# ON THE BASIS OF A COMPUTATIONAL MODEL FOR DECISION MAKING

*Thang N. Nguyen, College of Business Administration, California State University Long Beach, 1250 Bellflower Blvd, Long Beach, CA 90840, (562) 985-8995, tnnguyen@csulb.edu*
*Vanja Cvetojevic, Faculty of Science, University of Gothenburg, SE-405 30 Gothenburg, 46 (0)31-786 00 00, guscveva@student.gu.se*

## ABSTRACT

This article is about the first step towards a computational model for decision making and other higher mental abilities, which is biologically-inspired after the human neocortex. Our approach consists of (1) an reexamination of the human visual perception from light stimuli to the primary visual cortex (PVC) in order to understand and visualize what a sensory memory looks like, (2) a formulation of topological invariants and homeomorphisms towards associated homotopy and homology groups suitable for representing sensory memories, and memory operations such as store, recall, association, etc. and (3) an initial prototypical tool for proof of concepts and further insights into the intended, computational decision model.

## INTRODUCTION

To make a decision, one commonly needs facts and a good scheme of reasoning for selecting the right alternative. The facts (descriptive knowledge) are acquired via sensory memory to be retained in long-term memory, primarily in the neocortex, and the reasoning (procedural knowledge) must be learned before it can act upon the facts. To arrive at a computational model, the basis of the decision making centers around a simple question "What does a sensory memory look like in the brain?" and how to represent the sensory memory mathematically for the possibility to perform memory operations .
Arguably, the answer depends on the level of granularity we are looking at. Nevertheless, at a level suitable for decision making and other mental abilities we have found no direct/complete answer to the question asked in the textbooks or current literature, even when restricted to just one modality, vision.
In this paper, we will suggest an answer to the above question, limited to visual perception. Even within the scope of visual perception, we will further limit ourselves to the part of the visual process from the refracted light originated from an object/scene to the retina, to the lateral geniculate nucleus (LGN) in the thalamus, and to the PVC. Processing at higher cortical areas for modality integration and otherwise toward short-term memory and long-term memory via the hippocampus will be postponed. Furthermore, we will be dealing, initially, only with form, shape, shade of visual object/scene. Color, depth and motion will also be excluded at this time. For our initial prototype, we will be dealing with only simple geometric objects visually perceived.

## PRINCIPLES GOVERNING VISUAL PROCESS AND MEMORY

Vision is the best understood subject among the five human modalities: vision, sound, touch, taste and smell. We recall some major breakthroughs in vision as follows. We mark 1900 as the starting point where Ramon y Cajal discovered the different layers of the retina using Golgi staining technique [1]. It wasn't until half a century later, in 1950, that Stephen Kuffler [2] has successfully described the antagonistic on-off center-surround structure of the retina. About the same time, the mechanism of neural firing was found by the work of Tsuneo Tomita as mentioned in [1].

A decade later, David Hubel and Torsten Wiesel [1], identified the orientation feature of simple cells, ocular dominance and direction selectivity of complex cells. The understanding of bleaching was confirmed in the 70's by William Hagins and associates described in [1] and details on the physiological basis of memory storage in neurons became available by Eric Kandel [3]. Then Helga Kolb, Ralph Nelson and Edward Famiglietti [4] who determined that off-center cell dendrites terminated closer to the middle layer of the retina than those of the on-center cells added further details to the vision process. During the next 30 years, however no significant breakthroughs have been identified in the understanding of vision mechanisms beyond the primary visual cortex. Only very recently, by fMRI, researchers in UCSD were able to visualize the portion of vision in the PVC [5].
The vision process as we have known it consists of:

**(Visual Process):** Refracted light→**Retina {Photoreceptors, Bipolars, Ganglions}**→Lateral Geniculate Nucleus (LGN) in the Thalamus →**Primary Visual Cortex (V1)** → Higher cortical areas {V2, V3, V4, MT} → **Short-term memory** →Hippocampus→**Long-term memory**

In the following we detail a little more six principles governing the vision process based on the evidences gathered from the literature and on the work of most prominent researchers some of whom were Nobel laureates [1-8].
*Principle of summary (vertical integration).* This principle says that visual information is abstracted out from the refracted light originated from the object or scene to the PVC through each and every layer. Two concepts are essential for the understanding of this principle: *visual field* and *receptive field* defined by David Hubel [1]. The visual field depicts the visual environment as seen by the eyes. The receptive field is what is seen by a single cell. Intuitive evidence supporting the vertical integration is observed from the estimated number of cells at each layer, i.e. from the initial layer (photoreceptors) in the retina to the complex cell layer in the PVC. In fact, there are 125M photoreceptors (rods and cones)/eye, roughly 1-1.5M ganglion cells/eye, and roughly 1.5M in each LGN. Therefore, up to and including the cells in the LGN, the convergence occurs due to the large reduction of number of cells across the layers. From the 1.5M/LGN to some 200M cells (simple and complex) in the PVC, since the number of cells increases more than 100-fold, it is a fan-out or divergence. Thus, it appears that the convergence is counter-intuitive. By taking a closer look, however, there is a convergence for visual feature. It involves the conversion of the circular on-off center-surround to the rectangular (rectilinear) receptive field of orientation-specific and direction-selective cells in the PVC. It takes a larger number of simple cells to detect orientation-specific information from incoming center-surround LGN inputs. And again, it takes a larger number of complex cells (complex cells outnumber simple cells by a factor 3:1) to construct the visual image at the PVC because of the direction-selective abstraction of visual feature being performed.
*Principle of horizontal (lateral) integration.* It has been found that although individual cell's receptive field is commonly small in size, there are cells which provide larger, combined receptive fields on the object. In fact, the outer plexiform (between photoreceptors and bipolar) contains horizontal cells that synapse laterally with bipolar cells, and the inner plexiform (between bipolars and ganglions) contains amacrine cells which bipolar cells synapse with. Both horizontal cells (accumulating information from a wide field of cones) and amacrine cells (transmitting information from rods) interact with hundreds of bipolar cells in their outer and inner plexiforms respectively by lateral integration. The horizontal integration helps sharpen the boundary between centers and surrounds as described by Helga Kolb [4].
*Principle of parallelism.* There are multiple paths of projection from the retina to the PVC, therefore parallelism. First there are two paths as originated from the retina layer: direct and indirect path. The direct path is from the reflected light to the retina, to bipolar to ganglion to the LGN to the PVC. The indirect path involves the horizontal cells between the photoreceptors and the bipolar and the amacrine cells between bipolar and ganglions as we have presented above. As shown by Hubel and Wiesel [1], the direct path is highly specific and compact, while the indirect path is more diffused, extended due to lateral connections.

At the LGN, there are other parallel paths: magnocellular (M) and parvocellular (P) which land on different layers of each of the 6-layered LGN. These two major paths terminate in the PVC at different layers. These paths are kept segregated until after the PVC. In addition to M and P paths, there are interlaminar layers between the six cell layers in the LGN which project onto PVC blob columns in layer 2 and 3 of the PVC.

*Principle of focus (attention)*. For the same object, the eyes do not stay still due to microsaccades [4] even though one fixates on some part of the object/scene. Consequently, the subset of neurons also increases or decreases in terms of which cones/rods or on-off center surround cells, or orientation or direction-selective simple or complex cells participating in the building up of the image. The change can be minuscule, or it can be largely intense. The results constitute a family of subsets, each of which differs from the others in different contrasts, orientation or direction. They are the outputs of a family of mappings following the previous principles (vertical integration, horizontal integration, parallelism) from the object/scene to the different layers of the visual system. At any point in time, one sketch is in the foreground and more acute. The rest are dimmer.

*Principle of association (linking)*. The association is originated either externally or internally or both. It is generated at the molecular level, where some or all chemical and electrical processes at the synaptic clefs or at the membranes are initiated initially by stimuli or as recalls. As a result, a perceived image may contain part of the subsets which is associated with those which have been previously captured and stored. An example will clarify this. When we look at a circle or a rectangle, we may think of a marble, a soccer ball, a football, or a die, a cube and/or those of them of different size, or of similar shapes.

*Principle of recall (pattern recognition)*. This principle overlaps the previous principle (association) somewhat. Both association and recall are triggered externally or internally by previous stimuli at the molecular levels (dynamically resulting from what happens at the synapses) leading to patterns of sketches and surface filling-ins. The difference between this principle and the association principle is that association can be linked to some completely different percepts while recall cannot. Association is more of an aggregation nature while recall is sort of generalization and/or specialization.

## A FORMULATION ON WHAT A VISUAL MEMORY LOOKS LIKE IN THE PVC

In this section, we will trace the making of a visual memory from the stimuli (refracted light) onwards according to the visual process shown previously. Based on this process, we will formulate what a visual memory looks like at each of the three major stages: retina, LGN and PVC.

When photoreceptors are illuminated (Figure 1A), their membranes are hyperpolarized and cut down the neurotransmitter release which results in excitatory synapses to off-center bipolars (at an approximate 2:1 ratio) by excitation. The reverse, i.e. the on-center bipolars resulted from inhibitory synapses, occurs when light is absent. Thus, a sketch of the image by refracted light is delimited by contrasts (dark versus light) in the visual field as shown by the collection of on-off center and surround bipolar cells.

The horizontal cells will fine tune the image. These cells receive inputs from the receptors over a wider area, compared to the receptive field by a single bipolar. They are responsible for the receptive-field surrounds of the bipolar cells. If these cells connect directly to the bipolars, the synapses to on-bipolars are excitatory, and inhibitory otherwise. Thus, the bipolar cells have center-surround receptive fields where the center is supplied by the direct inputs from receptors, and the surround from the indirect path of receptors-to-horizontals to bipolar. Amacrine cells have a similar role to ganglion cells. With the help of horizontal cells and amacrine cells, the on-off center-surround ganglion cells are next hyperpolarized or depolarized.

Ganglion cells's receptive field similarly consists of circular on-center region surrounded by an off region interacted in an antagonistic fashion. When a large spot covers both regions the response is much weaker than when the spot shines on only the center. The off-center region works in opposite way. Thus

diffuse light has little effect on these on-center fields. A finer sketch is preserved by the ganglion cells. The impulses generated in their axons travel down the optic nerve, cross over at the optic chiasm to enter the two lateral geniculate nuclei.

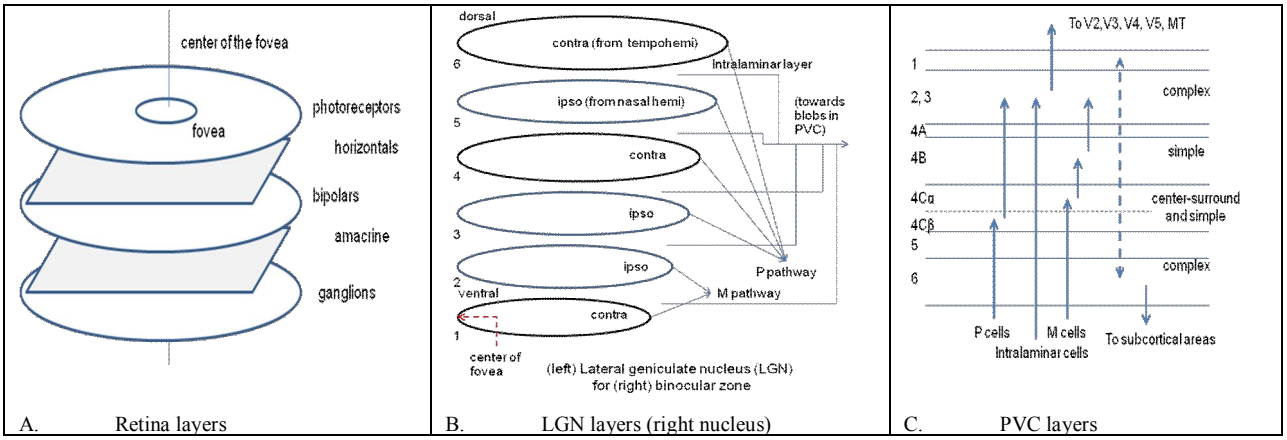



Figure 1: Retina, LGN and PVC Layers

At LGN stage (which also has the same on-off center-surround structure as the ganglion layer) the collection of receptive fields (similarly, circular structure) captures the line segments delimited by the contrast between light and dark just as the retina process does (Figure 1B). A part of the sketch is more acute as it corresponds to the fovea where the cells are dense. The sketch is visualized as a collection of broken line segments. Different sketches will be produced as the eyes make the tiny movements randomly across the visual field. Acuity of each in the family of sketches is higher as the eyes jump and focus on the portion of image corresponding to it.

At the PVC stage (Figure 1C), the collection of cells approximating the sketches, hyperpolarizes or depolarizes the simple cells according to their specific orientation (from 0 degree to 360 degrees), and the collection of simple cells wire up direction-selective complex cells. Both add curvature and smoothness features to the scene/object seen by the eyes. Surface filling-ins are realized based on the boundaries defined by the sketches.

Thus, the detection of image points by on-off center-surround cells from photoreceptors to LGN M's and P's, and of image lines by orientation-specific and direction-specific cells in the PVC suggests many layers of sketches (composed of points, lines) and surface filling-ins that are superimposed. Again, one of these layers appear more acute depending upon which part of the object the eyes pay attention to. The union of these sketches and surface filling-ins constitutes the end-state of the visual memory, as an abstraction of the image describing the object/seen features seen.

Let N is the set of all neurons (estimated as 100 billion cells), we postulate the end-state of a visual memory M as element of the power set $P_N$ of N, where $P_N$ is a topological space:

$$\boldsymbol{M = U \{\{layered/superimposed\ sketches\}\ and\ \{layered/superimposed\ surface\ filling\text{-}ins\}\}} \quad (1)$$

A particular visual memory layer seen is a subset of the power set $P_M$ of M (Expression 1) which in turn is a subset of $P_N$. This end-state formulation is both intuitive and logical. Intuitively, the family of sketches has to be layered due to the very nature of vision architecture (multi-layered retina, LGN and PVC). Only one sketch is acute at any point in time as part of the visual field focused is projected in the fovea. The remaining goes into the background, as one would experience it. If enough time is spent on each sketch, the sketch will be retained in short-term memory, transformed by the hippocampus, and ends up in the long-term memory. Logically, each sketch or surface filling-in has been observed by biological evidence as shown by D. Marr (primal sketch) [6], Hubel (maps) [1], Swindale (number of maps) [7].

Each sketch is actually dynamically generated by the process at the molecular level as described in Gerstner (the SRM model) [8]. If it is continuously and frequently done, the molecular process (chemical reactions and electrical pulses) is "remembered" and retained. When recall occurs, an element/subset of M is matched against those previously generated, therefore the recalled or remembered object/scene is never immediately clear or complete. Also during recall, additional details are filled-in by association and/or by the dynamic process that is re-generated.
The number of layers in sketches and surfaces in a view varies depending on the object perceived from a particular perspective fixated by the eyes and by a particular microsaccade at time t. Which of the layered and superimposed surface filling-ins are involved in the store, recall or being "remembered" follow the same logic as that of the sketches. This explains why we have different detailed views of the same object at different times or we see different images as a result of what is termed as optical illusions (e.g. contrasts give the image of a young girl as opposed to an old lady, etc.).
The formulation of a visual memory (Expression 1) is at the networked level of organization. The union (U) can be more than one-level deep, i.e. union of unions of unions, as one needs to express the details at the cellular level or molecular level. Each sketch (or surface filling-in) is visualized by a layer. It is formed by the union of cells at the said layer, i.e. at the cellular level. Each cell in turn is the union of all inputs received at the dendrites at the molecular level.

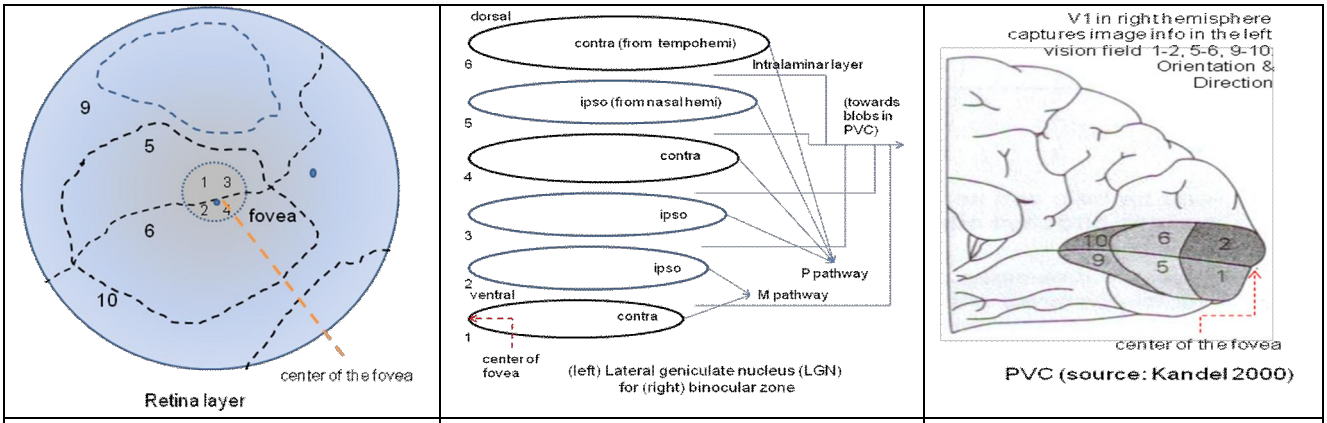
## TOPOLOGICAL SPACES AND INVARIANTS, HOMEOMORPHISMS AND HOMOTOPY/HOMOLOGY GROUPS

The idea of using a topological approach to vision and its link to cognition is not new. W.C. Hoffman [9] has suggested the consideration of point-set, differential and algebraic topology in studying visual form, meaning, and cognition. Hoffman argued that form is geometric and its meaning is a thought process connecting point neurons, therefore lines, triangles, tetrahehra of simplicial topology might be considered. Hoffmann also suggested that if looking at the visual process as maps and projections to and from the entire neocortex (total space) to a base space called consciousness, then algebraic topology for meaning and cognition may be useful. We explore topological ideas and formulation as follows.
*Topological spaces*. The power set of all neurons in the neocortex as well as the power sets of neurons in each and every layer (retina layers, LGN and PVC) are considered as topological spaces by definition (including the empty set and the set of all neurons of interest). The notion of receptive field for each cell is considered as the notion of *open sets* for each point x in the topological subsets of cells. The larger receptive field due to horizontal integration can be considered as an *open covering* defined on a particular part of the object. One such case is the set of cones hit by light photons which constitutes a topological space on which a topology is defined as the collection of *open sets* receptive fields seen by each cone cell. Thus a sketch is the boundary which is considered as the difference between two topological sets: the set of hyperpolarized cones and the set of depolarized cones within the fovea.
*Topological invariants*. Recall from the previous section that there are three major and different types of mappings, namely, (1) from retina to ganglions and to LGN as on-off center surround mechanism, (2) the frequency-regulated sequence of impulses along the optic tracks to PVC, and (3) the transformation from on-off center-surround cells configurations to orientation hyper-columns, direction-selective columns, and blobs. These mappings dramatically transform the original set of stimuli entering the eyes. Despite these complex biochemical and electrical transformations and processes as mappings applied to the input stimuli, a simple fact remains: we easily recognize a sound heard, an object seen, or something in the past that we thought was long gone. This strongly implies *invariance* throughout the process, i.e. some topological invariants, e.g. contrast, orientation and directional motion that are preserved in the topological spaces where a particular topology can be defined.

*Homeomorphisms.* The maps g(b(p)) from any subset of P (photoreceptors, $r_1$) → any subset of B (bipolar, $r_2$) → any subset of G (ganglions, $r_3$) can't be bijective or one-to-one due to vertical integration. But if we consider the mapping from any subset of P to (G, B, P) where each point x in the photoreceptor layer is mapped to a collection of three points defining the image as (x ϵ P, y ϵ B, z ϵ G) then it is a bijection and one-to-one mapping with an inverse. Thus this mapping is a homeomorphism. If we remove the center of fovea (considered as origin 0) in the 1-dimension manifold representing the photoreceptor layer, any ]0, $r_1$] is homeomorphic to a line outside the circle (a simple homeomorphism in any basic topology textbook). The latter is then homeomorphic to the ]0, $r_2$] of the 1-dimension manifold representing the bipolar layer (with the center of this disc removed). By transitivity, the ]0,$r_2$] of the bipolar layer (with the center removed) is homeomorphic to its outside line, which in turn is homeomorphic to the ]0, $r_3$] of the rightmost disc representing the ganglion layer (with the center of the disc removed). We have just established homeomorphisms between the three ]0, $r_i$] layers.

*Fixed point property*. There are roughly 6M cone cells in the fovea, mostly reds and greens, with a small portion of blues (2%) near the periphery of the fovea. *The center of the fovea of each eye is considered as a topological fixed point*. We refer to this as the fixed-point property. Since every synaptic clef between a ganglion axon and corresponding LGN, and between each LGN axon to connecting layers (mostly 4Cα and 4Cβ) in the PVC is pre-positioned in their structure, there is an image of this fixed point in each of the subsequent layers of the visual system (Figure 2).

*Homotopy and homology groups*. All the line segments and closed curves (after some triangulation and deconstruction) will form a 2-polygonal chain associated with the image sketch as seen by the fovea (and surface filling-ins). We can generalize to n-dimension chains. To manipulate these chains and to define the maps between these spaces and their associated homotopic/homologous groups, for example we will need to further decompose the 2-chains into 1-chains and 0-chains. The triangulation and deconstruction are applicable to include sketches formed in the fovea and in the larger ring surrounding the fovea. We can start building the homotopy groups associated with sketches and homology groups associated with surface filling-ins by an initial set of definitions. It is also extended to sketches projected to bipolar layer and to ganglion layers since bipolar, ganglion, LGN and the first sheet of the PVC (PCV has a few sheet of one-cell deep sub-layer) have the same on-off center-surround, antagonistic structure.




| A. Retina | B. LGN | C. PVC |
|---|---|---|

Figure 2: Topological elements and invariants in the LGN and in the PVC

The sketches are the union of the following basic types of line segments or closed curves on the fovea and on the ring surrounding it. These types are, as shown in Figure 2A: (a) the center of the fovea as fixed point, (b) a point, (c) a line segment that crosses the fixed point, (d) a line segment delimited by the retina, (d) a curve that is fully contained in the fovea, (e) a curve in the ring which surrounds the fovea, or (f) a curve surrounding the fovea. These types are topological elements of sketches.

As the eyes move about, there exists a continuous mapping that transforms the sketch while preserving some topological invariant to be further identified. We will be concerned with those sketches that are homotopic. Likewise, the transformations between surface filling-ins are homologous.
The six principles described previously will be explored to further guide the development of a set of definitions and rules or theorems governing the creating of the layered, superimposed sketches and surface filling-ins, and the mappings among them where topological invariants are preserved as we have generally discussed in this section. As we are proceeding, the prototypical tool to be presented in the next section will continue to provide more insights into suitable topological elements and invariants.

## A PROTOTYPICAL TOOL FOR PROOF OF CONCEPTS AND INSIGHTS INTO FURTHER INVESTIGATION

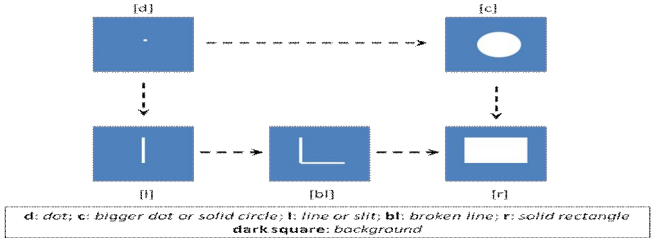


Figure 3: Simple objects on dark background

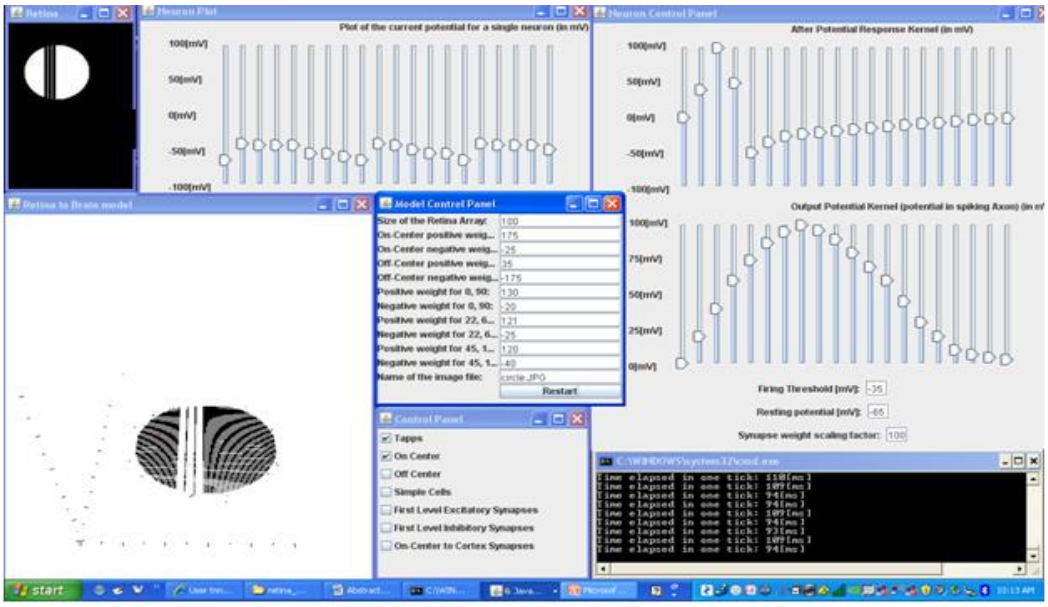


Figure 4: Sample screens of our prototypical Vision program

A prototypical tool is being built based on the current understanding of visual process published in the literature for proof of concepts and insights into the formulation and modeling of visual percepts in homology groups and eventually for higher cortical areas integration in form and shape. We initially limit ourselves to simple objects perceived visually (Figure 3). The prototype currently consists of the following layers of neurons: (1)photoreceptor layer (rod and cones), (2) On-center bipolar/ganglion cell layer and Off-Center bipolar/ganglion cell layer, and (3) Simple cell layer in the PVC.
Currently, the information moves only in one direction. Thus, there is currently no feedback loop in the model yet, as it will be addressed as the model matures. The Vision GUI (graphical user interface) (Figure 4) consists of four types of windows: input windows, output windows and two control windows: model control and neuron control. Clockwise from top left: input windows (on dark background), stage

processing control windows showing layer outputs, console window, and output window. Three output screens (layered sketches & surface filling-ins) are shown in Figure 5.

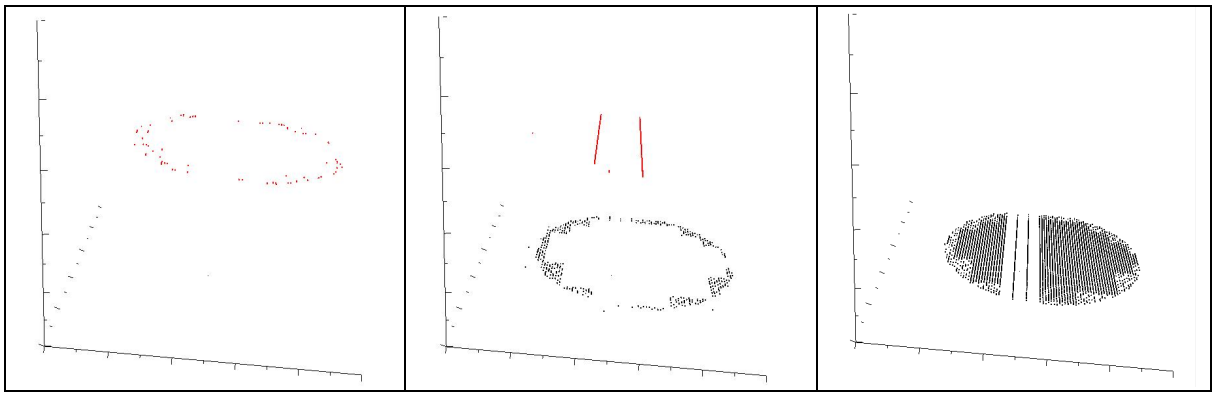


Figure 5: Layered outputs at different stage of the vision process

## CONCLUDING REMARK AND FUTURE WORK

The topological basis discussed this paper can be extended to more complex objects, beyond the PVC, more than one modalities for integration and representation of concepts (languages-driven). Concepts will give rise to learning, and to higher mental abilities such as planning, reasoning, and decision making. It's a daunting task and the road is long however. At the very least, it is a novel and viable alternative to current models of AI and neuron nets in the understanding and realization of the linking of the brain to the mind for many potential possibilities and applications.

## REFERENCES


[1] Hubel, David H. (1995). *Eye, Brain and Vision*, Freeman Press. Also, http://hubel.med.harvard.edu/

[2] Kuffler, Stephen (1976). *From Neuron to Brain: a cellular approach to the function of the nervous system*, Sinauer Associates, 1976

[3] Kandel, Eric, James H. Schwartz and Thomas M. Jessels (eds.) (2004). *Principles of Neuroscience,* 4e, Appleton & Lange. 2000

[4] Kolb, Helga Eduardo Fernandez and Ralph Nelson. "Webvision". http://webvision.med.utah.edu/

[5] Serences, John (2009). *Medical News Today*. http://www.medicalnewstoday.com/articles/139898.php

[6] Marr, D. (1982). Vision: *A Computational Investigation into the Human Representation and Processing of Visual Information*, WH Freeman. 1982

[7] Swindale, Nicolas (2000). How Many Maps Are There in Visual Cortex? *Cerebral Cortex*, V. 10, no. 7, July 2000

[8] Gerstner, W. and Kistler W. (2002). *Spiking Neurons Models: single neurons, populations, plasticity,* Cambridge University Press. 2002

[9] Hoffman, William C. *Topological Psychology* (2009). http://home.att.net/~topologicalpsychology/.