CONSENSUS-BASED GROUP DECISION MAKING

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ABSTRACT

The group decision making problem that is discussed in this paper is apportioning a finite resource among a number of involved parties. An approach is developed for computing the consensus solution, and explanation, based on the concept of bounded rationality, is provided for the discrepancy found in real situations between the theoretical consensus and the actual final decision arrived at by the decision makers. Finally, the effects of the real life phenomena of coalitions among members in group decision making is discussed. The results are illustrated using data from a real case study.

Keywords: Consensus; group decision making; bounded rationality; coalition, factions, and cliques.

THE PROBLEM

"In virtually all [complex] decision processes," argues [5] that "there are various actors (decision makers) who represent individual subjects (persons, countries, companies, etc.) and their respective interest groups. To reach a meaningful decision, opinions of all such actors must be taken into account or a given decision may not be implemented. Ideally, a decision would be made after a consensus between the parties involved had been attained. So consensus is a very desirable situation." In consensus decision making every member of the group "must be flexible and willing to give up something to reach an agreement." [1] Thus, the basic premise is that every member of a group says *"I am willing to compromise, but I don't want anybody in the group, including myself, to win or lose by too much compared to others."* [2] The problem is how to make this premise operational.

THE CONSENSUS MODEL

Let $i = 1,\cdots,m$ be the index of available policies; $j = 1,\cdots,n$ be the index of the members of group; and q_{ij} be the payoff to member *j* according to policy *i*. The sum of payoff, $\sum_{j=1}^{n} q_{ij}$, to all members of the group, for all policies $i = 1,\cdots,m$ add up to total amount of resources, a constant.

A consensus policy can be constructed as a convex combination of available policies, $i = 1,\cdots,m$, such that the payoff to member *j* is $x_j = \sum_{i=1}^{m} \alpha_i q_{ij}$, where $\sum_i \alpha_i = 1; \alpha_i \geq 0, i = 1,\cdots,m.$ α_i is the multiplier associated with each policy *i* in constructing the consensus policy. The regret of player *j*, when the consensus policy is adopted, is defined as $r_j = b_j - x_j, j = 1,\cdots,n,$ where $b_i = \max_{1\leq i\leq m} q_{ij},$ is the best policy for player j among all possible policies, $i = 1,\cdots,m$. The variance of regret of compromise policy is proportional to $\sum_j r_j^2$. Thus, in order to minimize variance of regret, it is sufficient to minimize

$\sum_j r_j^2 = \sum_j (b_j - x_j)^2$. That is, the members of the group can arrive at a consensus by computing a policy, which is the optimal solution to the following quadratic program:

$$\{ \min_{\alpha_i \geq 0, x_j \geq 0} \sum_j (b_j - x_j)^2 \mid \sum_{i=1}^{m} q_{ij} \alpha_i = x_j, \ \ j = 1, \cdots, n, \ \sum_i \alpha_i = 1. \} \qquad (1)$$

An Example: Budgeting in an Academic Institution

Consider the re-allocation of the salary budget among departments in a college of business administration. [4] This case involves three policies that were proposed by the five department chairs (members of the group). The five departments are Accountancy (j = 1), Finance (j = 2), Information Systems (j = 3), Management (j = 4), and Marketing (j = 5). The first of the three policies considered, A, (i = 1), gave equal weighing to three criteria: Full Time Equivalent Students (FTES), Full Time Equivalent Faculty (FTEF), and enrollments based upon the historical relationships during the most recent four semesters; each weighed equally. The second policy, B, (i = 2) was identical to the first except the time period considered was two semesters, weighed equally. The third policy, C, (i = 3) was in many ways a mere revision of the former strategy in that only two criteria, FTES and FTEF, each weighed equally, were to be considered, with equal weighing for the most recent two semesters.

The allocation amounts under the three policies are A, B, and C, for each of five members are shown in Table 1.

POLICY	ACCT	FIN	IS	MGMT	MKT	SUM
A	1,204.2	1,819.7	1,851.4	1,641.2	1,138.8	7,655.3
B	1,243.9	1,876.5	1,884.9	1,537.9	1,112.1	7,655.3
C	1,192.1	1,878.7	1,938.3	1,549.5	1,096.7	7,655.3

Table 1. Budget allocations to departments under each policy in $1,000s.

Solution of the quadratic program (1), results in the consensus solution shown in Table 2.

POLICY	ACCT	FIN	IS	MGMT	MKT	α_i
A	1,204.2	1,819.7	1,851.4	1,641.2	1,138.8	0.51
B	1,243.9	1,876.5	1,884.9	1,537.9	1,112.1	0.00
C	1,192.1	1,878.7	1,938.3	1,549.5	1,096.7	0.49
CONSENSUS	1,189.2	1,848.9	1,894.3	1,595.9	1,118.0	1.00

Table 2. Consensus solution with corresponding multipliers.

BOUNDED RATIONALITY

In the case of department chairs in the budget allocation case, the actual solution that the members of the group ended up with is given, together with consensus solution for comparison, in Table 3. [4]

POLICY	ACCT	FIN	IS	MGMT	MKT
Actual	1,225.3	1,828.3	1,872.9	1,597.2	1,131.6
CONSENSUS	1,189.2	1,848.9	1,894.3	1,595.9	1,118.0

Table 3. Actual solution arrived at by the department chairs.

Although the numbers are quite close, they are not identical. Next, we would like to determine which convex combination of policies resulted in the actual solution decided upon by the members of the group. Due to bounded rationality of real life decision makers, the actual solution may not fall in the convex hull of points defined by the policies; and hence, it may not be possible to represent the actual solution as an exact convex combination of the policies. In order to find the solution in the convex hull of policies that is closest to the actual solution, which we will refer to as *adjusted* solution, we make use of a goal programming formulation.

Recall that q_{ij} is the payoff to group member *j* according to policy *i*. Let $d_j, j=1,\cdots,n,$ be payoff to member *j* in the *actual* solution. The decision variables in the goal program are adjusted payoff to member *j* of the group, a_j, and the nonnegative deficiency variables, "under-" and "over-satisfaction", respectively, as $\{\gamma_j, \delta_j\}$, such that for all $j=1,\cdots,n,$

$$a_j + \gamma_j - \delta_j = d_j, and \sum_{i=1}^{m} q_{ij}\alpha_i = a_j.$$

That is, the convex combination of the policies, $\sum_{i=1}^{m} q_{ij}\alpha_i$, is equal to the *adjusted* solution, a_j, minus the over-satisfaction deficiency variable, δ_j, plus the under-satisfaction variable, γ_j, is equal to the target value, the payoff to member *j*, d_j, in the *actual* solution. Thus, given the actual solution that was decided upon, in order to determine, with minimal "discrepancy", the corresponding convex combination multipliers, α_j, one has to solve the following goal program (2):

$$\{Min_{\alpha_i, x_j \geq 0; \gamma_j, \delta_j \geq 0} \sum_{j=1}^{n} (\gamma_j + \delta_j) \mid \sum_{i=1}^{m} q_{ij}\alpha_i - x_j = 0, j=1,\cdots,n; \sum_{i=1}^{m} \alpha_i = 1; \alpha_j + \gamma_j, -\delta_j, = d_j, j=1,\cdots,n.\}$$

Using the actual solution in the budget re-allocation example, the optimal multipliers, (α_i), to this goal program is $\{\alpha_1,\alpha_2,\alpha_3\} = \{0.57, 0.43, 0.00\}$ with resulting discrepancy, $\sum_{j=1}^{n} (\gamma_j + \delta_j)$, of \$31,187 out of a total budget of \$7,655,300, which is less than half of one percent. Table 4 gives, in addition to convex combination multipliers, the adjusted, actual, and consensus solutions, for comparisons.

POLICY	ACCT	FIN	IS	MGMT	MKT	α_i
A	1,204.2	1,819.7	1,851.4	1,641.2	1,138.8	0.51
B	1,243.9	1,876.5	1,884.9	1,537.9	1,112.1	0.00
C	1,192.1	1,878.7	1,938.3	1,549.5	1,096.7	0.49
ADJUSTED	1,221.1	1,848.9	1,865.7	1,597.2	1,127.4	
ACTUAL	1,225.3	1,828.3	1,872.9	1,597.2	1,131.6	
CONSENSUS	1,189.2	1,848.9	1,894.3	1,595.9	1,118.0	

Table 4. Adjusted solution with corresponding multipliers, with actual and consensus solutions.

On the other hand, the consensus solution was $\{\alpha_1,\alpha_2,\alpha_3\} = \{0.51, 0.00, 0.49\}$, which seems to be quite afar from the actual solution of $\{\alpha_1,\alpha_2,\alpha_3\} = \{0.57, 0.43, 0.00\}$. In order to measure how far apart these solutions, if the Euclidean measure of distance is assumed, the distance in between two points, **v** $=\{v_1,\cdots,v_m\}$ and **w** = $\{w_1,\cdots,w_m\}$, in m-dimensional space is $\sum_{i=1}^{m} \sqrt{(v_i - w_i)^2}$. The distance between the compromise consensus solution and the actual solution is 0.66, which says they are 38% apart (the maximum distance in a unit cube is 1.73.)

COALITIONS, FACTIONS, AND CLIQUES IN DECISION MAKING

In analyzing this problem, [3] state that consensus ". . . is often masked by behind-the scene maneuvers involving consensus (or coalition) of subgroups, which then exert their influence (or threats) to advance the total team towards the masked consensus decision the subgroup had sought to institute. Finding the actual roles and motivations of parties in these behind-the-scene maneuvers is difficult." They introduce the term *"defining subgroup"*, which is a subset of members of the group. Attempt to arrive at a consensus is made only among this members of the group. The *defining subgroup* can be a single member, a minority, or a majority; or the entire group itself. In a group with n members, there are $\sum_{j=1}^{n} C_j^n = \sum_{j=1}^{n} \frac{n!}{j!(n-j)!}$ defining subgroups. Each defining subgroup may constitute a faction or a coalition. Its consensus solution, after adjusting for the bounded rationality of its members, may be the basis for the actual solution arrived at by the group. Therefore, given an actual solution, one can "reverse engineer" in order to figure out which coalition or faction is the predominant defining subgroup in molding the group's final decision. In [3], this is attempted by finding the defining subgroup whose consensus solution is closest to the adjusted solution of the group, where the closeness is measured in Euclidean sense in the space of convex combination multipliers of the policies. One can further argue that there can be more than one defining subgroup, each having differing degrees of influence on the actual solution finally arrived at by the group. In order to assess we need to find the set of defining subgroups whose convex combination define the adjusted solution. In computing this convex combination, priority should be given to the defining subgroups closer to the adjusted solution. One way of achieving this is to weigh each defining subgroup according to closeness rankings to the adjusted solution. Consider the following formulation: let $k = \sum_{j=1}^{n} \frac{n!}{j!(n-j)!}$ be the number of defining subgroups. Let the index $l = 1, \cdots, k,$ denote each subgroup ranked according to its closeness to the adjusted solution; $l = 1$ being the closest and $l = k$ being the farthest. The consensus solution of the defining subgroup l, can be expressed as the convex combination of m policies. Let $[\alpha_{1,l}, \cdots, \alpha_{m,l}]$ denote the corresponding convex combination multipliers that make up the columns of the following matrix:

$$A = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,k} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m,1} & \alpha_{m,2} & \cdots & \alpha_{m,k} \end{bmatrix}$$

and the convex combination multipliers corresponding to each defining subgroup is denoted by $\beta = [\beta_1, \ldots, \beta_k]$. Then the following linear program (3) will provide us the desired convex combination multipliers of the defining subgroups:

$$\{Min_{\beta \geq 0} \sum_{l=1}^{k} l\beta_l \mid A\beta = \alpha^*; \sum_{l=1}^{k} \beta_l = 1\}, \qquad (3)$$

where α^* is the column vector of convex combination multipliers of the policies corresponding to the adjusted actual solution.

Using the data from budget re-allocation among departments, the solution of the linear program (3), provides the β values shown in Table 5. Clearly, ACCT is the dominant player in the decision making, as appearing in all defining subgroups. The results show that ACCT in coalition with FIN and MGMT account for about 71% of the decision, and in coalition with MKT account for 29% of the decision, while IS did not play a role in the decision.

CLOSENESS RANKING, ℓ	THE DEFINING SUBGROUP					β_ℓ
	ACCT	FIN	IS	MGMT	MKT	
1	1	1	0	1	0	0.71
3	1	0	0	0	1	0.29

Table 5. The defining subgroups that make up the actual adjusted solution with their relative weights**.**

SUMMARY AND CONCLUSIONS

A perfect environment for complete consensus is very rare in practice. Formation of coalitions, factions, or cliques is quite common among members of a group. Given the actual decision arrived at in a particular situation, the approach presented in the previous section provides a way to identify the factions or cliques that mold the final decision by the group. A new idea which requires further research and development involves "engineering" a sustainable consensus-based group decision making. As demonstrated in our consensus model, when the problem is viewed in game theoretic framework, the solution can be interpreted as the Nash equilibrium which we labeled it to be a "win-win" solution for the group. In other words, if the actual or final implemented solution is the "win-win" solution (or one that is close to it), we postulate the solution will be a stable and sustainable one. On the contrary, the greater the departure the actual implemented solution is from the "win-win" solution, the less stable and unsustainable it is. Therefore, to engineer the "win-win" solution, it would be helpful to not only know what the "win-win" solution is, but also the closeness or distance of the other subgroup solutions from the "win-win" solution. Constructing a table that shows these distances from the "win-win" solution will provide us with those sub-groups that are closest to the "win-win" solution. Armed with this knowledge, it will allow the dean of the college (in our case example) to know which of the department chairs to influence and how to steer the department chairs towards the actual solution that will be stable and sustainable in the long run.

REFERENCES

[1] Butler, C. T. and Rothstein, A. On conflict and consensus: A handbook on formal consensus decision making, 2001. Available from World Wide Web: http://www.consensus.net/.

[2] Chong, P. S. and Benli, O. S. Consensus in team decision making involving resource allocation. *Management Decision*, 43(8):1147–1160, 2005.

[3] Chong, P. S. and Benli, O. S. Benchmarking the efficacy of team decisions using game theoretic approach. *Total Quality Management and Business Excellence*, 20(3):349–361, 2009.

[4] Chong, P. S. & Runyon, L. R. In search of a consensus team decision budget agreement. *International Journal of Education Management*, 18(5):297–303, 2004.

[5] Kacprzyk , J., Nurmi, H. & Fedrizzi, M. (eds). *Consensus under Fuzziness*. Kluwer Academic Publishers, 1997.