A MATRIX APPROACH TO SUPPORT DEPARTMENT RECIPROCAL COST ALLOCATIONS

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ABSTRACT

The Jordan Kekoa Company case presents a spreadsheet matrix approach to computing and allocating reciprocated costs of many support departments. The case builds upon algebraic expressions for support department reciprocated costs commonly presented in accounting textbooks. By converting them to an equivalent matrix relationship, spreadsheet matrix functions can then easily compute the reciprocated cost of support departments and the allocated cost to user departments.

Keywords: accounting for support departments, reciprocal cost allocation, matrix algebra application

INTRODUCTION

In today's business organizations, there is an increase in the cost of support departments, an increase in the number of support departments, and an increase in the amount of services that they provide to other support departments. With a more complex business environment, the allocation of support department costs becomes more challenging. Hence, the allocation of support department costs to operating departments is an important concept taught in most cost accounting courses.

The overall objective of support department cost allocations is to have more accurate product, service and customer costs. Textbook authors [1] [2] identify the reciprocal method as the most accurate support department cost allocation as it captures services provided to other support departments and operating departments. However, many accounting instructors continue to emphasize the direct and step-down methods over the reciprocal method for support department cost allocations. Furthermore, textbook authors present oversimplified problems for the reciprocal method using just two support department that are solved by algebraic substitutions.

Even though the reciprocal method is better suited to meet a changing business environment, accounting textbook authors have been hesitant to illustrate more realistic and complex scenarios among support departments. Spreadsheet matrix techniques for solving reciprocated costs are seldom used to solve simultaneous equations for modeled interrelationships of support departments.

The next section is a brief overview of three common support department cost allocation methods. The concluding section presents the Jordan Kekoa Company case that utilizes matrix functions to easily represent and solve reciprocated costs of support departments.

COST ALLOCATION METHODS

Horngren et al. [2] present three methods to allocate support department costs: direct, step-down, and reciprocal. The most widely used cost allocation procedure is the direct method and it is also the simplest to use. It allocates support department costs only to operating departments. The direct method allocates support department costs to operating departments based on the percentages of use by operating departments. Its major drawback is that it disregards support services provided by other support departments.

An improvement to the direct method is the step-down method of allocating support department costs. The step-down method is characterized by partial recognition of support services provided to other support departments. The step-down method is called that because a closed support department cannot receive allocations from remaining support departments. The partial recognition of support services and different sequences in closing support departments are drawbacks to the step-down method [2]. The step-down method would calculate the percentages of use for the remaining support departments and operating departments at each step, and then allocate the support department's cost based on the percentages.

The reciprocal method is a significant improvement over the direct and the step-down methods as it fully recognizes support services to all departments. Complete interdepartmental support services or reciprocated costs are explicitly defined as a support department's own cost plus any interdepartmental costs allocated to it from other support departments. The reciprocal method requires at least n reciprocated cost variables for the n support departments and that this set of independent linear equations be solved simultaneously. The simultaneous equations can be solved using algebraic techniques such that $n-1$ variables are methodically eliminated until a single variable is solved from one equation [2]. If there are three or more support departments, this can be a trying exercise for students and time consuming for instructors. Hence, most textbook authors illustrate the reciprocal method with just two support departments.

SPREADSHEET MATRIX APPROACH TO RECIPROCATED COST ALLOCATIONS

Spreadsheet matrix function will easily solve independent simultaneous equations formulated by the reciprocal method. A matrix approach is not limited as to the number and complexity of relationships among many support departments and operating departments. Furthermore, the computed reciprocated costs of each support department can be easily allocated to the other support and operating departments using a matrix function. The matrix approach facilitates the students' solving for reciprocated costs of support departments. When illustrated on a spreadsheet, students are likely to better understand the concept of reciprocated cost allocations even though some students may not fully understand the matrix functions required to solve the set of linear equations. Another benefit to the students is that they learn matrix functions that enhance their spreadsheet skills. The following Jordan Kekoa Company case demonstrates the matrix approach for reciprocal cost allocations of support departments.

JORDAN KEKOA COMPANY

Background

A manufacturer of electronic naval reconnaissance equipment, Jordan Kekoa Company incurs significant costs in support Departments A, B, C and D. In the past, management used the direct method to allocate support department costs to operating Departments X, Y and Z. However, with an expected increase in government contracts, management anticipates audits by government agencies and recognizes improvements to its cost accounting system are necessary to reflect more accurate product costs. President Jordan Kekoa reviews the accounting literature and recognizes the step-down and reciprocal methods for support department cost allocations as improvements to the direct method. He concludes from his reading that his company should adopt the reciprocal cost allocation method. The examples in cost accounting textbooks illustrate how reciprocated costs for two support departments should be calculated. Jordan Kekoa is able to set up algebraic formulas for the reciprocated costs of the four departments A, B, C and D; but, he is unable to solve the set of simultaneous equations using an algebraic approach. Furthermore, he knows that even more support departments will be needed as they expand into government work. He rereads the accounting textbooks and determines that a spreadsheet matrix approach to solving his problem is available. However, the textbooks do not illustrate this matrix approach to reciprocal cost allocations. Jordan Kekoa emails the cost accounting staff hoping to find this skill.

Daxia Nalani is a recent employee in the accounting group. She had been taught in a cost accounting course in the graduate accounting program to use the matrix approach for the reciprocal method. Daxia Nalani responds to the email. She is immediately sent information and asked to present a spreadsheet matrix solution to the current cost allocations.

Departmental Data for Reciprocated Allocations

The costs for four support departments A, B, C, and D and three operating departments X, Y, and Z are presented below. In addition, the percent of support services provided by departments A, B, C and D to the other departments is displayed. For example, Department B provides 0.08 and 0.40 of its services to Departments A and X. Support services provided to its own department are not necessary as they are contained within the amount for reciprocated cost [2].

The total percentage of reciprocated services provided by a support department to all other departments is equal to +1.00. Since all of a support department services will be allocated to other user departments, the **-1.00** listed for each support departments represents their **allocated** reciprocated services. Hence, the total for each support department is equal to 0.00, which is the -1 allocated reciprocated services netted with the +1 total percentage of services provided.

	Dept A	Dept B	Dept C	Dept D	Dept X	Dept Y	Dept Z	Total
Depart. Costs:	500,000	400,000	300,000	200,000	3,600,000	2,000,000	1,000,000	8,000,000
Support by:								
Dept A	**-1.00**	0.10	0.08	0.07	0.30	0.25	0.20	0.00
Dept B	0.08	**-1.00**	0.06	0.06	0.40	0.15	0.25	0.00
Dept C	0.06	0.05	**-1.00**	0.04	0.25	0.40	0.20	0.00
Dept D	0.05	0.05	0.15	**-1.00**	0.30	0.30	0.15	0.00

Simultaneous Equations for Reciprocated Costs

In the following algebraic expressions, A, B, C and D represent the reciprocated costs for corresponding departments. Equation 1a for Department A states that the reciprocated cost 1.00A is equal to its own cost of $500,000 and 0.08 of Department B reciprocated cost, 0.06 of Department C and 0.05 of Department D. Equation 1b for Department A expresses the relationship in matrix format. An equivalent algebraic expression for support departments B, C and D can be similarly obtained. The four equations for departments A, B, C and D represent the set of independent simultaneous equations in matrix format necessary to solve for reciprocated costs of each department.

Department A $$+1.00A = 500{,}000 + 0.08B + 0.06C + 0.05D \tag{1a}$$
$$+1.00A - 0.08B - 0.06C - 0.05D = 500{,}000 \tag{1b}$$

Department A $+1.00A - 0.08B - 0.06C - 0.05D = 500{,}000$

Department B $-0.10A + 1.00B - 0.05C - 0.05D = 400{,}000$

Department C $-0.08A - 0.06B + 1.00C - 0.15D = 300{,}000$

Department D $-0.07A - 0.06B - 0.04C + 1.00D = 200{,}000$

Matrix Relationship for Reciprocated Costs

The following equation (2) multiplies two matrices such that S x X = K. This matrix relationship is equivalent to the set of four simultaneous equations. For example, equation 1b is equivalent to multiplying the first row of the S matrix with the first and only column of the X matrix and then setting it equal to the constant 500,000. The S matrix (4x4) represents reciprocated services among support departments. The X matrix (4x1) represents the support departments' unknown reciprocated costs as variables A, B, C, and D. The K matrix (4x1) represents the individual cost of each department.

$$\underset{\mathbf{S}}{\begin{vmatrix} +1.00 & -0.08 & -0.06 & -0.05 \\ -0.10 & +1.00 & -0.05 & -0.05 \\ -0.08 & -0.06 & +1.00 & -0.15 \\ -0.07 & -0.06 & -0.04 & +1.00 \end{vmatrix}} \quad \underset{\mathbf{x}}{\text{x}} \quad \underset{\mathbf{X}}{\begin{vmatrix} A \\ B \\ C \\ D \end{vmatrix}} \quad \underset{\mathbf{=}}{=} \quad \underset{\mathbf{K}}{\begin{vmatrix} 500{,}000 \\ 400{,}000 \\ 300{,}000 \\ 200{,}000 \end{vmatrix}} \tag{2}$$

Each value within a matrix can be identified by specifying the matrix and its row and column. For example, ($s_{1,3}$) is equal to -0.06 as it is found in the S matrix at row 1 and column 3. An example of an array of numbers is noted as ($s_{1,1}$:$s_{4,4}$), which is equivalent to the S matrix.

Reciprocated Cost Solution Using Matrix Functions

The reciprocated cost solution for each department is computed mathematically by multiplying both sides of the matrix equation (3) with the inverse of S or S^{-1}. The S^{-1} matrix multiplied with the S matrix equals the identity matrix I; the identity matrix I multiplied with the X matrix equals just the X matrix.

$$\begin{aligned} \mathbf{S \times X} &= \mathbf{K} \\ \mathbf{S^{-1} \times S \times X} &= \mathbf{S^{-1} \times K} \\ \mathbf{I \times X = X} &= \mathbf{S^{-1} \times K} \end{aligned} \tag{3}$$

The following EXCEL formula (4) multiplies S^{-1} with the K matrix to solve for X, which is a matrix for reciprocated costs of each department. After selecting a range (4x1) for the solutions, enter the formula and press **Ctrl + Shift + Enter** keys together. The solution matrix for the reciprocated costs for departments A, B, C and D is shown below.

EXCEL formula: **=mmult(minverse(S),K)** or **=mmult(minverse($s_{1,1}$:$s_{4,4}$),$k_{1,1}$:$k_{4,1}$)** (4)

$$\begin{vmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \end{vmatrix} \quad \begin{matrix} = \\ = \\ = \\ = \end{matrix} \quad \begin{vmatrix} \mathbf{578{,}935} \\ \mathbf{493{,}184} \\ \mathbf{418{,}937} \\ \mathbf{286{,}874} \end{vmatrix}$$

Reciprocated Cost Allocations Using Matrix Functions

The allocation of reciprocated costs for each support department to all other departments is performed by equation (5) which multiplies two matrices R x P = *A*. The EXCEL formula (6) for this matrix multiplication is shown below. The R matrix (4x4) is the reciprocated costs for each department and is it is multiplied by the P matrix (4x7) which is the table of services provided by support departments.

The resultant A matrix (4x7) is the allocation of reciprocated costs of support departments to all other departments.

$$\underset{\mathbf{R}}{\begin{vmatrix} 578{,}935 & 0.00 & 0.00 & 0.00 \\ 0.00 & 493{,}184 & 0.00 & 0.00 \\ 0.00 & 0.00 & 418{,}937 & 0.00 \\ 0.00 & 0.00 & 0.00 & 286{,}874 \end{vmatrix}} \ \mathbf{x} \ \underset{\mathbf{P}}{\begin{vmatrix} -1.00 & 0.10 & 0.08 & 0.07 & 0.30 & 0.25 & 0.20 \\ 0.08 & -1.00 & 0.06 & 0.06 & 0.40 & 0.15 & 0.25 \\ 0.06 & 0.05 & -1.00 & 0.04 & 0.25 & 0.40 & 0.20 \\ 0.05 & 0.05 & 0.15 & -1.00 & 0.30 & 0.30 & 0.15 \end{vmatrix}} = \boldsymbol{A} \qquad (5)$$

EXCEL formula: **=mmult(R,P) or =mmult(($r_{1,1}$:$r_{4,4}$),($p_{1,1}$:$p_{4,7}$))** = A (6)

The resultant ***A*** matrix is placed in the cost allocation table below. The reciprocal cost allocation method has transferred all support department costs to the operating departments and at the same time recognized all services performed by support departments.

	Dept A	Dept B	Dept C	Dept D	Dept X	Dept Y	Dept Z	Total
Depart. Costs:	500,000	400,000	300,000	200,000	3,600,000	2,000,000	1,000,000	8,000,000
Allocations:								
Dept A	***-578,935***	***57,893***	***46,315***	***40,525***	***173,680***	***144,735***	***115,787***	0.00
Dept B	***39,455***	***-493,184***	***29,591***	***29,591***	***197,274***	***73,977***	***123,296***	0.00
Dept C	***25,136***	***20,947***	***-418,937***	***16,758***	***104,734***	***167,575***	***83,787***	0.00
Dept D	***14,344***	***14,344***	***43,031***	***-286,874***	***86,062***	***86,062***	***43,031***	0.00
Total	0	0	0	0	4,161,750	2,472,349	1,365,901	8,000,000

CONCLUSION

The Jordan Kekoa Company case illustrates the ease in using spreadsheet matrix functions to solve reciprocal cost allocations of support departments. This spreadsheet approach for the preferred reciprocal cost allocation method can be easily adopted for use in the classroom. Students are more likely to understand and appreciate the benefits of the reciprocal cost allocation method when using a spreadsheet matrix approach. This case demonstrates another example of how technology can improve accounting instruction and better prepare students for a more complex business environment.

REFERENCES

[1] Hilton, R.W., Maher, M.W. & Selto, F.H. *Cost Management: Strategies for Business Decisions,* McGraw-Hill/Irwin, New York, 2003.

[2] Horngren, C.T., Foster, G. & Datar, S.M. *Cost Accounting: A Managerial Emphasis,* Prentice-Hall, Upper Saddle River, New Jersey, 2006.