A MODEL TO OPTIMIZE WATER DISTRIBUTION

Raquel Salazar, Universidad Autónoma Chapingo, Km 38.5 Carr. México-Texcoco, Chapingo Edo. México, 56230, raquels@correo.chapingo.mx
Ferenc Szidarovszky, Department of Systems and Industrial Engineering, The University of Arizona, Tucson, Arizona 85721-0020, USA, szidar@sie.arizona.edu
Abraham Rojano, Universidad Autónoma Chapingo, Km 38.5 Carr. México-Texcoco, Chapingo Edo. México, 56230, abrojano@correo.chapingo.mx

ABSTRACT

A Multiobjective Optimization Model is developed to find the most satisfying water distribution strategy for the State of Mexico. Four objective functions are considered: the water shortages of agricultural, industrial, and domestic users and the subsidy paid by the National Water Commission. Distance based methods with the L_1 and L_∞ distances were applied in order to consider the extreme cases of possible compensation among the objectives. The application of the L_1 distance requires the application of the simplex method. In the case of the L_∞ distance, the nondifferentiable objective function can be transformed into a linear objective by adding linear constraints to the original constraints of the problem. Therefore this case also could be solved by the simplex method. The results with 10 different priority alternatives are computed and compared.

INTRODUCTION

Mexico is classified as having low water availability, 56% is an arid region, the annual average rainfall is about 711 mm. Moreover increased rainfall shortage has resulted in increased use of groundwater to satisfy the domestic, agricultural and industrial water demands. These sectors consume 13%, 77% and 10% of the total available water in the country respectively. There is an increasing competition between these sectors for the scarce water resources. While more water is needed in cities because of the expanding populations and industries, water availability per person have been dropped to 57% in a period of 40 years.

The purpose of this paper is to develop a water distribution model for Mexico city and its metropolitan area, considering the government as one decision maker facing the problem of satisfying the domestic, agricultural and industrial water demands from available surface, groundwater and treated water and at the same time minimizing water subsidy. In this study our objective is to determine how much groundwater can be withdrawn without unacceptable damage to the environment and how the available surface, groundwater and treated water should be allocated among the three users.

STUDY AREA

Mexico City and its metropolitan area is one of the country's most critical area as far as the water shortage situation concerns. It is located in the center of the nation with an area of 3,773 km². The population in this area is over 18 million with an expected growth of 380 thousands inhabitants per year with an estimated average water consumption of 230 lt/person/day (http://www.urbanage.net/03_conferences/conf_mexicoCity.html, February 2006). The water shortage situation for the state of Mexico is worsening due to increasing domestic, industrial and agricultural water demands which raise the necessity of transporting surface water from the Balsas River basin and groundwater from the Alto Lerma System. These transfer schemes are likely to grow rising the disputes over water. Besides water scarcity, the quality of surface and groundwater resources is worsening due to pollution. The capacity of the waste water treatment facilities is not enough to treat the current water discharge.

This area of the country is an industrial centre with a variety of economic activities. According to the Water National Commission from the amount of available water resources in the Mexican Valley (state of Mexico, Mexico city and part of the states of Hidalgo and Tlaxcala) 48% is consumed by domestic users, 34% by irrigation, 5% by industry and 13% is transferred to Mexico City. The main sources of water supply in the State of Mexico are nine aquifers, six of which are shared with Mexico City. The groundwater resources are overexploited beyond their capacity. In addition, deforestation has reduced the infiltration rate and recharge of the aquifers.

The main institution involved in the competition for water in the state of Mexico is the National Water Commission who delivers water in bulk to the state water utility (Comisión Estatal de Agua y Saneamiento). The state water utility is responsible for accepting the water, treating it, and distributing it to various counties in the state. The surface water sources undergo chemical coagulation, filtration, and chlorination. Ground water is normally treated only by chlorination.

The demand for water exceeds the sustainable yield of aquifers and rivers in the state. The gap between the continuously growing use of water and the sustainable supply is widening each year, making the water supply more and more difficult. The amount of subsidy given by the government is 34% of the total National Water Commission budget, and it is expected to increase in the next few years. Therefore there is a need in the state of Mexico to make choices about how this resource should be allocated among competing users and on the other hand how to minimize Government subsidy in this area.

LITERATURE REVIEW

Donevska et al. [2] investigate a similar situation to ours with water demands for agricultural and non-agricultural users in the Republic of Macedonia. The conflict situation due to the urbanization and industrialization arises by the competition of water between irrigation and other water users. This conflict is analyzed with engineering and non-engineering measures as reducing water losses along the distribution networks, as well as the application of irrigation methods with higher efficiency, construction of more facilities for storing water in wet years for use in dry years, transporting water long distances from areas of surplus water to areas of water shortages and also efficient watershed management. A similar study was done by Jensen et al. [3] who analyze multiple uses of irrigation water, they conclude that future scenarios to maintain a sufficient minimum supply of domestic water requires cooperation rather than competition between the irrigation and water supply sectors. Also, much stronger institutional links between the irrigation and the water supply sectors are needed to avoid negative health implications for the multiple users of irrigation water. The main weakness of these works is their lack of the quantitative approach for analyzing the water competition problem.

Much less research has been carried out on the development and use of game theoretical techniques in water resources management. Lund and Palmer [4] mention that game theory has not been extensively incorporated into quantitative analysis of water resource conflicts to suggest promising strategies for one party or promising solutions for a group of stakeholders. Coppola and Szidarovszky [1] applied Game Theory in a real-world case in Toms River New Jersey. Several conflict resolution methods were applied to a two-person conflict between a community and a water supply company, where the two payoffs were health risk and quantity of supplied water. The obtained pumping policies agree with the physical dynamics of the modeled groundwater supply system.

DESCRIPTION OF THE MATHEMATICAL MODEL

There are three water users: agriculture, industry and domestic. Each of them uses surface, ground and treated water. The objectives and constraints for the three users and the National Water Commission are as follows:

The first three objectives represents the minimization of water shortage for farmers, industry and domestic users

$$Min \quad D_1 - (s_1 + g_1 + t_1 + s_1^* + g_1^*) \tag{1}$$

$$Min \quad D_2 - (s_2 + g_2 + t_2 + s_2^* + g_2^*) \tag{2}$$

$$Min \quad D_3 - (s_3 + g_3 + t_3 + s_3^* + g_3^*) \tag{3}$$

For National Water Commission the objective is to minimize subsidy, which is the difference of cost and revenue:

$$Min \quad \{(g_1 + g_2 + g_3)c + (g_1^* + g_2^* + g_3^*)tr_g + g_3 cl_g + (s_1^* + s_2^* + s_3^*)tr_s + s_1 c_d + (s_2 + s_3 + s_2^* + s_3^*)tr_p + s_3 cl_s\}$$

$$- \{p_f(s_1 + g_1 + t_1 + s_1^* + g_1^*) + p_i(s_2 + g_2 + t_2 + s_2^* + g_2^*) + p_d(s_3 + g_3 + t_3 + s_3^* + g_3^*)\} \tag{4}$$

The constraints are as follows. The supplied water amount for farmers must not exceed demand:

$$s_1 + g_1 + t_1 + s_1^* + g_1^* \leq D_1 \tag{5}$$

Also, there is a minimum amount of water to be supplied to farmers

$$s_1 + g_1 + t_1 + s_1^* + g_1^* \geq S_1^{min} \tag{6}$$

The overall groundwater percentage must not be less than needed by crops irrigated with only groundwater:

$$\frac{g_1 + g_1^*}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \geq \frac{\sum_{i \in G} a_i w_i}{W} \tag{7}$$

The treated water percentage cannot be larger than in the case when all crops in T are irrigated only with treated water:

$$\frac{t_1}{s_1 + g_1 + t_1 + s_1^* + g_1^*} \leq \frac{\sum_{i \in T} a_i w_i}{W}. \tag{8}$$

For industry, the supply must not be larger than demand:

$$s_2 + g_2 + t_2 + s_2^* + g_2^* \leq D_2 \tag{9}$$

The water amount for industry has to be at least $S_2^{\min}$

$$s_2 + g_2 + t_2 + s_2^* + g_2^* \geq S_2^{\min} \tag{10}$$

Since having more groundwater improves quality but more treated water makes quality worse, we added two addition constraints:

$$\frac{g_2 + g_2^*}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \geq B_g \tag{11}$$

and

$$\frac{t_2}{s_2 + g_2 + t_2 + s_2^* + g_2^*} \leq B_t, \tag{12}$$

For domestic users supply must not exceed demand:

$$s_3 + g_3 + t_3 + s_3^* + g_3^* \leq D_3. \tag{13}$$

There must be at least 200 lts/person/day to satisfy the minimum water requirements. For the total population, the minimum supply becomes $S_3^{\min}$ =1343 mill m^3

$$s_3 + g_3 + t_3 + s_3^* + g_3^* \geq S_3^{\min} \tag{14}$$

Since treated water can be used only for certain areas, we need an addition constraint:

$$\frac{t_3}{s_3 + g_3 + t_3 + s_3^* + g_3^*} \leq B_d \tag{15}$$

The Water National Commission gives a subsidy at least one peso for each cubic meter of water supply:

$$\{(g_1 + g_2 + g_3)c + (g_1^* + g_2^* + g_3^*)tr_g + g_3 cl_g + (s_1^* + s_2^* + s_3^*)tr_s + s_1 c_d + (s_2 + s_3 + s_2^* + s_3^*)tr_p + s_3 cl_s\}$$

$$- \{p_f(s_1 + g_1 + t_1 + s_1^* + g_1^*) + p_i(s_2 + g_2 + t_2 + s_2^* + g_2^*) + p_d(s_3 + g_3 + t_3 + s_3^* + g_3^*)\} \geq 4000 \tag{16}$$

The total surface and groundwater water supplies in the state and from other states are

$$s_1 + s_2 + s_3 = S_s \tag{17}$$

$$g_1 + g_2 + g_3 = S_g \tag{18}$$

$$s_1^* + s_2^* + s_3^* \leq S_s^* \tag{19}$$

$$g_1^* + g_2^* + g_3^* \leq S_g^* \tag{20}$$

All decision variables must be non negative, where:

D_1 = water demand of farmers

s_1 = surface water available for farmers

g_1 = groundwater available for farmers

t_1 = treated water available for farmers

s_1^* = surface water imported from other places available for farmers

g_1^* =groundwater imported from other places available for farmers.

S_1^{min} = minimum water amount to be supplied to farmers.

G= set of crops that can use only groundwater

a_i = ratio of crop i in agriculture area

w_i = water need of crop i per ha.

$W = \sum_{all\ i} a_i w_i$ = total water need per ha.

T= set of crops which can use treated water.

D_2= Industry water demand =1000 mill m3

s_2= surface water available for industry

g_2= groundwater available for industry

t_2= treated water available for industry

s_2^* = surface water imported from other places available for industry

g_2^* =groundwater imported from other places available for industry.

B_g= minimum proportion of groundwater that industry can use

B_t= maximum proportion of treated water that industry can use.

D_3= domestic water demand

s_3= surface water available for domestic use

S_3^{min} = minimum water amount to be supplied to domestic users =1343 mill m^3

g_3= groundwater available for domestic use

t_3= treated water available for domestic use

s_3^* = surface water imported from other places available for domestic use

g_3^* =groundwater imported from other places available for domestic use

B_d= maximum proportion of treated water that can be used for domestic usage.

c= unit pumping cost of groundwater

tr_g = unit transportation cost of groundwater from other state

cl_g= unit cleaning cost of groundwater for domestic usage

tr_s = unit transportation cost of surface water from other state

c_d= unit water cost for surface water to farmers

tr_p= unit water pressurizing cost of surface and groundwater to domestic and industry users

cl_s= unit cleaning cost of surface water for domestic usage

p_f= water price for farmers

p_i= water price for industry

p_d= water price for domestic users

S_s = total surface water supply in the state

S_g= total groundwater supply in the state

S_s^* = total surface water supply from other states

S_g^* = total surface water supply from other states

The solution of this problem has two stages. First the minimum and maximum values of the objective functions have to be determined. In the second stage compromise programming is used to find the most satisfying solution. Since all objectives and constraints are linear, in the first stage linear programming was used to find the minimum (G_i*) and maximum (G_{*i}) value of the objectives. After obtaining these values a single-objective optimization problem was formulated, where the objective function was given by equation (22) with different weights selections and with $p = 1$ and $p = \infty$.

$$L_p(w,x) = \left(\sum_{i=1}^{m} w_i^p \left(\frac{g_i(x) - G_i^*}{G_{*i} - G_i^*} \right)^p \right)^{1/p} \tag{22}$$

In the case of $p = 1$ the composite objective function is linear, so linear programming was used to solve the problem. In the case of $p = \infty$ the absolute value and maximum operators make the objective function nonlinear, and even nondifferentiable. However by introducing new variables the problem can be transformed into a linear programming problem, so the simplex method can be used again. . The composite objective function is

$$L_\infty(w,x) = \max_i \left\{ w_i^\infty \left(\frac{g_i(x) - G_i^*}{G_{*i} - G_i^*} \right) \right\},$$

and if we introduce it as an additional decision variable L, then the objective is to minimize L subject to the original constraints and we have to add four additional linear constraints

$$w_i^\infty \frac{g_i(x) - G_i^*}{g_{*i} - G_i^*} \le L \quad (i=1,\ 2,\ 3,\ 4).$$

Since the new objective function and the additional constraints are all linear, the problem remains a linear programming problem.

RESULTS

We applied the method with $p = 1$ and $p = \infty$ for ten different sets of weights. They are presented in Table 1. The Distance based Multiobjective Techniques were implemented using these sets of weights.

Table 1. Set of weights applied to Distance based Technique

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
Objective 1	0.25	0.5	0.2	0.2	0.2	0.4	0.4	0.4	0.1	0.1
Objective 2	0.25	0.2	0.5	0.2	0.2	0.4	0.1	0.1	0.4	0.1
Objective 3	0.25	0.2	0.2	0.5	0.1	0.1	0.4	0.1	0.4	0.4
Objective 4	0.25	0.1.	0.1	0.1	0.5	0.1	0.1	0.4	0.1	0.4

The results with the L_1 distance are summarized in Table 2. The optimal water distribution for most of the cases implies no external sources of groundwater and surface water supplies, which make sense since water importations are expensive for the NWC (National Water Commission).

Table 2. Water supply and shortage L_1 distance based results for different scenarios (mill m^3) and the subsidy in pesos

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
Farmers supply	1733.8	2643.4	1459.6	1733.8	2643	2643	1734	2644	1459.6	1733.8
Industry supply	642.2	712	685	642.1	712	712	642	711.9	685	642.1
Domestic supply	2350.6	1343	2350.6	2350.6	1343	1343	2351	1343	2350.6	2350.6
Total Water Supply	4726.6	4698.4	4495.2	4726.5	4698	4698	4727	4698	4495.2	4726.5
Subsidy	3998	3998.2	4003.6	4000.3	4000	4002	4000	4000	4003.64	3998.19

Notice that for scenarios 1, 4, 7 and 10 the farmers receive only 1733 mill m^3 and the maximum possible amount for scenarios 2, 5, 6 and 8. The amount of subsidy by the NWC is always around 4000 million pesos. This result can be interpreted as noticing that the compromise solution is not sensitive to the weight of the objective function of the NWC.

Table 3. Water supply and shortage L_∞ distance based results for different scenarios (mill m^3) and the subsidy in pesos

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
Farmers supply	1412,1	2914,7	2188,3	1497,3	2123,5	2716,9	1494,4	2158,6	1498	1417,8
Industry supply	321,3	282	736,9	448,3	281,9	595,5	421	281,9	420,5	309,6
Domestic supply	2350,7	1343	1343	2350,5	1343	1342,9	2350,6	1343	2351	2350,6
TWS	4084,1	4539,7	4268,2	4296,1	3748,4	4655,3	4266	3783,5	4269	4078
Subsidy	4476,3	12376	3998,9	6031,5	4001,6	6028	6285,8	4002,8	6322	4357

The results with distance L_∞ are shown in Table 3. In cases 1 and 10, the farmers can receive less water than in the worst case of the application of the L_1 distance, and in cases 2 and 6 they can get more water than in the best case of applying distance L_1. The subsidy shows a large variation between 3998.9 and 12376 million pesos in contrary to the case of the distance L_1.

CONCLUSIONS

In this paper we developed a water distribution optimizing model for Mexico City and its metropolitan area, taking into account that all of the water users, the farmers, the industry and domestic users get at least a minimum amount to operate. The Water National Commission has to subsidize around one peso per each cubic meter. Additional environmental constraints were added like the maximum of groundwater extracted from the aquifer to maintain aquifer sustainability. Water quality concerns were considered by bounding the percentage of ground water and treated water.

Two multiobjective methods were applied for the water distribution problem, the L_1 and L_∞ distance based multiobjective technique, in order to cover the extreme case of compensability of the objectives. For both cases we tried 10 different priority orders. In using L_1, in all scenarios surface water is not used for industry, and no surface water and groundwater transported from other places for domestic use. In the case of L_∞ distance, the scenarios 1, 4, 6, 7, 9 and 10 use up all available water sources and the subsidy has to increase in some cases.

This study shows the feasibility of using multiobjective techniques for solving water distribution problems given the importance of the users and the given costs of water supply. Our study presents the current situation of the water management in the state of Mexico, and the presentation and results will be largely improved after further information will be available.

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