

TEACHING PORTFOLIO OPTIMIZATION USING QOS-15, EXCEL AND DATA FROM YAHOO! FINANCE

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ABSTRACT

Portfolio optimization is best taught as a process that starts with gathering security price and dividend data, estimating the parameters of the joint distribution of asset returns, and then using a portfolio optimizer to construct a mean-variance efficient portfolio. This case study describes two optimization exercises that walk students through the entire process, and prepares them for the more complex portfolio optimization problem formulations used in practice.

INTRODUCTION

The mean-variance framework for portfolio selection, developed by Markowitz in 1952 [8], continues to be the most popular method for portfolio construction. It is best taught as a process that starts with gathering price and dividend data for a set of stocks, estimating the parameters of the joint distribution of asset returns, and then using a portfolio optimizer to construct mean-variance efficient portfolios.

This case study shows how to use freely available data sources, such as Yahoo!Finance, to get time series adjusted price data for the stock market, use *Excel* for estimating the parameters that will be used as inputs to the optimization, and then use *QOS-15*, a free academic version of the *Quadratic Optimization System* available at www.financiometrics.com to construct optimal portfolios. The overall project requires students to manage a five-stock portfolio for the duration of the course. For the optimization component of the project, each student uses two pairs of stocks from the five selected stocks. The data setup and simple optimization exercises described below are designed to instill an understanding and appreciation of the role of expected returns, risk and correlation in portfolio optimization. These exercises prepare students to set up more complicated optimization problems with asset weight constraints, group constraints (such as sector, style and portfolio beta), variable transactions costs, turnover constraints, penalty functions, and gross leverage constraints for long-short portfolio optimization that are used in practice.

ESTIMATING THE JOINT DISTRIBUTION OF ASSET RETURNS

The first task of the project is to download adjusted price data for the five companies in the portfolio for calculating monthly stock returns for the most recent two and a half years, i.e., 30 monthly returns. For the purposes of this project students can download monthly *Historical Prices* for one stock at a time from Yahoo! Finance, and use them to estimate the joint distribution of asset returns. Table I shows some of the rows of the joint return distribution for IBM, Exxon Mobil, Newmont Mining, Procter & Gamble, and Starbucks for the months January 2005 through June 2007.

After the data for all five stocks has been downloaded students construct the joint return distribution for the stocks. The easiest way to do that is to calculate the time series of 30 monthly returns for each stock separately in its own spreadsheet, and then create a new spreadsheet for the joint distribution of returns

for the five stocks. Table I shows some of the rows of the joint return distribution for IBM, Exxon Mobil, Newmont Mining, Procter & Gamble, and Starbucks for the months January 2005 through June 2007.

Table I. Joint Distribution of Asset Returns with Summary Statistics

Date	IBM	XOM	NEM	PG	SBUX
6/1/2007	0.0126	0.0086	0.0375	0.0372	0.0892
5/1/2007	0.0470	0.0523	0.0243	0.0131	0.0712
4/2/2007	0.0843	0.0520	0.0069	0.0253	0.0108
3/1/2007	0.0142	0.0526	0.0668	0.0053	0.0149
•	•	•	•	•	•
•	•	•	•	•	•
1/3/2005	0.0523	0.0068	0.0637	0.0292	0.1341
Minimum	0.1642	0.1164	0.1642	0.0681	0.1341
Maximum	0.1268	0.2329	0.1945	0.1014	0.1457
Average	0.0048	0.0201	0.0001	0.0058	0.0030
Std. Dev.	0.0584	0.0642	0.0879	0.0361	0.0754

THE CORRELATION AND COVARIANCE MATRICES

To estimate the correlation matrix of stock returns, use Excel's *Data Analysis ToolPak*. Click on Tools|Data Analysis, and select *Correlation* in the dialog box for Data Analysis. Select all five columns of monthly returns data for the five stocks including the labels in the first row, check the *Labels in First Row* box, and then click OK. Excel will create a new worksheet containing the lower triangle of the symmetric 5x5 correlation matrix for returns of the five stocks. Table II shows the correlation matrix of asset returns for the five stocks in our example. Students can also estimate the covariance matrix of returns by repeating this procedure with the *Covariance* calculator available in the Data Analysis menu.

Table II. Correlation Matrix of Asset Returns

	IBM	XOM	NEM	PG	SBUX
IBM	1.0000				
XOM	0.2334	1.0000			
NEM	0.0523	0.4987	1.0000		
PG	0.2761	0.2006	0.3438	1.0000	
SBUX	0.3179	0.2011	0.0857	0.1317	1.0000

OPTIMIZATION

The algorithm used in QOS-15 is based on the *Active Set Method* of quadratic programming and takes advantage of the structure of portfolio optimization problems as described in Best and Kale [5], and Best and Hlouskova [3, 4]. The algorithm is extremely fast and compact, and works well for solving portfolio optimization problems with several thousand assets on personal computers.

Students create optimal portfolios consisting of just two stocks at a time, with different parameters and then compare the results. For the first optimization exercise they select the riskiest stock and the stock with the highest correlation with the first stock, and use the historical average returns for the two stocks as estimates for expected return. The data is entered into QOS-15 by using the data entry screens shown in Figures 1 through 3.

Figure 1. Objective Function Screen

Financometrics QOS Case 32 - Case Information

File

Optimize Display Reports Close Case

Objective Asset Data Attribute Constraints Covariance Matrix

Optimization Parameters

- Given Risk Aversion, Optimize Portfolio
- Given Risk Tolerance, Optimize Portfolio
- Given Expected Return, Minimize Risk
- Given Standard Deviation, Maximize Expected Return
- Given Threshold Return, Maximize Sharpe Ratio
- Given Sharpe Ratio, Maximize Threshold Return

Starting Value Ending Value No of Frontier Points

100 0 101

Case Constraints

- Turnover Constraint 0.1
- Maximum Assets in Portfolio 15

Figure 2. Asset Data Screen

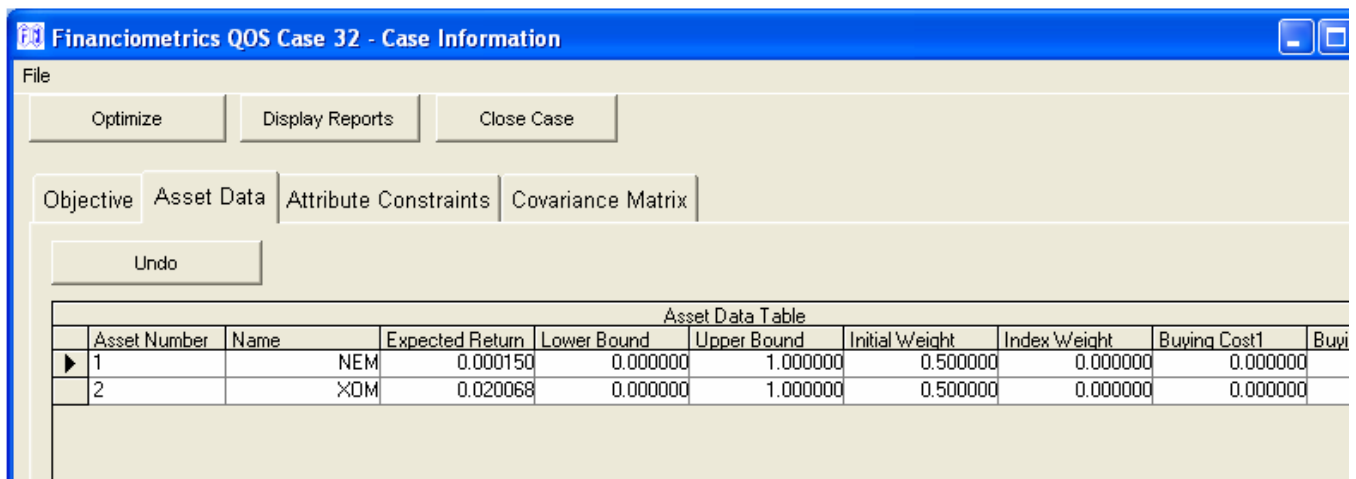
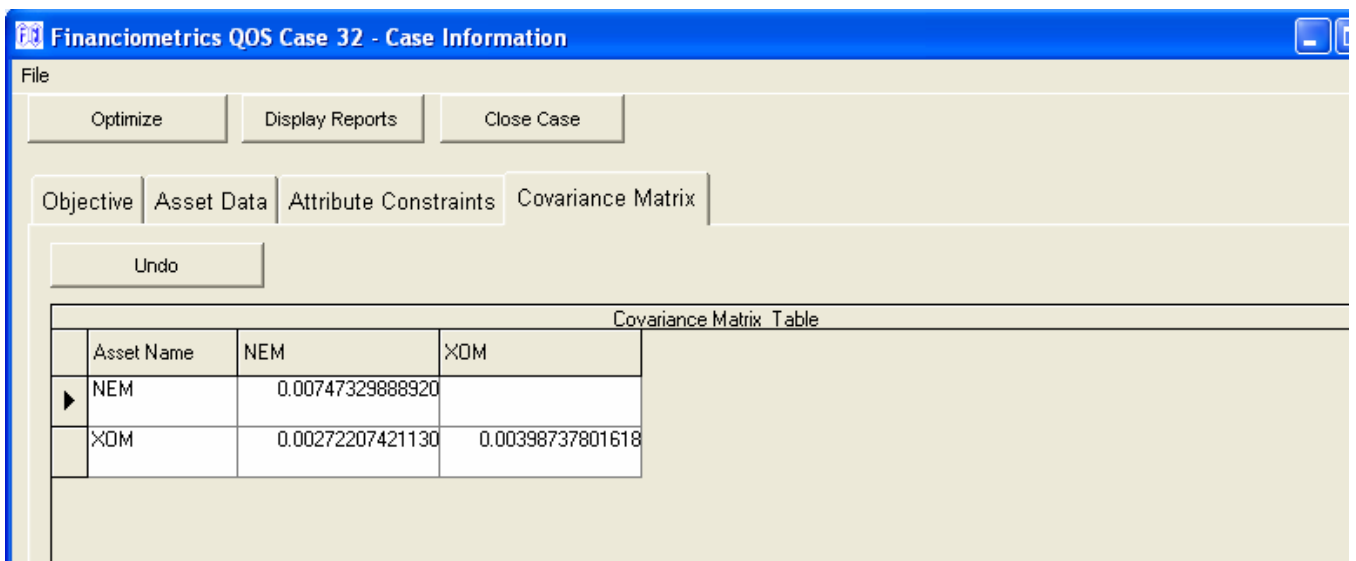


Figure 3. Covariance Matrix Screen



Once the data entry is complete, clicking the *Optimize* button generates the optimal portfolios. In our example QOS-15 constructs 101 optimal portfolios on the efficient frontier and displays a chart of the frontier. The expected return and standard deviation of the optimal portfolios are shown in Figure 4, and the composition of these optimal portfolios is shown in Figure 5.

Figure 4. Mean-Variance Efficient Frontier Table

The screenshot shows a software window titled "Financiometrics QOS Case 32 - Reports". It has three buttons at the top: "Back to Case Information", "Print Excel Reports", and "View in Excel". Below these are three tabs: "Optimal Weights", "Efficient Frontier", and "QOS Log". The "Optimal Weights" tab is active, displaying a table with the following data:

Frontier Point	Expected Return	Standard Deviation	Active Expected Return	Active Standard Deviation	Lambda	T-Factor	Threshold Return
1	0.016539	0.061056	0.016539	0.061056	100.000000	0.010000	-0.356249
2	0.016545	0.061057	0.016545	0.061057	99.000000	0.010101	-0.352527
3	0.016552	0.061059	0.016552	0.061059	98.000000	0.010204	-0.348806
4	0.016559	0.061060	0.016559	0.061060	97.000000	0.010309	-0.345085
5	0.016566	0.061061	0.016566	0.061061	96.000000	0.010417	-0.341363

The least risky portfolio is at Frontier Point No. 1, where the coefficient of risk aversion, λ , is equal to 100. It consists of an investment of just over 80% in Exxon Mobil. The large investment in Exxon Mobil is a result of its higher expected return and lower stand-alone risk when compared to Newmont Mining. However, Newmont Mining's investment weight is still substantial, at close to 20%, because the correlation between the two stocks is significantly lower than 1, and Newmont Mining provides diversification benefits to the portfolio (Markowitz [8]). As Table I shows, the standard deviation of return for this portfolio is 0.0611, or 6.11%, which is lower than the standard deviation for Newmont Mining (8.79%) and Exxon Mobil (6.42%).

Figure 5. Optimal Investment Weights Table with NEM and XOM

The screenshot shows the same software window as Figure 5, but with the "Efficient Frontier" tab selected. It displays a table titled "Optimal Investment Weights at Each Frontier Point (%)" with the following data:

Asset Name	1	2	3	4	5	6	7	8
NEM	17.7198	17.6864	17.6522	17.6174	17.5819	17.5456	17.5085	17.4706
XOM	82.2802	82.3136	82.3478	82.3826	82.4181	82.4544	82.4915	82.5294

As the value of λ drops, the optimizer generates portfolios with higher risk and higher expected return. When $\lambda = 0$, the optimizer constructs the most risky portfolio on the efficient frontier. It has 100% invested in Exxon Mobil since there is no penalty for risk, and Exxon Mobil has a higher expected return than Newmont Mining. For this portfolio, the diversification benefits associated with low correlation between stocks have no effect on the optimization.

For the second optimization exercise students replace the second stock with the stock which has the lowest correlation with the first stock, and then generate the efficient frontier with these two stocks. In our example, the two stocks are Newmont Mining and Starbucks. The changes in the composition of the optimal portfolios generated for the second optimization exercise are dramatic. The standard deviation of return for the least risky portfolio with this combination of stocks is 0.0539, or 5.39%, which is significantly lower than the standard deviation of 6.11% in the first optimization exercise.

In addition to the two optimization exercises described above, students estimate betas for the stocks, use the Capital Asset Pricing Model to estimate expected returns for the stocks, replace the average historical monthly returns with these CAPM expected returns, and then rerun the two optimization exercises. Typically the new estimates for expected return result in a big change in the optimal portfolios, and students can immediately appreciate the difference between historical average returns and possible future returns for stocks.

CONCLUSION

The optimization exercises described in this case study have proved to be very useful for introducing students to Markowitz-style mean-variance optimization. The process of gathering data, estimating the parameters of the joint distribution of asset returns, and then using QOS-15 to construct mean-variance efficient portfolios bridges the divide between theory and practice very effectively.

ENDNOTES

For a detailed literature survey and references, please contact the authors.