OUTSOURCING CONTRACT DESIGN: PRICE AND TIME MODEL

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ABSTRACT

Outsourcing is currently being used as an important strategy for many companies to achieve cost savings and speed up delivery time. We consider a company, or a buyer, that outsources her works to a supplier in order to reduce costs and time-to-market. A game theoretic model is used to design the optimal contracts between the buyer and the supplier under two types of information scenario. Under the full information case, the buyer shares her private cost information with the supplier. Under the asymmetric information case, the buyer does not share her private cost information with the supplier. In both cases we find the optimum outsourcing price, outsourcing time, and retail price.

INTRODUCTION

Outsourcing is one of the best management solutions of reducing costs and time-to-market. In this research, we consider a company, or a buyer, that outsources part of its service to a supplier in order to reduce the cost and time-to-market. We use a game theoretic model to design the optimal contracts between the buyer who seeks outsourcing and the supplier under two types of information scenario. Under the full information case, the buyer shares her private cost information with the supplier. Under the asymmetric information case, the buyer does not share her private cost information with the supplier. In both cases we find the optimum outsourcing price, outsourcing time, and retail price.

LITERATURE SURVEY

Outsourcing has been widely adopted, like manufacturing outsourcing, call center outsourcing, IT outsourcing, and BPO (Business Process Outsourcing). In the call center literature, several papers model the call center as a queue, as reviewed in details by [1]. [2] studies two types of call center contract: a volume-based (a per-call pricing) and a capacity-based (a per-agent-per-hour) contract. IT outsourcing through empirical analysis has been studied in the following papers. [3] performs an empirical analysis of offshore software development, where the choice between fixed price and time-and-materials type contracts are explored. We will study how an optimal outsourcing contract can be designed under cost asymmetric information. Recently, a number of studies came out on asymmetric information.[4] study different types of contracts to coordinate a supply chain for both complete information and asymmetric information asymmetry cases. [5] finds that in case of asymmetric information, optimal order quantity is smaller and optimal selling price is higher than for the case with full information. [6] studies the asymmetric information issue in a mixed channel setting. [7] studies the quality management for outsourcing contract with buyer’s cost asymmetric information. In contrast to any of the above research streams, our paper combines the issue of two major outsourcing benefits (cost reduction and time-saving) and information asymmetry. We focus on the effect of information on the outsourcing contract design.

THE MODEL SCENARIO

We consider a single supplier and single buyer in a supply chain. A buyer outsources her work to a supplier and resells the outsourced product to potential customers. We consider a Stackelberg game where the supplier acts as the leader and the buyer acts as a follower. First, the supplier decides the unit outsourcing price *s* and outsourcing time *t*. Then the buyer announces the retail price *p* to the final customers. The decision variables in our model are *s* and *t* for the supplier and retail price *p* for the buyer, each maximizing his or her own profit functions. We will use game theory to study the relationships among outsourcing price, outsourcing time, and retail price.

<u>Demand Function</u>

$$d = a - bp - rt \tag{1}$$

where $a>0$, $b>0$, and $r > 0$ are known parameters. As we see that the demand *d* is decreasing in price charged *p* and time-to-market (outsourcing time) *t*. *a* is the base demand. *b* is the price elasticity. *r* is the sensitivity of outsourced product with respect to the time-to-market. From the demand function, we can see that the demand *d* will increase with lower retail price *p* and shorter time-to-market *t*. Time-to-market is an important factor to impact the retail price, market share and revenue. Including time factor in the demand function is the one of major contributions for this research.

<u>Profit Functions</u>

The supplier's profit function is:

$$\pi_S = (s-c)d - \frac{L}{2}\left(\frac{T-t}{T}\right)^2 \text{ or } \pi_{S\_} \tag{2}$$

The supplier receives *s* from the buyer for each unit of outsourcing work. The costs for offering outsourcing work include variable cost *c* per unit and fixed cost $\frac{L}{2}\left(\frac{T-t}{T}\right)^2$. *T*, pessimistic estimate time, is the maximum time required for completing outsourcing work under adverse conditions. *t*, the span of completing outsourcing work, is the decision variable for the supplier. *L* is a fixed cost parameter. *T* and *L* are common knowledge.

The buyer's profit function is:

$$\pi_B = (p-s-m)d \quad \text{ or } \quad \pi_{B\_} \tag{3}$$

The profit margin for each outsourcing work is decided by using retail price *p* subtracting by outsourcing price *s* and buyer's internal variable cost *m*. We assume that $p-s-m>0$. *m* is a private information of the buyer. In general, the supplier does not know *m*. The supplier assumes the buyer holds a prior cumulative distribution *F(m)* with density function *f(m),* defined on $[\underline{m}, \overline{m}]$, where $0 \le \underline{m} \le \overline{m} \le \infty$.

The reservation profit levels of the supplier and the buyer are $\pi_{S\_}$ and $\pi_{B\_}$, respectively.

THE CONTRACTS

We will study contracts under two information structures, full information scenario and asymmetric information scenario.

Contracts under Full Information (F)

The buyer shares her private variable cost information *m* to the supplier. The moves of buyer and supplier follow a Stackelberg type of game. The supplier acts as a leader, proposing outsourcing unit price *s* and finish time *t*. The buyer acts as the follower, announcing retail price *p* after that. We first derive the buyer's best response function as following,

$$p^{BR} = \frac{a - tr + bs + bm}{2b} \tag{4}$$

The supplier substitutes this best response function (4) into his profit function. Then he will obtain the optimization solution by maximizing his profit function. Proofs are available by request.

<u>Proposition 1.1:</u>
The equilibrium outsourcing price, outsourcing time, retail price, and profits under full information are given by:

$$s_F = -\frac{-2aL + T^2r^2c + 2TrL + 2bmL - 2cbL}{4bL - T^2r^2}$$

$$t_F = \frac{T(-Tra + Trbm + Trcb + 4bL)}{4bL - T^2r^2}$$

$$p_F = -\frac{-3aL + T^2r^2m + T^2r^2c + 3TrL - bmL - cbL}{4bL - T^2r^2}$$

$$\pi_{BF} = \frac{L^2\,(-a + Tr + bm + cb)^2\, b}{\left(4bL - T^2r^2\right)^2}$$

$$\pi_{SF} = \frac{1}{2}\,\frac{L\,(-a + Tr + bm + cb)^2}{4bL - T^2r^2}$$

<u>Proposition 1.2:</u>

i. *The optimal outsourcing unit price* s_F is increasing in *a* and decreasing in *m;*
ii. *The optimal outsourcing time* t_F is increasing in *m* and *c*, and decreasing in *a*;
iii. *The optimal retail price* p_F is increasing in *a;*
iv. *The buyer's profit* π_{BF} *is less than or equal to the supplier's profit* π_{SF} *if* $T^2r^2 \le 2bL$

Contracts under Asymmetric Information (A)

The supplier typically does not know the buyer's internal variable cost *m*. We assume that the supplier only knows the prior density function *f(m)* and cumulative distribution *F(m)* of the buyer's true cost *m*. *f(m)* and *F(m)* are defined on $[\underline{m}, \overline{m}]$, where $0 \le \underline{m} \le \overline{m} \le \infty$. The supplier acts as a leader, proposing outsourcing unit price *s* and finish time *t* without knowledge of *m*.

The buyer acts as the follower, announcing retail price *p* after that. In the next proposition, we derive the equilibrium outsourcing price, outsourcing time, and retail price for the buyer and supplier under Asymmetric Information in the next proposition.

<u>Proposition 2.1:</u>
The outsourcing price, outsourcing time, retail price, and cut-off point under asymmetric information, at the cut-off point M, are given by:

$$s_A = -\frac{-2aL + T^2r^2c + 2TrL + 2b\dfrac{\int_{\underline{m}}^{M} mf(m)dm}{F(m)|_{\underline{m}}^{M}}L - 2cbL}{4bL - T^2r^2}$$

$$t_A = \frac{T(-Tra + Trb\dfrac{\int_{\underline{m}}^{M} mf(m)dm}{F(m)|_{\underline{m}}^{M}} + Trcb + 4bL)}{4bL - T^2r^2}$$

$$p_A = \frac{a - t_A r + bs_A}{2b} + \frac{m}{2}$$

M satisfies $(s_A - c)(a-b)(\frac{a - t_A r + bs_A + bM}{2b}) - rt_A - \frac{L}{2}(\frac{T - t_A}{T})^2 = \pi_{S\_}$

The corresponding profits at equilibrium are given as:

$$\pi_{SA} = (s_A - c)d - \frac{L}{2}(\frac{T - t_A}{T})^2$$

$$\pi_{BA} = (p_A - s_A - m)d$$

We obtained closed form solutions for the special case at cut off point M, where $\pi_{SA}(M) = \pi_{S\_}$. There is no closed form solution for the general case.

<u>Proposition 2.2:</u>

i. *The optimal outsourcing unit price* s_A is increasing in *a;*
ii. *The optimal outsourcing time* t_A is increasing in *c*, and decreasing in *a*;

We get similar findings for the full information case.

NUMERICAL EXPERIMENTS

In this section, we will report the results of our extensive numerical experimentation.The numerical values used in this experiment are given in Table 1.

a	b	r	L	T	c	m	$\underline{m}$	$\pi_{S\_}$
30	3	0.1	0.2	1.00	2	2	1	3

Table 1: *numerical values in the experiments*.

We assume a Uniform distribution for *m* with $f(m) = \frac{1}{\overline{m} - \underline{m}}$ and $F(m) = \frac{m - \underline{m}}{\overline{m} - \underline{m}}$ over the interval $[\underline{m}, \overline{m}]$ for the asymmetric information case. From Figure 1, we see that the outsourcing time t_A and t_F are all increasing with the buyer's internal cost *m*. It is also interesting to find that the outsourcing time *t* is shorter under asymmetric information (A) than full information (F) case. The managerial insights we gain here is that the buyer can reduce the time-to-market by sharing her internal cost *m* with the supplier. After getting the buyer's internal cost, the supplier will reduce his outsourcing time from t_A to t_F .

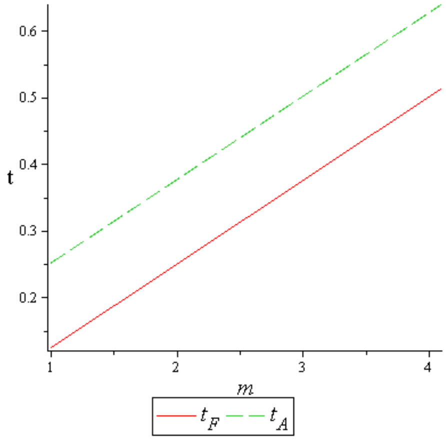

Figure 1: *m* versus the outsourcing time under (F) and (A) case.

From Figure 2 and Figure 3, we observe that π_{BF} , π_{BA}, π_{SF} , and π_{SA} are all decreasing with *m*. This result is expected. We also observe that $\pi_{BF} + \pi_{SF} > \pi_{BA} + \pi_{SA}$ (the graph is omitted), i.e., the total supply chain profits are always higher under (F) than under (A). The profit realized under supply chain coordination is always higher than the sum of the retailer's and the manufacturer's profits without such coordination.

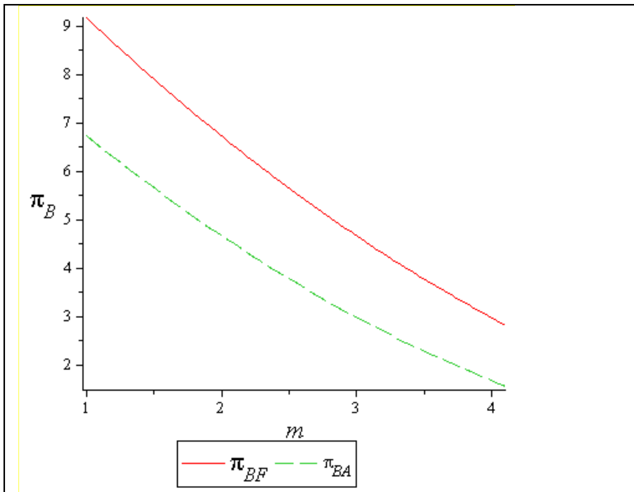

Figure 2: *m* versus the buyer profit π_{BF} and π_{BA} under (F) and (A) case.

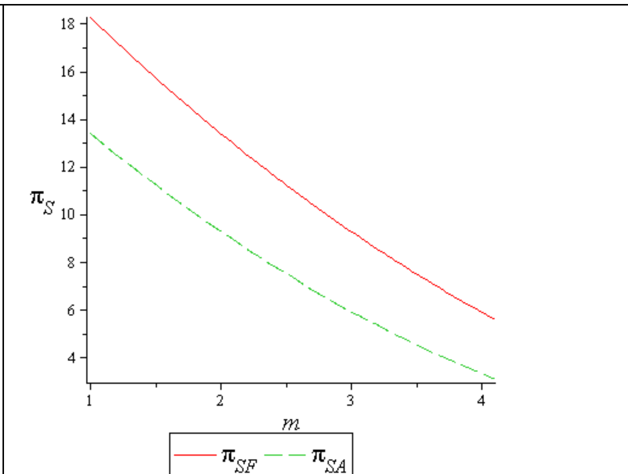

Figure 3: *m* versus the supplier profit π_{SF} and π_{SA} under (F) and (A) case.

MANAGERIAL IMPLICATIONS AND CONCLUSION

In this paper, we consider an outsourcing scenario where a buyer outsources product or service to a supplier. We derived the equilibrium outsourcing price, outsourcing time, retail price, cut-off point, and profits under these two cases, full information and asymmetric information. Numerical experiments are conducted. The main insights from this paper are as follows. The supplier's decision variable s_F, outsourcing unit price, is increasing in base demand *a* and decreasing in buyer's internal cost *m.* The other supplier's decision variable t_F, optimal outsourcing time, is increasing with his own internal cost *c* and buyer's internal cost *m*, and decreasing in base demand a. The value of information is reflected in the gap between the cases of full information (F) and asymmetric information (A), as shown in Figure 2 and Figure 3. We also found that the supplier, buyer, and total supply chain would get higher profit under (F) than under (A).

APPENDIX

Proof of Proposition 1.1

First, the supplier can derive the best response function of the buyer in terms of *p* (as in Lemma 1) by maximizing buyer's profit π_B, given in Equation (3). Then in stage 1 of the game, the supplier maximizes his own profit by cooperating the buyer's best response function to derive the optimal outsourcing price and time.Last, in stage 2 of the game, the buyer derives the optimal retail price by bring optimal outsourcing price and time into Lemma 1.

Proof of Proposition 2.1

The Lagrangian for the supplier's problem as follows. Assume π_s is decreasing function in *m*.

$$L = \int_{\underline{m}}^{M} \pi_s f(m)dm + \int_{M}^{\overline{m}} \pi_{s\_} f(m)dm + \lambda\left(\pi_B - \pi_{B\_}\right)$$

Where λ is the Lagrangian variable.

Using KKT-condition, we set $\frac{\partial L}{\partial s} = 0$, $\frac{\partial L}{\partial t} = 0$ and $\frac{\partial L}{\partial M} = 0$.

s_A, t_A,and *M* can be solved simultaneously.

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