STRATEGIC INVENTORIES IN A TWO-PERIOD STACKELBERG DUOPOLY WITH VERTICAL CONTROL

Vijayendra Viswanathan, Department of Industrial Engineering, University of Wisconsin-Milwaukee, 3200 N Cramer St, Milwaukee, WI 53211, 414-229-5786, viswana2@uwm.edu
Jaejin Jang, Department of Industrial Engineering, University of Wisconsin-Milwaukee, 3200 N Cramer St, Milwaukee, WI 53211, 414-229-5643, jang@uwm.edu

ABSTRACT

We investigate the role of strategic inventory between periods, in a one-manufacturer, two retailer, two-period ordering Stackelberg Duopoly. One of the retailers is the Stackelberg leader and the other, follower. We find that strategic inventories strengthen the competitive advantage of the leader firm and weaken that of the follower.

Keywords: Strategic Inventories, Vertical Control, Stackelberg Duopoly, two-period ordering

INTRODUCTION AND LITERATURE REVIEW

Decentralized Supply Chain (SC) coordination deals with the problem of aligning the operational decisions of individual SC entities who are free to make their operational decisions to serve their own individual interest to the best interest of the SC as a whole in order to ensure the product reaches the consumer at the least possible cost.

Recently, there's been increased interest in strategic inventories, which are inventories carried by SC entities for purely strategic reasons, even in the absence of the "traditional" reasons to hold inventory. Traditional reasons for holding inventory at a SC entity (e.g., manufacturer, retailer and distributor) have been economies of scale in production resulting in cycle inventories, to hedge against production or distribution delays resulting in pipeline inventories, inventories held as safety stock, inventories held to hedge against price fluctuations, termed speculative inventory [1]; and also inventories held to smooth production and thus lower production costs [4].

Most discussions on inventory in SC coordination literature focus on the development of coordinating contracts. Tsay [13] reviews supply chain contract literature focusing on stochastic and deterministic demand. Cachon [2] presents a literature review on uncertain demand models that reviews five different types of contracts – buyback, quantity flexibility, sales rebate, wholesale price and revenue sharing elucidating their advantages and drawbacks. He also addresses important classes of SC coordination problems such as coordination with horizontal competition, multi-period ordering, asymmetric information, information sharing and two-location base-stock models with forecast updating.

Anand et. al. [1] is one of the first papers that study strategic inventories and vertical control together in a multi-period ordering environment. Keskinocak et. al. [7] extend the basic one manufacturer, two-retailer, two-period model in [1] to the cases in which the manufacturer's first period capacity is limited. Earlier economics literatures study either vertical control in a non cooperative environment ([3], [12]) or in horizontal competition ([6], [9], [10]). But none before [1] study inventories and vertical control together.

Matsumura [5] studies strategic inventories in a Cournot Duopoly without vertical control. Mujumdar et. al. [11] study a two-period ordering Cournot Duopoly with no vertical control (competing retailers in the same echelon) and study managerial incentives to discourage strategic inventory holding.

Viswanathan and Jang [14] extend the discussion on strategic inventories with vertical control from a one-manufacturer, one-retailer case as in [1] and [7] to the case with downstream retailer competition. They show that incentive to hold strategic inventories is diminished when there are two retailers competing against each other as a Cournot Duopoly and also that strategic inventories are only optimal in a narrow range of reservation prices. In this paper, we extend these results to a Stackelberg Duopoly to find the effects of "leader-follower" type competition on strategic inventories in a two-period model with vertical control.

THE MODEL

The Stackelberg competition model is a strategic game in economics in which a leader moves first and then the followers move sequentially; competing on the quantity of product each one sells in a market. Firms may engage in Stackelberg competition if one of them has some sort of advantage enabling it to move, or make a decision, first.

The model under consideration here consists of one manufacturer supplying to two downstream retailers over two periods. Retailer 1 is assumed to be the "Stackelberg Leader" and moves first in both periods. Retailer 2 observes Retailer 1's decision and then formulate his/her own decision in both periods; being the "Stackelberg Follower".

Retailers can carry inventory between the periods. The product's unit price in the market is given by the price-dependent linear demand function $p(q) = a-bq$, where q is the total quantity being sold in the market in a period, and a and b are positive parameters. Both retailers are assumed to be homogeneous, i.e., sell identical products, with similar holding cost structures. Both retailers can sell the quantities they want at the price determined by the market.

The steps of the game in the first period are:

1. The manufacturer quotes the wholesale price for the 1st period.
2. Retailer 1 (leader) sets the quantity he will sell in period 1 and the inventory he wants to carry into period 2.
3. Retailer 2 (follower) sets his corresponding selling and inventory quantities for period 1, after observing the leader's move.
4. Retailer 1 and 2 buy their set quantities at the wholesale price set by the manufacturer.
   Both retailers sell their first period selling quantities at the market price determined by the total selling quantity for the period, realizing their profits or losses.

Similarly, in the second period, the steps of the game are:

1. The manufacturer quotes the retail price per unit for the 2nd period.
2. Retailer 1 sets the quantity he will sell in period 2 considering its inventory from period 1.
3. Retailer 2 sets his corresponding 2nd period selling quantity considering its inventory from period 1.
4. Retailer 1 and 2 buy the quoted quantities at the wholesale price set by the manufacturer.
5. Both retailers sell their second period quantities-on-market (quantity ordered + inventory from 1st period) at the market price for the period.

ANALYSIS

q_{ij} → order quantity of retailer *j* in period *i* (i = 1,2; j = 1, 2)
I_j → inventory carried by retailer *j* from period 1 to period *2*
w_i→ wholesale price set by the manufacturer in period *i*
h → holding cost of either retailer to carry one unit of inventory from period 1 to period 2
Πr_{ij}→ Profit of retailer *j* in period i
Πm_i→ Profit of the manufacturer in period *i*

We solve this game for sub-game perfect Stackelberg Duopoly Equilibrium to get closed form expressions for order quantities and inventory.

Second period decisions:
The second period profit function of retailers 1(leader) and 2 (follower) are as follows:

$$\Pi r_{21} = \ a - b(q_{21} + I_1 + q_{22} + I_2)](q_{21} + I_1) - w_2 q_{21}\ . \quad (1)$$
$$\Pi r_{22} = \ a - b(q_{21} + I_1 + q_{22} + I_2)](q_{22} + I_2) - w_2 q_{22}\ .$$
(2)

Taking the 1st derivative of (2) with respect to q22, we get the optimal second period order quantity for the follower as:

$$q_{22} = \frac{a - w_2}{2b} - I_{12} - \frac{q_{21} + I_{11}}{2} \quad . \quad (3)$$

Now, retailer 1 conjectures retailer 2's output decision is (3). Substituting (3) into (1) and setting the first derivative of (1) with respect to q_{21} to zero, we have retailer 1's 2nd period order quantity as:

$$q_{21} = \frac{a - w_2}{2b} - I_1\ . \quad (4)$$

Substituting (4) into (3) we have retailer 2's order quantity:

$$q_{22} = \frac{a - w_2}{4b} - I_2\ . \quad (5)$$

Now, manufacturer's 2nd period profit function is

$$\Pi m_2 = w_2(q_{21} + q_{22})\ . \quad (6)$$

Substituting (4) and (5) into (6) and setting the first derivative of (6) with respect to w_2 to zero, we obtain the 2nd period manufacturer's equilibrium wholesale price as

$$w_2 = \frac{a}{2} - \frac{2b}{3}(I_1 + I_2)\ . \quad (7)$$

Substituting (7) into (4) and (5) we obtain the optimal order quantities in period 2:

$$q_{21} = \frac{a}{4b} + \frac{1}{3}\left(I_2 - 2I_1\right) \; ; \; q_{22} = \frac{a}{8b} + \frac{1}{6}\left(I_1 - 5I_2\right). \tag{8}$$

First period decisions:
The first period problem for the retailer is minimizing his profit of the first and second periods:

$$\text{Max } \Pi r_{12} = [\,a - b(q_{11} + q_{22})](q_{12}) - w_1(q_{12} + I_2) - hI_2 + \Pi r_{22.}$$
$$= a - b(q_{11} + q_{12})](q_{12}) - w_1(q_{12} + I_2) - hI_2 + \frac{a^2}{64b} + \left(I_1 + I_2\right)\left(\frac{a}{24} - \frac{2b}{3}I_2\right) + \frac{b}{36}\left(I_1 + I_2\right)^2 + \frac{a}{2}I_2 . \tag{9}$$

By setting the first derivatives of (9) with respect to q_{12} and I_{12}, respectively, to zero we get

$$q_{12} = \frac{a - w_1}{2b} - \frac{q_{11}}{2} \; ; \; I_2 = \frac{39a}{92b} - \frac{18}{23b}\left(w_1 + h\right) - \frac{11}{23}I_1 . \tag{10}$$

Retailer 2's 1[st] period problem is maximizing

$$\Pi r_{12} = \left(a - b\left(q_{11} + q_{12}\right)\right)\left(q_{11}\right) - w_1\left(q_{11} + I_1\right) - hI_1 + \Pi r_{12.}$$
$$= \left(a - b\left(q_{11} + q_{12}\right)\right)\left(q_{11}\right) - w_1\left(q_{11} + I_1\right) - hI_1 + \frac{+a^2}{32b} + \left(I_1 + I_2\right)\left(\frac{a}{12} - \frac{2b}{3}I_1\right) + \frac{b}{18}\left(I_1 + I_2\right)^2 + \frac{a}{2}I_1 \quad . \tag{11}$$

Substituting (10) into (11) and setting the first derivative of (11) with respect to q_{11} and I_1 to zero, we get

$$q_{11} = \frac{a - w_1}{2b} \; ; \; I_1 = \frac{21a}{44b} - \frac{9}{11b}\left(w_1 + h\right) - \frac{5}{11}\left(I_2\right). \tag{12}$$

From (10) and (12) we obtain

$$I_1 = \frac{1}{2b}\left(\frac{a}{2} - \left(w_1 + h\right)\right); \; I_2 = \frac{1}{46b}\left(\frac{28a}{2} - 25\left(w_1 + h\right)\right). \tag{13}$$

Now, the manufacturer determines the first period wholesale price w_1 by

$$Max \; w_1 \left(q_{11} + q_{12} + I_1 + I_2\right). \tag{14}$$

Substituting (10) and (12) into (14) and setting its first derivative with respect to w_1 to zero, we get

$$w_1 = \frac{1}{50}\left(37a - 24h\right). \tag{15}$$

Substituting (15) into (13), we get the optimal inventory levels:

$$I_1 = \frac{1}{50b}\left(37a - 13h\right); \; I_2 = \frac{-\left(9a + 26h\right)}{92b}. \tag{16}$$

Also, substituting (15) into (10) and (12), we get the optimal order quantities:

$$q_{11} = \frac{1}{100b}(13a + 24h); \quad q_{12} = \frac{1}{200b}(13a + 24h). \tag{17}$$

Substituting (13) into (7), we have the optimal 2nd period whole sale price:

$$w_2 = \frac{1}{1725}(124a + 26h). \tag{18}$$

Also, substituting (16) into (8), we have the optimal 2nd period order quantity:

$$q_{21} = \frac{1}{3450}(923h - 952a); \quad q_{22} = \frac{1}{6900}(2276a + 1325h). \tag{19}$$

DISCUSSION

We note that the leader (retailer 1) has a big advantage over the follower (retailer 2) in this Stackelberg game. The leader's 1st period order quantity which is the sum of his 1st period selling quantity and strategic inventory carried into the 2nd period (q_{11}+I_1) is significantly greater than the follower's corresponding order quantity. (from equations 16 and 17). The leader's 1st period selling quantity is higher than the follower (comparing q_{11} and q_{12} from (10) and (12)). Also, it should be noted that the inventory quantity for the follower is negative, implying that the follower will not carry any inventory into the 2nd period.(equation 16).

We also observe that the second period order quantity for the leader (q_{21}) is positive only for a < 0.97h, and usually, we have a>>h. So, we can say, over a wide-range of holding costs and reservation price (*a*), the leader carries enough inventory from the first period to fully meet 2nd period demand, without needing to order in the 2nd period. We see that this is an advantageous strategy for the leader, since we also note that the equilibrium wholesale price set by the manufacturer in the second period is higher than the first. The follower is now forced to buy all his second period selling quantity at this higher wholesale price thus further lowering his profits since he is not carrying any inventory from the 1st period.

So, strategic inventories increase the advantage the leader inherently has over the follower in a two-period ordering leader-follower Duopoly. Strategic Inventory carriage further increases the disadvantage that the follower has in a leader-follower model.

On the other hand, the follower could negotiate some kind of transfer payment structure with the leader, or negotiate some kind of arrangement that caps the inventory the leader can carry to allow for the follower to survive and still make profit rather than risk being quickly swallowed up by the leader. Design of effective contracts for these situations would be another interesting avenue for further research.

REFERENCES

[1] Anand, Krishnan, Anupindi, Ravi, Bassok, Yehuda. Strategic Inventories in Vertical Contracts. Management Science, 54: 1792-1804

[2] Cachon, G. 2003. SC coordination with contracts. Handbooks in Operations Research and Management Science: SC Management, Eds. Steve Graves and Ton de Kok, North-Holland.

[3] Deneckere, R., Howard P.Marvel, and James Peck (1996).Demand Uncertainty, Inventories and Resale Price Maintenance, The Quarterly Journal of Economics 111(3), 885–913.

[4] Holt, C., F. Modigliani, J. Muth, H. Simon. 1960. Planning Production, Inventories, and Work Force. Prentice Hall, Inc.
[5] Matsumura, Toshihiro, 1999. Cournot Duopoly with Multi-period Competition: Inventory as a Coordination Device, Australian Economic Papers, Blackwell Publishing, vol. 38(3), pages 189-202
[6]Mollgaard, H. Peter, Sougata Poddar, Dan Sasaki. 2000. Strategic inventories in a two-period oligopoly. Department of Economics, University of Exeter, Exeter, UK.
[7] P. Keskinocak, K. Charnsirisakskul, and P. Griffin (2008), "Strategic Inventory in Capacitated Supply Chain Procurement," Managerial and Decision Economics, Vol. 29, No. 1, 23-36.
[8] Pal, Debashis, 1991. "Cournot Duopoly with two production periods and cost differentials," Journal of Economic Theory, Elsevier, vol. 55(2), pages 441-448, December.
[9] Rotemberg, Julio J., Garth Saloner. 1989. Cyclical behavior of strategic inventories. Quarterly Journal of Economics 104(1) 73–97.
[10] Saloner, Garth. 1986. The role of obsolescence and inventory costs in providing commitment. International Journal of Industrial Organization 4 333–345.
[11] Sudesh Mujumdar, Debashis Pal, Strategic managerial incentives in a two-period Cournot Duopoly, Games and Economic Behavior, Volume 58, Issue 2, February 2007, Pages 338-353
[12] Tirole,J., The Theory of Industrial Organization, MIT Press, Cambridge, MA , (1990)
[13] Tsay, A.A., S. Nahmias, and N. Agrawal, "Modeling Supply Chain Contracts: A Review," [14] Quantitative Models for Supply Chain Management (Volume 17 of International Series in Operations Research and Management Science), S. Tayur, R. Ganeshan, and M. Magazine (Eds.), Kluwer Academic Publishers, Boston, MA, 1998, pp.299-336.
[15] V. Viswanathan and J. Jang, "Strategic Inventories in a Cournot Duopoly" in Proc. Of the 2009 Annual Conference of the Productions and Operations Management Society, Orlando, FL, May 1st - 4th 2009