THE ESTABLISHMENT OF A SYSTEM OF PUNITIVE FINES ON THE BASIS OF AN UNBIASED ESTIMATOR: FAIR OR NOT?

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ABSTRACT

This paper discusses the process by which unbiased sample estimators of the mean of overcharges by an ambulance company is used to establish the fine levied against that firm for the population of charges levied by that company. The sample overcharge mean may be used to estimate the total overcharge of the firm, and this figure used as the amount fined the ambulance company. The sample mean has a 50% chance of overestimating the population mean, and a 50% chance of underestimating the population mean. The end result is that there is a 50% chance that the fine is larger than it should be, assuming that the goal is to have the company cough up the monies it overcharged. Could not this approach be improved by using sound statistical practice?

Background: An Example

A Health Care Excel auditor took a sample of 146 claims from the population of 7,932 XYZ Care Inc. claims. They found 68 claims with exceptions identified resulting in an initial overpayment of $1,735.25.

A re-audit of the 68 claims from the sample of 146 claims was conducted after additional documentation was provided by XYZ Care. The initial overpayment of the sample of 146 claims of $1,735.25 was reduced to $1,145.00.

This is the same as a sample average of ($1,145.00/146) = $7.842. If one uses the sample mean to estimate the population mean, then it is logical to conclude that a point estimate of the total amount of overpayment is equal to the total number of claims times the sample average or $7.842 times 7,932 claims (the total number of claims during the time period) = $62,206.44 overcharge.

The Shortcoming

It is also true mathematically to say that if one uses this process repeatedly to estimate the amount of overpayment and the resultant fine due ambulance companies, then the probability that the true overpayment is less than this total is 0.50. That is; half the audits result in overpayments, and half result in underpayments.

It is understood, that fines are estimated one company at a time. It does an individual company no good to understand that in the long run, the average size of the fine is justified. An individual company wants assurance that its specific fine is justified. And our system of justice is predicated on assurance that the concept of individual justice is embedded within the process.

Short and Simple

Understand that the sample mean of $7.84 is only an estimate of the true average overcharge. I will show that the ‘standard’ error associated with that estimate is close to a dollar and a half. That is to say, it would not be unusual at all for the correct mean (the true/population mean) to be $1.50 or more below (or above that specific sample mean). It would be somewhat unusual for the true mean to be more than 2 standard errors ($3.00) away from the sample mean. Therefore if trans-care were fined close to an average of $4.84 per customer (or a total of $38,410), there would be a reasonable probability that this fine were not in excess. Much more careful attention will be paid to the development of this approach as I proceed.

The Development

It is necessary to utilize a statistical process such that the probability an individual company were fined in excess would be reasonable small. One needs to use basic statistical theory associated with the distribution of sample means in order to accomplish this goal.

One can develop a probability statement that ensures if a specific process is carried out that the probability that the company is paying too large a fine is 1 out of a hundred (.01), or 1 out of 20 (.05), or 1 out of 10 (.10). The process that is used now guarantees that the probability that the company is paying too large a fine is 1 out of 2 (.50).

Since the size of the fine is directly tied to the estimate of the population mean, the above probability statements need to be properly linked to the estimate of the population mean. If one develops a probability statement that ensures that the probability that the sample mean is being overestimated is only .01, that would ensure that the probability the company were paying an excess fine would be .01 instead of 0.50. We can also do the same using the value of .05, or a value of .10. After obtaining these 3 estimates of the lower boundary of the mean, we can obtain the resulting estimate of the fine.

$$P\ (\mu < \overline{X} - t_{\alpha} \cdot s_{\overline{X}}) = \alpha \tag{1}$$

The above is such a statement. The value α (alpha) would be set equal to 0.01. All we need to find this estimate of the mean (on the right hand side of the inequality) is the value of t associated with alpha, and associated with n-1 degrees of freedom. This value can be obtained for 145 degrees of freedom using the statistical software associated with the excel package.

The symbol $s_{\overline{X}}$ is known as the estimate of the standard error of the statistic. An accepted estimate of the size of their deviation is called the standard error of the mean. The standard error of the mean is equal to the standard deviation of the variable divided by the sample size used to obtain the mean. The estimate of the standard error of the mean is the estimate of the standard deviation of the variable divided by the sample size used to obtain the mean.

We are forced to use the estimate of the standard error of the mean, rather than the standard error of the mean because we do not have the standard deviation of the variable. We only have its estimate. We also should employ the finite population correction factor in our estimate of the standard error of the mean since we are dealing with a finite population. (This will make a small difference). We also have to recognize that since we are dealing with an estimate of the standard deviation of the variable, we must work with the t distribution, rather than the z (the standard normal) distribution. (Again, in this case, that will only make a small difference since the sample size is 146).

I have first obtained the estimate of the standard deviation of the variable(s) using the data from the re-audit. Recall that in the initial audit, there were 78 cases that had no overcharge (a value of zero), and in the original 68 cases with overcharges, about 30 of them were reduced down to zero after the re-audit. Regardless, the sample standard deviation of this data set turned out to be $18.25. The estimate of the standard error of the statistic

$$s_{\bar{X}} = (s/\sqrt{n}) \cdot \sqrt{\frac{N-n}{N-1}} \tag{2}$$

where small n is 146, and capital N is 7,932; turns out to be $1.496 or about a dollar and a half. So it would not be unusual at all for the true mean to be off by more than $1.50 from the sample mean in this specific situation.

The Computations

$$P(\mu < \bar{X} - t_\alpha \cdot s_{\bar{X}}) = \alpha \tag{3}$$

P(μ < $7.897 - 2.352·$1.496) = P(μ < $4.376) = .01; resulting fine $34,714.
If XYZ-care pays a fine of $34,714, there is a 1/100 chance they have been fined too much.

$$P(\mu < \bar{X} - t_\alpha \cdot s_{\bar{X}}) = \alpha \tag{4}$$

P(μ < $7.897 - 1.655·$1.496) = P(μ < $5.419) = .05; resulting fine $42,986
If XYZ-care pays a fine of $42,986, there is a 1/20 chance they have been fined too much.

$$P(\mu < \bar{X} - t_\alpha \cdot s_{\bar{X}}) = \alpha \tag{5}$$

P(μ < $7.897 - 1.287·$1.496) = P(μ < $5.970) = .10 resulting fine $47,354
If XYZ-care pays a fine of $47,354, there is a 1/10 chance they have been fined too much.

Of course, other probability statements can be developed if there is a need for fine-tuning.

SUMMARY

It may very well be that society would not accept a probability of .99 that a firm had paid too little, but I also imagine that a statistically educated society would have difficulty accepting a process where the probability that a firm paid too much is also .50. This should be a topic worth investigating. Frankly, I wonder why companies have not pursued this investigation. I would think that the existing process would generate a high level of distrust and resentment.