A GENERALIZED SEQUENTIAL IMPERFECT PREVENTIVE MAINTENANCE POLICY WITH AGE-DEPENDENT MINIMAL REPAIR COST

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ABSTRACT

This paper presents a generalized sequential imperfect preventive maintenance (PM) policy with age-dependent minimal repair cost for repairable systems. The imperfect PM model proposed in this study incorporates with improvement factors in hazard rate function. After each PM, the life distribution of the system changes in such way that its hazard rate function increases with the number of PMs carried out. PM is performed in a sequence of intervals. The objective is to determine the optimal PM schedule to minimize the expected cost per unit time for an infinite-horizon case. It is shown that under certain reasonable assumptions, a sequential PM policy has unique solution.

INTRODUCTION

Preventive maintenance (PM) is an important activity to restore or keep the function of a repairable system in good state. A PM policy specifies how PM activities should be scheduled. One of the popular PM policies is the Periodic PM, which specifies that a system is maintained at integer multiples of some fixed period [4]. Another popular PM policy is Sequential PM, in which the system is maintained at a sequence of intervals that may have unequal lengths [8][9]. Periodic PM is more convenient to schedule, whereas sequential PM is more realistic when the system requires more frequent maintenance as it ages.

A PM model describes the effects of a PM activity. Perfect PM (or simply replacement) models assume that the system is 'as good as new' immediately after PM. Minimal repair models assume that the system hazard rate function was not disturbed by any minimal repair of failures, i.e., the system is 'as bad as old' after minimal repair [1]. The more realistic assumption is that the system after PM lies somewhere between as good as new and as bad as old. Applying this assumption to PM is known as the imperfect PM model [3][7]. Pham and Wang [10] summarized and discussed various methods and optimal policies for imperfect maintenance. One way of modelling an imperfect PM activity is to consider that PM is equivalent to minimal repair with probability p, and equivalent to replacement with probability $1-p$ [2]. Another approach is to model the PM effects directly by using the system effective age or the hazard rate function [5][9].

Nguyen & Murthy [9] considered the sequential imperfect PM models. Their basic assumption was that the life distribution of the system changes after each of the maintenances in such way that its hazard rate function increases with the number of maintenances carried out. Malik [5] introduced improvement factors to model the effect of maintenance actions. Nakagawa [6] considered improvement factors in hazard rate function and effective age after PM. Nakagawa [8] adopted improvement factors in hazard rate function: the hazard rate function in the next PM interval becomes $ar(t)$ when it was $r(t)$ in the previous PM interval, where $a \geq 1$ is the improvement factor and $t \geq 0$ represents the time from the previous PM. Nakagawa [8] also assumed that the hazard rate function right after PM reduces to zero and the rate of increase of the hazard rate function gets higher after PM.

In this paper, a generalized sequential imperfect preventive maintenance policy with age-dependent minimal repair cost for repairable systems is proposed and analyzed. The model incorporates with improvement factors in hazard rate function. The major assumption is that the life distribution of the system changes after PM in such a way that its hazard rate function increases with the number of PMs carried out. PM is performed in a sequence of intervals.

GENERAL MODEL

First, we assume that a system deteriorates with time as well as with the number of PMs carried out. After PM, the system is operational but its failure characteristic is altered as follow:

1. Let $r_i(t)$ denote the hazard rate at time t of the system subjected to $i-1$ PM(s) for $i = 1, 2, \cdots, N$, where t is the time from the last PM, viz., $t \in (0, T_i)$, and $r_1(t) = r(t)$. We assume that $r(t)$ is a continuous and positive increasing function for $t \geq 0$ and $r(0) = 0$.
2. The hazard rate $r_i(t)$ after the $(i-1)$ th PM becomes $a_{i-1} r_{i-1}(t)$, where a_{i-1} is the improvement factor in hazard rate after the $(i-1)$ th PM and $1 = a_0 < a_1 \leq \cdots \leq a_{N-1}$.

We consider a generalized sequential imperfect preventive maintenance model in which minimal repair, maintenance or replacement take place according to the following schemes:

1. A system has tow types of failure when it fails. A type □ (minor) failure is corrected by a minimal repair, whereas a type □ failure (catastrophic failure) is removed by an unplanned PM (or unplanned replacement if $i = N$) for a system subjected to $i-1$ PM(s).
2. Replace the system after $N-1$ PM(s).
3. For a system subjected to $i-1$ PM(s), it is preventive maintained (or replaced if $i = N$) at the time of type □ failure or at age T_i (T_i is the time from the last PM or replacement), whichever occurs first. A PM (or replacement) occurring at the time of type □ failure is identified as an unplanned PM (or unplanned replacement), while a PM (or replacement) occurring at the planned age T_i is identified as a planned PM (or planned replacement).

4. If the failure of a system subjected to $i-1$ PM(s) is at time $t\ (<T_i)$, it undergoes either unplanned PM with probability p_i (type □ failure) or minimal repair with probability $q_i(=1-p_i)$ (type □ failure). It is reasonable to assume that p_i increases with the number of PMs.

We define the following various costs:

1. Let c_0 = cost of planned PM and c_B = additional cost to unplanned PM or replacement, then, the cost of unplanned PM = c_0+c_B. Let c_R = cost of planned replacement, then the cost of unplanned replacement = c_R+c_B. We assume that $c_R>c_0>0$.
2. Let $C_i(t)$ = the cost of the minimal repair of a system subjected to $i-1$ PM(s) at age $t\ (<T_i)$. Suppose that $C_i(t)$ in the ith period of PM is a random variable with distribution $L_i(x)$, density function $l_i(x)$ and finite mean $E[C_i(t)]$.
3. Finally, the following hypotheses are made:
4. After a replacement the procedure is repeated. Replacement are made perfectly and do not affect the system's characteristics.
5. All failures are instantly detected and repaired. All replacement, maintenance and minimal repair times are negligible.

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