BINARY LINEAR PROGRAMMING AND SIMULATION FOR CAPITAL BUDGEETING

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ABSTRACT

Binary linear programming is often used to maximize total NPV for a set of feasible projects given a capital constraint. The BLP optimal combination of projects can be further analyzed with simulation and a probability distribution for total NPV. In contrast, when a statistic is first specified for the total NPV output, the BLP selection of projects can be required to meet the specified output statistic. Hence, managers are better able to examine alternative strategies in maximizing total NPV. For example, when a risk-averse strategy is specified, a different set of projects are found for the BLP solution. A case is presented to illustrate the concurrent use of BLP and simulation for capital budgeting.

INTRODUCTION

Capital budgeting analysis benefits from spreadsheet add-ins for linear programming and risk analysis by simulation. Binary linear programming (BLP) will select a combination of projects that maximizes total NPV given a capital constraint. Simulation performs risk analysis for capital budgeting with probability distributions for key inputs and NPV outputs. However, BLP and risk simulation cannot be performed concurrently unless a statistic is specified for the NPV output variable. The statistic specified for NPV allows for management strategies to be examined for a capital budgeting decision. The Sasha Case presents a risk-averse strategy which leads to a set of capital projects that differs from the BLP solution. This case may be used within an MBA managerial accounting course,

Capital Budgeting and Binary Linear Programming in Accounting

Linear Programming is an optimization model in which an objective function is optimized given scarce resources or other requirements [4]. A typical objective function in accounting is a performance measure such as contribution margin to be maximized or costs to be minimized. Constraints or restrictions on the set of allowable decisions are often in the form of physical and economic limitations. Within accounting, constraints for LP models include sales requirements and scarce resources for manufacturing direct materials, direct labor and machine hours.

Binary linear programming (BLP) models are used to indicate logical or dichotomous decisions (e.g., on/off, true/false, or accept/reject) with integer variables (usually 1 and 0). Models for scheduling, financial portfolios, capital rationing environments, and production planning [4] are common applications for BLP. Binary linear programming is seldom discussed in accounting education; and, the topic of linear programming is often presented with a graphical approach for just two projects within an appendix [2]. Furthermore, students utilizing LP models will often round to the nearest integer even when Integer LP techniques are readily available [1]

Simulation and Risk Analysis for Capital Budgeting

Risk analyses often begin with estimates of uncertain input variables in evaluating the expected output values of modeled relationships. In capital budgeting, input variables for net annual cash savings, interest rates, and project lives are commonly used for sensitivity analysis or scenario analysis. Simulation add-ins to spreadsheets (e.g., @Risk and Crystal Ball) have replaced these traditional techniques and are better suited for risk analysis [3] [6]. Simulation models have key input variables as probability distributions. When a simulation is completed, a targeted cell such as NPV will have an output distribution that identifies the range of outcomes and its likelihood of occurrence.

OPTIMIZING A SIMULATION

Linear programming is unable to optimize models that have probability distributions as input variables. Yet, when a statistic is specified for an output variable, add-ins to spreadsheets (e.g., RiskOptimizer and OptQuest) will optimize models having probabilistic input. An optimized simulation finds a set of values that meets both the constraints of the LP model and the desired simulation statistic of the output variable [5]. Hence, managers can specify a statistic for the output variable that will examine various risk strategies. The following Sasha Company example highlights binary linear programming and optimized simulation as tools for risk analysis for a capital rationing decision. This example has been presented to students in a graduate managerial accounting course after discussing capital budgeting and linear programming.

Sasha Company: An Example for Optimized Simulations

Sasha Company is examining five feasible capital projects A, B, C, D, and E for investment in the coming year. NPVs are first calculated for each project from their annual net cash savings and investment cost. Binary linear programming solves for the optimal mix of projects that maximizes Total NPV given a capital constraint. Uncertainty is added to the decision with probability distributions for annual net cash savings. An optimized simulation is performed in which a risk-averse strategy that maximizes the 30th percentile for Total NPV leads to a project mix that differs from the BLP solution.

Project Data

Managers at Sasha Company recognize the uncertainty surrounding key input variables to a capital budgeting model and prefer a probability distribution instead of a single estimate. Hence, an expected value used by deterministic models is the mean of an underlying distribution. In Panel A of Table 1, each project’s cash flows are presented - NPV, equipment cost, and annual net cash inflow with its PV, mean and underlying probability distribution.

<Insert Table 1>

In Panel B of Table 1, the PV of the net annual cash inflows is first computed, and NPV for each project is found after deducting the cost of the equipment. Assume a 6% required rate of return and a four-year life for all projects.

Deterministic BLP Solution

Binary linear programming and the NPVs for the five projects are used to solve for a mix of projects which maximizes Total NPV subject to the capital constraint of $2,000,000. Also, assume that three projects must be accepted (e.g., to keep the full-time employees working). In building the spreadsheet model, the decision variables (“1” or “0”) are to be included as a multiplicative factor for NPV, minimum projects requirement, and the equipment cost constraint.

The BLP solution is presented in Panel B of Table 1. The BLP decision shows that projects B, C and E are to be selected. Total NPV will be maximized at $186,000 while equipment costs of $1,875,000 will be less than the $2,000,000 constraint. From the BLP solution set, a simulation is performed for Total NPV and the output distribution is shown in Panel A of Table 2. It identifies a 14.2% percentile having a negative NPV, with a minimum of <$336,000> and a maximum of $714,000.

<Insert Table 2>

Optimized BLP Simulation Solution

From the project distributions for net cash savings presented in Table 1, an optimizing simulation is performed for a risk-averse solution specifying that Total NPV be maximized at the 30^{th} percentile. This risk-averse strategy will select projects that reduce the likelihood of a negative NPV. Panel C of Table 1 finds that projects A, B and D will have Total NPV of $165,000 at its 30^{th} percentile, while incurring equipment costs of $1,775,000. The output distribution is found in Panel B of Table 2, having a maximum of $381,000.

Comparing BLP and Optimized Simulation Results

The risk-averse strategy of Panel B in comparison to Panel A of Table 2 indicates (a) Total NPV of has been reduced by $21,000 ($186,000 less $165,000), (b) equipment costs has been reduced by $100,000 ($1,875,000 less $1,775,000), (c) the likelihood of having a negative NPV has been nearly eliminated, and (d) the range of maximum NPVs are reduced with the risk-averse strategy. The expected net savings of $79,000 ($100,000 less $21,000) is projected for the risk-averse strategy; however, the tradeoff is that expected maximums have been curtailed.

REFERENCES

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TABLE 1: PROJECT DATA, BLP AND OPTIMIZED SOLUTIONS

Panel A: Project Cash Flows (in thousands)

Project	NPV	Equip. Cost	Annual Net Cash Inflow PV	Mean	Probability Distribution
A	$56	$550	$606	$ 250	Triangular, $165 minimum, $170 most likely, $190 maximum
B	$60	$650	$710	$ 205	Normal, $205 mean, $15 standard deviation
C	$68	$625	$693	$ 200	Uniform, $150 minimum, $250 maximum
D	$49	$575	$624	$ 180	Histogram, $170 minimum, $189 maximum, divided with probability ratios of 1, 4, 3, 2
E	$58	$600	$658	$ 190	Triangular, bottom points $140 and $240 with 10% probability, most likely $190 with 90%
		$3,000			

Panel B: BLP Solution (in thousands)

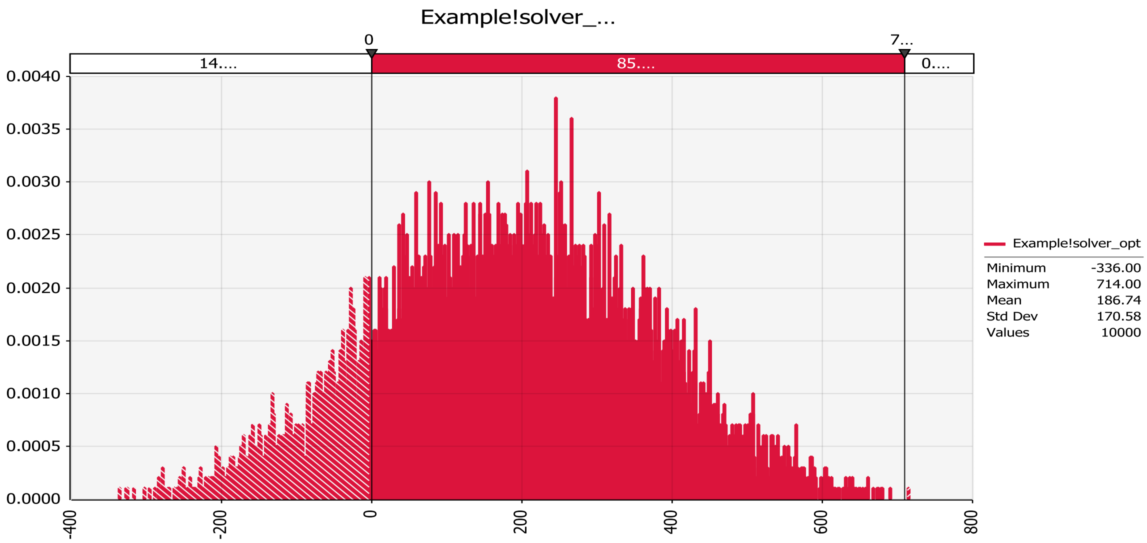
	Proj. A	Proj. B	Proj. C	Proj. D	Proj. E	Total		
Decision	0	1	1	0	1	3		
NPV	0	$60	$68	0	$58	$186		
Minimum projects	0	1	1	0	1	3	>=	3
Equipment cost	0	$650	$625	0	$600	$1,875	<=	$2,000

Panel C: BLP Optimized Solution (in thousands)

	Proj. A	Proj. B	Proj. C	Proj. D	Proj. E	Total		
Decision	1	1	0	1	0	3		
NPV	$56	$60	0	$49	0	$165		
Minimum projects	1	1	0	1	0	3	>=	3
Equipment cost	$550	$650	0	$575	0	$1,775	<=	$2,000

TABLE 2: NPV DISTRIBUTIONS FOR BLP AND OPTIMIZED SIMULATION SOLUTIONS

Panel A: NPV Distribution for BLP Solution



Panel B: NPV Distribution for Optimized Simulation - Total NPV Maximized at 30th Percentile