LOCATION ROUTING PROBLEM WITH TRAFFIC IMPACT

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ABSTRACT

Substantial variation in travel time during different hours of a day is a result of traffic congestion which has significantly influenced the travelling decisions. Variations in time and consequently energy consumption are two significant consequences of the traffic congestion which has not been investigated thoroughly in literature of Location Routing Problem. Hence, in this paper a formulation of Time Dependent Location Routing Problem presented in which travel times are arbitrary functions of time. The objective function of the problem is to minimize total network cost incurred from transportation, depot establishment, and vehicle departures. The formulation is illustrated with an example.

INTRODUCTION

The problem under investigation in this paper is a Location Routing Problem (LRP). LRP is the combination of two problems Location-Allocation Problem (LAP) and Vehicle Routing Problems (VRP) [3]. LAP is the problem of locating a set of potential facilities and allocating customers to the locations with an objective of cost minimization [6]. On the other hand, VRP is the problem of finding a set of routes originating from a set of depots to serve a set of customers with known demands. Each customer must be visited only once and all vehicles return to the depot from which they departed. Also, cumulative customer demands in a route should not exceed the vehicle capacity [2]. Since the location of a Distribution Center (DC) impacts the routing of vehicles, LAP and VRP are investigated together in a more comprehensive problem called LRP.

Time Dependent LRP (TDLRP) is a variant of LRP in which the travel times between nodes in the network is not constant and may change depending on the time at which the travel occurs. Even though LRP has been vastly investigated in literature, research on TDLRP is very scarce [5][9]. Existing research work usually addresses only the VRP with time dependent travel times and do not approach the location and routing problem simultaneously. Orda and Rom [14] proposed an algorithm for the shortest problem in which an arbitrary function for a link delay is allowed. The objective of the work by Orda and Rom is to find the shortest path and minimum delay under different waiting constraints. Ahn and Shin [1] developed a heuristic for VRP with time window constraints and time-varying congestion. The heuristic is a modification of the saving, insertion, and local improvement algorithms. Hill and Benton [8] presented a method for estimating the time dependent travel speed and a heuristic to solve the Time Dependent Vehicle Routing Problem (TDVRP). Malandraki and Daskin [11][12] presented a Mixed Integer Linear Programming (MILP) model for the TDVRP with time window constraints. They developed a heuristic algorithm using the nearest neighborhood heuristic for VRP without time windows. In addition, a mathematical heuristic for the TDVRP with time windows was also developed. In this work, waiting time is allowed at nodes. The step functions considered for the travel times are symmetric. Ichoua, Gendreau, and Potvin [9] presented a heuristic solution methodology based on Tabu-search algorithm. Figliozzi [5] presented a flow-arc formulation for the TDVRP with hard and soft time windows along with heuristic algorithms for solving the problem. Most recent efforts regarding the

TDVRP has focused on developing a heuristic/meta-heuristic for solving the problem [4][7][15]. The main weakness of methods presented for TDVRP in literature can be categorized into two groups:

1. The travel time function is usually considered as a discrete step function. Due to this assumption, waiting times have to be permitted at customer location in order to get feasible solutions.
2. The effort toward applying a continuous travel time function is very rare and the formulations presented are too complicated and intricate to be analytically solved. A continuous travel function will allow improved modeling of the travel times, especially when the time intervals are for a day or shorter. This also allows easier analysis of the travel function, when the time changes from a highly congested to less congested or vice-versa.

The goal of this paper is to develop a formulation for TDLRP when travel times are a function of time (discrete or continuous) and no waiting time is allowed at a customer location. The travel time function can be developed from historical data of traffic congestion.

PROBLEM STATEMENT

In this section, notations used for the formulation of the problem are presented. There are *N* customers and *M* depots in the problem. The collective set of DCs and customers in the network is represented by nodes. Nodes 1 to N represent customers and nodes N+1 to N+M represent DCs. The decision variables in the formulation, provides the assignment of customers to vehicles, vehicles to DCs, as well as the sequence of visits by each vehicle. When nodes are assigned to a vehicle, a route is formed. Thus, a route is formed by a set of nodes. Position of a node in the route, is the order in which the node is visited by the vehicle. For instance, if node *g* is in position 2 of vehicle 1's route, it implies that node *g* is the second node visited by vehicle 1. *D* is the set of possible positions that a customer can take in a route. A vehicle may be assigned to visit at the most all N customers. Thus, the maximum number of possible positions for a customer is equal to *N*. The decision variables, $X_{mgv} = 1$, implies that node g (a customer or a DC) is the m^{th} node visited by vehicle *v*. Or in other words node g is the m^{th} node in the route assigned to vehicle *v*. Thus, in the definition of the notations, vehicle and route are used alternatively since they represent the same concept.

N	Total number of customers
M	Total number of DCs
K	Total number of vehicles
I	Set of customers, $I= \{1, 2, \ldots, N\}$
J	Set of DCs $J= \{N+1,2,\ldots, N+M\}$
D	Set of possible positions that a customer can take in a route, $D= \{1, 2,\ldots, N\}$
V	Set of Vehicles, $V= \{1,2,\ldots,K\}$
Y_v	Capacity of vehicle v
g/h	Index used for all nodes
X_{mgv}	$\begin{cases} 1 \text{ if node } g \text{ is in position } m \text{ of the route } v; \\ 0 \text{ otherwise} \end{cases} \quad \forall g \in \{I \cup J\}; \forall m \in D; \forall v \in V$
P_{mv}	$\begin{cases} 1 \text{ if } m \text{ is the last taken position of route } v; \\ 0 \text{ otherwise} \end{cases} \quad \forall m \in D; \forall v \in V$
O_g	$\begin{cases} 1 \text{ if there is any vehicle assigned to node } g; \\ 0 \text{ otherwise} \end{cases} \quad \forall g \in J$

z_{vh} $\begin{cases} 1 \text{ If vehicle } v \text{ is assigned to node h;} \\ 0 \text{ otherwise} \end{cases}$ $\forall h \in J; \forall v \in V$

C Cost per unit of time (Labor cost, vehicle cost etc.)

q_g Fixed cost for establishing node g, $\forall g \in J$

d_g Demand of node g, $\forall g \in I$

S_g Service time at node g, $\forall g \in I$

t_{mv} Cumulative departure time from position m on route v, $\forall m \in D; \forall v \in V$

T_{mv} Departure time from position m on route v (between 0:00 and 24:00), $\forall m \in D; v \in V$

$F_{gh} \langle t \rangle$ Travel time function between nodes g and h, $\forall g, h \in \{I \cup J\}$, $g \neq h$

B_g Departure cost from Distribution Center g, $\forall g \in J$

To ensure that the vehicles' depart always from a DC, position zero of each route is reserved for a DC. Hence, only DCs can be assigned to position 0 and DCs cannot take any other position in a route.

Each customer $g \in I$ has a demand d_g which is less than the vehicle capacity, Y_v. The travel time between each pair of nodes in the system is a function of time, $F_{gh}\langle t \rangle$, which is derived from historical data. There are K heterogeneous vehicles available and each vehicle departs DCs fully loaded. In addition, M locations are available for the establishment of DCs. If a DC is established, it incurs a cost of q_g, $g \in J$ in the system. There is only one type of product in the system. The objective of the problem is to find the location for establishment of DCs and routing plan for vehicles in order to minimize the system cost.

X_{mgv}'s are the decision variables defined to model the problem. X_{mgv} is a binary variable for $\forall g \in \{I \cup J\}, \forall m \in D, \forall v \in V$ which takes a value of 1, if node g (a customer or a DC) is placed in order (position) m of vehicle v's routing plan; and otherwise it is 0. There are additional terms defined to simplify the representation of the objective function and the constraints in the problem formulation. P_{mv}, presented in equation (1) is used to determine the final position taken on route v.

$$P_{mv} = \begin{cases} \sum_{g \in I} X_{mgv} & m = N \\ \prod_{m'=m+1}^{N} (1 - P_{m'v}) \sum_{g \in I} X_{mgv} & \forall m \in \{D / \{N\}\} \end{cases} \quad v \in V \tag{1}$$

Position 0 of each route is reserved for a DC. If the binary variable, $X_{0gv} = 1, \forall g \in J, \forall v \in V$, it means that vehicle v is assigned to distribution center g. However, distribution center g is not opened until a customer is also assigned to the vehicle v. Thus**,** g will be vehicle v's *DC* only if there is a link between the depot and a customer in the network. z_{vh} defined in (2) ensures that a connection exists between a DC and a customer in the system. z_{vh} is similar to the connectivity constraint between LAP and VRP in traditional formulation of LRP, and connects the location decision to routing decisions.

$$z_{vh} = \sum_{g \in I} X_{0hv} . X_{1gv} \qquad \forall h \in J; v \in V \tag{2}$$

The value of t_{mv} is calculated through a set of recursive equations given by (3). The value of t_{mv} can be considered to be 0 if the start time is set as zero.

$$\begin{cases} t_{0v} = 0 \\ t_{mv} = \left(\sum_{m'=1}^{m} \sum_{h \in I} \sum_{g \in \{I \cup J\}} \left(F_{gh} \langle t_{(m-1)v} \rangle + S_h \right) X_{m'-1gv} X_{m'hv} \right) \left(\sum_{g \in I} X_{mgv} \right) \end{cases} \quad \forall m \in D; \forall v \in V \tag{3}$$

If the travel time function repeats itself after H units of time, equation (3) must be replaced by equation (4) to calculate the t_{mv} $\forall m \in D; \forall v \in V$.

$$\begin{cases} T_{ov} = t_{0v} = 0 \\ t_{mv} = \left(\sum_{m'=1}^{m} \sum_{h \in I} \sum_{g \in \{I \cup J\}} \left(F_{gh} \left\langle T_{(m-1)v} \right\rangle + S_h \right) X_{m'-1gv} X_{m'hv} \right) \left(\sum_{g \in I} X_{mgv} \right) \quad \forall m \in D; \forall v \in V \\ T_{mv} = Mod(t_{mv}, H) \end{cases} \quad (4)$$

Where, Mod function returns the reminder of dividing t_{mv} by H. For instance, if the travel time functions are represented in unit of hour and they repeat every day, the value of H will be 24. In the following section, the mathematical formulation of the problem is presented.

MATHEMATICAL FORMULATION OF THE PROBLEM

The MINLP programming of the problem defined in the previous section is presented below:

Min

$$\sum_{v \in V} \sum_{m \in D} C * P_{mv} \left(t_{mv} + \sum_{h \in I} \sum_{g \in J} \left(F_{hg} \left\langle T_{mv} \right\rangle * X_{0gv} * X_{mhv} \right) \right) + \sum_{v \in V} \sum_{g \in J} z_{vg} B_g + \sum_{g \in J} q_g O_g \quad (5)$$

Subject to:

$$\sum_{v = V} \sum_{m \in D} X_{mgv} = 1 \quad \forall g \in I \quad (6)$$

$$\sum_{g \in I} X_{mgv} \leq 1 \quad \forall m \in D; v \in V \quad (7)$$

$$\sum_{g \in I} \sum_{m \in D} d_g X_{mgv} \leq Y_v \quad \forall v \in V \quad (8)$$

$$\sum_{v \in V} \sum_{g \in J} \sum_{m \in D} X_{mgv} = 0 \quad (9)$$

$$\sum_{v \in V} \sum_{g \in I} X_{0gv} = 0 \quad (10)$$

$$\sum_{g \in \{I \cup J\}} X_{m-1gv} \geq \sum_{g \in \{I \cup J\}} X_{mgv} \quad \forall m \in D; v \in V \quad (11)$$

$$\sum_{g \in J} z_{vg} \leq 1 \quad \forall v \in V \quad (12)$$

$$O_g \leq \sum_{v \in V} z_{vg} \leq K O_g \quad \forall g \in J \quad (13)$$

$$1 \leq \sum_{v \in V} \sum_{g \in J} z_{vg} \leq K \quad (14)$$

$$X_{mgv}, z_{hv}, P_{mv}, O_g \in \{0,1\}$$

Equation (5) is the objective function which minimizes the total travel time, the establishment cost of DCs, and the vehicle dispatching cost from DCs. Constraint (6) ensures that each customer appears in only one route, i.e., only one route is assigned to each customer. Constraint (7) enforces that each position of a route will not be taken by more than one customer. Constraint (8) makes sure that the total demand of customers assigned to a route is less than the vehicle capacity. It is assumed that position zero of each route is reserved for DCs. This assumption implies that DCs cannot take any other position in routes, and also customers cannot take position 0 of their assigned route. These conditions are imposed by equations (9) and (10). Constraint (11) ensures that position $m+1$ of a route cannot be taken unless position m is taken. Constraint (12) ensures that a vehicle is not assigned to more than one DC. Constraint (13) determines whether a DC is open or is close. Constraint (14) keeps the total number of the vehicles between one and the number of available vehicles.

ILLUSTRATIVE EXAMPLES

A two layer network problem is used to illustrate the proposed mathematical model. The problem consists of 2 DCs and 4 customers. Nodes 1, 2, 3, and 4 represent the customers and nodes 5 and 6 represent potential DCs, respectively. For the detail definition of travel time functions between each pair of nodes one can refer to [13]. There are two vehicles available with capacities of 50 and 70 units of product respectively. The departure costs from node 5 and node 6 are $45 and $50 respectively. The fixed cost of DCs in the planning horizon of the problem is $250 for node 5 and $200 for node 6.
Customers' demands for customers 1, 2, 3, and 4 are 40, 25, 20, and 10 respectively. Service times at customer's location are zero. The unit time (hour) cost for service is $3.
The mathematical formulation was solved using LINGO 12.0 optimizer software on a Pentium D CPU 3.2GHz, and 3.25 GB of RAM. The result obtained is shown in Table 3.1.

TABLE 1 RESULT OBTAINED FOR TDLRP

Objective Value	619.06
Computation Time	00:07:36
Variables with Value of 1	X_{112}, X_{131}, X_{221}, X_{242}, X_{061}, X_{062}, z_{16}, z_{26}, P_{21}, P_{22}

The result presented in the table implies that there are two routes in the network. The first route is 6-3-2-6 assigned to vehicle 1 with capacity of 50 units and the second route is 6-1-4-6 assigned to vehicle 2 with capacity of 70 units. Both routes are assigned to depot 2 (node 6).

CONCLUSION

Traffic congestion is a normal phenomenon especially in urban areas. Traffic rush hours in the morning and evening typically result in higher travel times. Thus, traffic congestion influences the time taken to travel. The main drawback of existing formulations of TDLRP is that the waiting time at customers' location is used to take care of time window violations. In the formulation, the time windows are eliminated and a TDLRP formulation that eliminates the waiting time at customer locations is first developed. Depending on the type of travel time function, different solution methods can yield different computational time, and this is a subject of future research.
In addition, since the problem is NP-hard, heuristic or meta-heuristic algorithms for solving large problems are required. A closer look at constraints (13) and (14) reveal that they can be decoupled from route/vehicle, and hence column generation or bender decomposition for finding the exact solution of larger size problems is possible and can be further investigated.

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