**APPLICATION OF MULTIPLE DISCRIMINANT ANALYSIS (MDA) TO PREDICT UTILITY BOND RATINGS**

*Steven V. Le, College of Business, California State University Long Beach, 1250 Bellflower Blvd. Long Beach, CA 90840, (562) 985-1608 or 4569,svle@usa.net*

**ABSTRACT**

Financial ratio analysis is a useful but limited tool in analyzing the performance of a business. A major limiting characteristic is the univariate nature of the technique. To overcome this limitation, analysts have used multiple discriminant analysis to develop multivariate predictive models. The purpose of this paper is to extend the multiple discriminant technique to predict bond ratings.

**INTRODUCTION**

This study is analogous to Altman's bankruptcy case. All members of the sample are members of one of two a priori groups and a linear equation can be developed to classify each member into its proper group. In the bond rating case, the sample would consist of utilities having one of two adjacent bond ratings, say an 'A' rating or a 'BBB' rating. The model developed using multiple discriminant analysis would classify each member of the sample into either the 'A' rating or 'BBB' rating group.A rigorous application of the multiple discriminant technique involves collection of financial ratios, perform calculation, and the results which analyzed to determine a combination that effectively classified the sample members into the correct priori groups.

**Multiple Discriminate Analysis**

Multiple discriminant analysis (MDA) used to classify an observation into one of several a priori groupings based upon the observation's individual characteristics. After the group classifications are established and financial data collected, MDA then attempts to derive a linear combination of the characteristics which best discriminates among the groups. The linear equation is of the form

Z = V1X1 + V2X2, (1)

Where V1 and V2 are the discriminant coefficients and X1 and X2 are independent variables.

The problem is to determine V1 and V2 through the use of historical data that makes Z useful as an index for discriminating among members of the two groups. To minimize the number of misclassifications, we need to separate the Z values of the two groups as widely as possible relative to the variations of Z within the groups. To do this, we find values of V and V that maximize the function.

$G=\frac{\left(\overbar{Z}\_{1}-\overbar{Z}\_{2}\right)^{2}}{\sum\_{i=1}^{2}\sum\_{j=1}^{n}\left(Z\_{1j}-\overbar{Z}\_{i}\right)^{2}}$ (2)

The numerator represents the separation of the two groups and the denominator represents the within group variation. V1 and V2 can be determined using the following equations (derived by partial differentiation):

V1S11 + V2S12 = d1 (3)

V1S21 + V2S22 = d2 (4)

Where:

$S\_{pq}=\sum\_{i=1}^{2}\sum\_{j=1}^{n}\left(X\_{pLj}-\overbar{X}\_{pL}\right)\left(X\_{qij}-\overbar{X}\_{qi}\right)$ (5)

$$d = \left(\overbar{X}\_{pi}-\overbar{X}\_{pL}\right)$$

p = 1, 2

q = 1, 2

**Development of the Model**

Previous MDA models of bond ratings typically predicted ratings with 60-70% accuracy. Although an improvement over random prediction, it is arguable whether this is an improvement over the results which would be obtained through use of an expert financial analyst. Perhaps one reason for the results obtained is the non-homogeneity of the sample. The intent of course is to develop a model that is applicable to a wide range of situations but the internal assumption that the rating services apply identical criteria regardless of the debtor's size. To obtain a homogeneous sample for use in this study, the initial list of all companies with an 'A' or 'BBB' rating was screened to eliminate the very large (greater than $1 billion in annual revenues) and the very small (less than $100 million) firms as well as any companies operated by municipalities. From the firms remaining, five were eliminated from the sample because accurate data on construction expenditures could not be obtained in the time frames available. One last screen was conducted to eliminate firms whose bond rating changed from 'A' to 'BBB' or from 'BBB' to 'A' within six months after the sample date. This screen eliminated one firm. A total of 26 firms remained after this screening process, 13 with an 'A' rating and 13 with a 'BBB' rating. Although the initial plan was to perform a stratified sample of the companies comprising the population, given the manageable size of the 26 firm samples it was decided to develop the MDA model using all of the remaining firms. The average size of the firms comprising the 'A' rating group and the 'BBB' rating group was comparable. After the initial groups were defined and firms selected, financial data were collected. A list of eight ratios was compiled. Ratios used included profitability, leverage, liquidity, activity, and cash flow. Two ratios were selected as doing the best job of classifying the companies into the correct groups and are discussed below.

X1 - AFUDC/Net Income. A principle used in establishing electric utility rates is that current rate payers should pay only the cost of providing current service. Electric generating plants are expensive and take years to construct. Large amounts of funds are tied up in construction work in progress yet regulators will not allow a current return on the assets. If no recourse were available to the companies, it is doubtful that generating plants could continue to be built.

X2 - Cash Flow/Construction Expenditures. Cash flow is calculated as net income plus depreciation less AFUDC. Standard and Poor’s has indicated that a utility's construction program is a key component of S&P's assessment of the utility's credit worthiness. As noted above, electric generating plants are expensive and time consuming to build, so the risk of default increases with the relative magnitude of ongoing construction work. A utility usually requires outside financing funding construction, but like any other business, a measure of the utility's long term survival potential is the level of funding that can be provided from internal sources. With regard to establishing bond ratings this ratio provides a good indicator of the funds that will be available to retire debt issues. A high level of internally generated funds should be viewed by the bond holder as a positive factor and, conversely, a low level of internally generated funds should be viewed as a negative factor by the bond holder and should be associated with lower rated bonds. The discriminant function calculated using these ratios is:

Z = -0.0014X1 - 0.1236X2  (6)

An ‘F’ test tests the discriminating ability of the individual variables and of the discriminate function. The null hypothesis is that the two groups are identical. If the F-value is less than the expected F-value for a given significance level, then the null hypothesis is supported. In Table 1, X1 and X2 are significant at the .001 level so the null hypothesis is rejected. The Z values are statistically significant and presumably useful for classifying observations into the different bond rating groups.

**Table 1**

|  |  |  |
| --- | --- | --- |
|  | Group Means |  |
| Variable | A Rating | B Rating | F Ratio |
| X1 | 46.72 | 2.11 | 16.56 |
| X2 | 1.39 | 0.39 | 24.23 |
| Z | -0.24 | -0.05 | 20.18 |

F[1,20] @ 0.001 = 14.82

**Empirical Results**

The results below displayed in a two by two classification matrix that allow assessment of the overall accuracy of the discriminant equation as well as the level of Type I and Type II error.

|  |  |
| --- | --- |
|  | Predicted Group Membership  |
| Actual Group Membership | A Rating | B Rating |
| A Rating | correct | Type I |
| BBB Rating | Type II | correct |

The calculated Z values has a range of 0.0224 to 0.4321 with the A rating group having the higher values. The highest score in the BBB group is .1066 and the lowest score in the A group is .0572. The arithmetic average of these two extreme values is 0.0819. Observations with a Z value greater than .0819 are assigned to the A group and observations with a Z value less than .0819 are assigned to the BBB group.

The classification matrix for the sample is as follows:

**Table 2**

|  |  |
| --- | --- |
|  | Predicted Group Membership  |
| Actual Group Membership | A Rating | B Rating  |
| A Rating | 11 | 2 |
| BBB Rating | 1 | 12 |

**Table 3**

|  |  |  |  |
| --- | --- | --- | --- |
| error | # correct | % correct | % error |
| Type I | 11 | 85% | 15% |
| Type II | 12 | 92% | 8% |
| TOTAL | 23 | 88% | 12% |

Based on the above, the model is highly accurate in assigning observations to the correct a priori grouping. Type I error proved to be 15% and Type II error an even more impressive 8%. The overall accuracy of the model is 88%.

**CONCLUSION**

The MDA model developed is characterized by its ease of calculation because of the use of only two variables, yet proved highly accurate in predicting the correct bond rating of 88% of the firms in the initial sample. The study indicates that ratios used in either univariate or multivariate analysis should be uniquely tailored to the specific characteristics of the industry under study.