function of the input parameter (in this case duration). The reason for the concavity of the cost function is the presence of the “economies of scale”. This fact and its effects on the model network cannot be easily captured because the overall project cost would be a concave function. As a result, a theoretical complexity is added to the overall model, making it extremely difficult to solve using existing software tools. This latter area of research is known as “Global Optimization” and currently is the frontier of theoretical research in mathematical programming (e.g. see [2]). As a consequence, a more realistic estimation of the project’s completion time and further assessments on time-cost trade-off require a more comprehensive analysis not provided in the commercially available simulation software packages and conventional tools.

CONCLUDING REMARKS

Schedule simulation should be used on any large or complex Engineering project since traditional mathematical analysis technique such as the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) do not account for path convergence and thus tend to underestimate project duration. It is easier to demonstrate, with a simulation, how task(s) not on the critical path may end up on the critical path due to deviations from the plan and derail a project. Also, with simulation one can illustrate the negative impact of parallel paths converging at critical points. This paper was geared toward this end.

REFERENCES

[1] Galway, L., “Quantitative Risk Analysis for Project Management: A Critical Review,” WR-112-RC, Rand Corporation, February 2004.

[2] Moshirvaziri, K. and Amouzegar, M.A. A Cutting Plane Algorithm for Linear Reverse Convex Programs *Annals of Operations Research*, 105 :201-212, Kluwer Academic Publishing, 2002.

[3] Armour, J. M., and Buffa, E. S., A Heuristic Algorithm and Simulation Approach, *Management Science* 9:294-309, 1963.