INVENTORY CONTROL SUBJECT TO RANDOM STOPPAGE IN SUPPLY AND NON-NEGLIGIBLE REPLENISHMENT LEAD TIME

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ABSTRACT

Supply interruptions are known for their negative impacts on managing inventories. This paper presents an overview of a replenishment model for an inventory system facing uncertain demand that orders from an unreliable supplier. Some numerical results are also discussed.

INTRODUCTION

Managing inventories when the replenishment process—in addition to the inherent volatility of demand— is subject to various sources of uncertainty is an arduous task which often requires the use of sophisticated supporting tools such as analytical models to incorporate the impacts of such uncertainties into the decision-making process. Naturally, the uncertainty of a supply process can be an attribute of various elements such as production yield and/or capacity, product quality, replenishment lead time, supplier's availability, or a combination of thereof. This paper is focused on supplier's availability (and its ensuing impact on supply disruptions) in the presence of a random replenishment lead time.

While much of the emphasis on supply chain management seems to be on efficiency or reducing costs, a recent study indicates that overemphasis on efficiency can make the supply chain brittle and more susceptible to the risk of disruptions. The study documents that based on a sample of 885 disruptions ('*glitches*') reported by various companies, firms that experience disruptions face on average 6.92% lower sales growth, 10.66% growth in cost, and 13.88% growth in inventories ([1], [2]). A wide variety of events (e.g., equipment failures, products rationing, strikes, storms, embargoes, terrorist attacks, war, and political crises, to name a few) are known to have caused unexpected disruptions in supply of various products by rendering a source of supply unavailable for an uncertain duration of time. For example, high-tech companies such as Apple, Motorola and Sony have periodically reported problems with meeting their demands due to shortages of key components caused by unforeseen circumstances ([4]). Numerous other examples of such phenomenon in the automotive and computer manufacturing industries have been cited in the literature (see [5], among others). From an inventory control standpoint, a typical exposition of the supply-disruption problem involves an *unreliable* supplier whose states alternates randomly between an available (*on*) and an unavailable (*off*) state. When the supplier is in the *on* state, the system functions as an ordinary inventory system with a fully reliable (always available) supplier. However, when the supplier switches from the *on* to the *off* state, it cannot accept any orders for the duration of the *off* period. Most papers in this category focus on incorporating an unreliable supplier into the classic *EOQ*-type inventory setting (i.e., deterministic demand and negligible lead time) by using various forms of probability distributions to characterize the *on* and *off* periods. Several studies have introduced the concept of an unreliable supplier to inventory models with random demand and negligible lead time. There are, however, only a few analytical treatment of the supply-disruption problem reported in the literature that allow for a replenishment lead time in the context of an inventory system. This is mainly due to the fact that the introduction of lead time adds yet another layer of difficulty in modeling an already complex problem. This note presents an overview of an inventory model as part of an on-going research effort to broaden the scope of analytical treatment of inventory systems with random supplier availability and non-negligible lead time.

In what follows, we first present an expository description of our model, and proceed with reviewing a sample of our numerical results.

INVENTORY MODEL

Consider a continuous-review inventory system with an unreliable supplier where demand follows a compound Poisson process; i.e., demand occurs at random epoch according to a Poisson process and demand sizes form an independent identically distributed (iid) sequence of random variables. The unreliable supplier alternates randomly between two possible *on* and *off* states according to an irreducible continuous-time Markov chain. All shortages, including the excess demand when it exceeds the inventory level (stock-on-hand), are considered to be lost.

We allow for an (s,Q)-type control policy with a maximum of one order outstanding at any time. According to this policy, a replenishment order of size Q is triggered as soon as the inventory level drops to or below the reorder level s $(0 \leq s < Q)$ at a demand occurrence epoch. If the supplier is found in the *on* state at this epoch, the order is immediately 'accepted;' otherwise, the triggered order is placed on 'hold' for the remainder of the *off* period until such time that the supplier switches back to the *on* state—upon which the order-on-hold is accepted instantaneously. Furthermore, unlike the previous studies, the model assumes that once an order is accepted, the processing of such outstanding order is disrupted at every epoch where the supplier switches from the *on* to the *off* state and remains halted for the duration of the *off* period. We also assume that the processing of such order is 'restarted' from the outset when the supplier switches back to its *on* state. In other words, the model assumes that every disruption caused by the supplier's switch from the *on* to the *off* state results in a complete loss of the partial work already completed on an outstanding order. Note that such impact of disruptions on order processing is common in chemical, pharmaceutical, food, casting, pulp and paper, and similar types of process industries. We assume that the lead time L, defined as the processing time of an order by a fully reliable (always available) supplier, follows a k-stage Erlang distribution with mean $E(L)$. It should be clear that due to the supplier's potential repeated alternations between the *on* and *off* states, the 'actual' lead time experienced by an order from the time of its acceptance to the point of its delivery is likely to be longer than L.

From an analytical standpoint, our model focuses on the steady-state behavior of the inventory system described above and provides an explicit closed form expression for the long-run average total cost per unit time function for a case where demand sizes are exponentially distributed. The total cost per unit time is defined as a function of the control policy parameters s and Q, and is equal to the sum of average ordering, carrying and shortage (including lost sales opportunity) costs per unit time.

NUMERICAL RESULTS

This section contains a sample of our numerical results. For expository purposes, we compared the results of the present model (labeled as 'Restart' here-in-after) with those of an earlier model [1] (labeled as 'Resume') under the same parameter settings. It should be noted that the main difference between these two models lies in how each model treats an outstanding order when the supplier switches from the *on* to the *off* state; namely, under the 'Resume' scenario, the processing of the disrupted order is 'resumed' at the conclusion of an *off* period without any loss of work already completed on the order.

Our numerical results validate that when k=1 (i.e., the lead time is exponentially distributed), the 'Resume' and 'Restart' models are identical. This is clearly because of the memoryless property of the exponential distribution. Furthermore, the average cost, stock-out-risk (i.e., probability of a stock-out at a demand occurrence epoch) and optimal order quantity f the 'Restart' model for all other shapes of the lead-time distribution (k>1) are considerably larger than those of the 'Resume' model. This can be attributed to the fact the 'actual' lead time experienced by an order under the 'Restart' model is more likely to be longer than that of the 'Resume' model under similar circumstances. Interestingly enough, the two models display opposite behaviors with respect to these measures as the shape parameter of the lead-time distribution increases beyond k=1. More specifically, while the average cost, stock-out risk and order quantity resulted from the 'Resume' model decrease rapidly as the shape of the lead-time distribution changes from exponential (k=1) to multi-stage Erlang (k>1) distribution, similar measures for the 'Restart' model display an increasing pattern as k increases. Given that the coefficient of variation of the lead-time distribution decrease as k increases, this may be viewed as a somewhat counter-intuitive result. However, this behavior can be explained by noting that the density function of the Erlang distribution has a considerable positive skew for smaller k values while shifting towards a more symmetrical shape as k increases. Finally, the reorder levels for both models increase as k increases beyond one.

CONCLUDING REMARKS

Supply disruptions have become increasingly known for their significant negative impact on effective management of inventories. Hence, analytical treatment of replenishment models that can take into account such phenomenon has emerged as a resurrected area in the field of inventory control. This note presented an overview of an inventory model as part of an ongoing research effort which examines the confluence of supplier's availability and lead-time uncertainty on replenishment decisions.

It should be pointed out that the form of the optimal control policy for the class of inventory systems considered here is an open research question. As such, in addition to eliciting analytical tractability in the modeling process, the selection of an (s,Q)-type control policy in this work is mainly attributed to the popularity of quantized ordering policies in practice.

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