# FUZZY MULTI-ATTRIBUTE DECISION SUPPORT SYSTEM

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## ABSTRACT

This article proposes a Fuzzy Multi-attribute Decision Support System (FMDSS). The FMDSS provides interactive linguistic sessions to elicit reliable subjective judgment, and effective procedures for detection and correction of inconsistencies. Nakamura's requirements for additive fuzzy utility differences are incorporated in the FMDSS.

## INTRODUCTION

Articulating subjective judgment from human experts is a very complex task, as suggested by psychologists and researchers in decision-making ([4] [9] [16]). Due to inconsistencies associated with expert or decision-maker judgment ([1] [14]), available multi-attribute decision methods have not found successful applications in many practical problems.

Qualitative (ordinal) evaluation and comparison of different alternatives are however much more appropriate than performing quantitative techniques on subjective judgment. Preference cones ([6] [7] [9] [13] [15]), Artificial Intelligence-based methods [3] are popular qualitative evaluation methods.

In order for multi-attribute methods to be successful, they should employ reliable information forms [12], and effective (interactive and using natural forms) means to detect and correct inconsistencies. Researchers confirm that reliable information forms are necessary for the representation of human judgment. Means of verification and correction of the inconsistency of information contents of those forms are required to support the elicitation process. Those means should be interactive and expressed in natural forms [9].

## FUZZY COMPUTATIONAL ENVIRONMENT

The article proposes a computational environment for fuzzy multi-attribute decision-making where the decision maker finds support in all steps needed to reach a solution. Subjective information is examined for inconsistencies before any step is processed. Intransitivity situations are identified when Attribute Wise Fuzzy Utility Differences (AFUD) is assessed, when Compound Fuzzy Utility Differences (CFUD) are obtained by aggregating AFUD, and when fuzzy preferences are induced from CFUDs. All the alternatives involved in those inconsistencies are returned to the decision maker. The proposed computational environment provides interactive procedures to support the decision maker in reassessing his/her new AFUDs. This process is reiterated until inconsistencies are no longer a threat to the decision process.

[*] The authors wants to thanks Professor Bel Raggad from Pace University

Let $X = \{x_i, i = 1, N\}$ be a finite set of alternatives. Also let $R$ be a fuzzy preference relation on $X^2$ ([10]). That is, $R$ is a binary fuzzy relation on $X$, whose compatibility function of any pair $(x_i, x_j)$ in $X^2$ is expressed by $r(x_i, x_j)$ in $[0,1]$. The value of $r(x_i, x_j)$ may be understood as a confidence grade whereby the alternative $x_i$, is preferred to the alternative $x_j$.

On the other hand, let $Y = \{y_i, i = 1, M\}$ be a finite set of attributes associated with the set of alternatives $Y$. The set of attributes forms the criteria for preferential judgment.

The literature provides two modeling concepts based on fuzzy utility differences, referred to as Type-I and Type-II concepts in [10].

In Type-I, given an attribute $y_k$, a utility value of every alternative $x_i$ is expressed as an ordinary number in the psychological space $U$. If the utility differences between any two alternatives $x_i$, and $x$ given a fixed attribute $y_i$ in $Y$ are small, the fuzziness in the attribute wise binary preferential judgment for $(x_i, x_j)$ will be derived from the difference.

Type-II associates a fuzzy number expressing an attribute wise utility value to every alternative in $X$. A pairwise comparison of fuzzy numbers will then lead to attribute wise binary preferential judgments.

Tversky [17], Tversky and Russo [18], and Nakamura [10] showed that both Type-I and Type-II encounter a serious difficulty at the aggregation process. Furthermore, Nakamura [10] confirmed that modeling with constant relational factors in the framework of the fuzzified additive utility difference structure might result in inconsistent preferential judgment. Nakamura suggested that the conditions 1) that the utility fuzzy numbers are of common shape and have attribute wise constant relational factors, and 2) that the utility fuzzy numbers are mutually F-non interactive on the alternative set, are at the origin of the inconsistency associated with Type-I and Type-II concepts.

This study however adopts the framework of the fuzzified additive utility difference structure where attribute wise relational factors are functions of attribute wise utility differences. Nakamura required 1) that fuzzy utility numbers be of common shape; and 2) that relational factors be strictly decreasing functions of mean values of fuzzy utility differences. Nakamura's corrections on this framework are also adopted.

This article does not elicit crisp utility values, as in Type-I, nor does assess fuzzy utility values, as is type II, but employs a heuristic procedure for the purpose of assisting the decision-maker in estimating real values representing pairwise fuzzy utility differences. Our assumption that the fuzzy utility differences of pair of alternatives are of semi-common shape stems from Nakamura's first assumption (1), since the fuzzy utility numbers are of common shape.

Nakamura's second assumption (2) requires that those pairs of alternatives of small differences should be computed using strictly smaller interactive relation factors. In other words, it is required that the resulting fuzzy utility differences that are smaller should be associated with smaller spread factors (less fuzziness).

Our approach does not really assess the decision maker's utility values of alternatives but pairwise fuzzy utility differences. We hence do not compute differences of fuzzy numbers for which assumption (2) is required. Nevertheless, the property that smaller fuzzy utility differences (i.e., smaller mean values) have smaller spread factors is very desirable to have and very easy to detect and correct, through a dialog between the decision maker and the system.

The system assists the decision maker in assessing his/her attribute wise fuzzy utility differences as a fuzzy number for every pair of alternatives. The system then aggregates the attribute wise fuzzy utility differences over the attributes to produce the compound fuzzy utility differences between paired alternatives. The system then develops a fuzzy preference relation on the set of alternatives.

The fuzzy utility difference scheme is therefore represented by M square matrices $\Pi_k, k = 1, M$. The matrix $\Pi_k$ contains fuzzy utility differences for pairs of alternatives $(x_i, x_j), i, j = 1, N$, given the attribute $y_k$. Entries of all the matrices $\Pi_{kij}$ are fuzzy numbers of mean values $\overline{\Pi}_{kij}$.

## THE MODEL

The FMDSS supports the decision maker in six phases:

1. Eliciting utility differences for each attribute (AFUD)
2. Attribute wise utility difference inconsistency management
3. Computation of compound utility differences (CFUD)
4. Computation of fuzzy preferences on the alternatives
5. Preference inconsistency management
6. Choice making

The design of the FMDSS is shown in Figure 1.

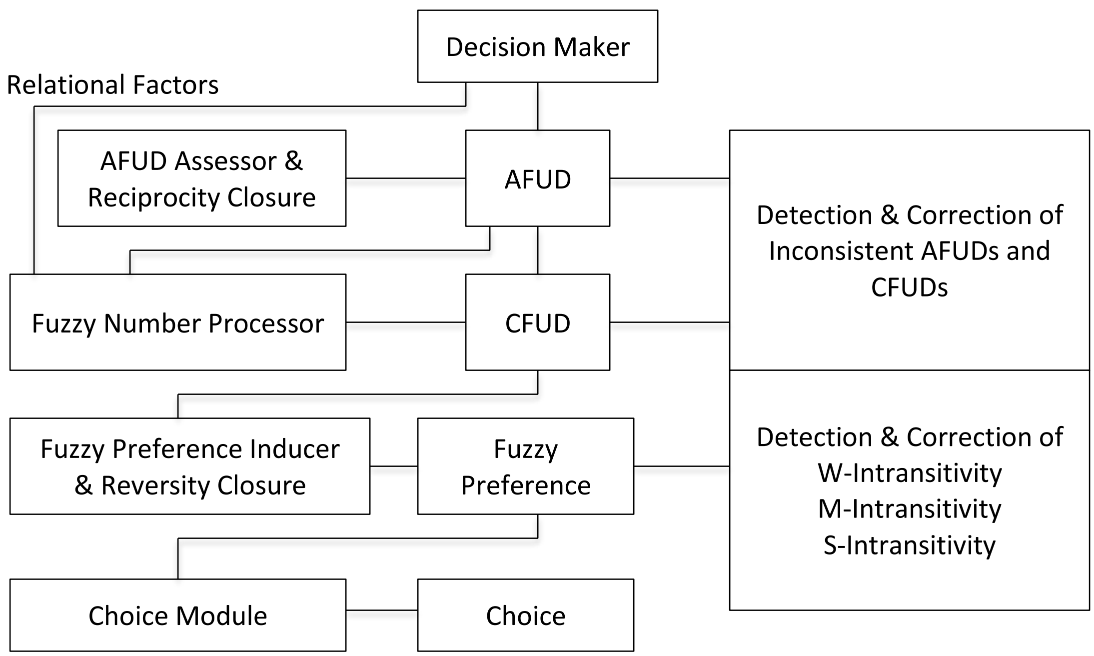


**Figure 1. Design of the FMDSS**

## ELICITATION OF UTILITY DIFFERENCES

Before explaining the process of eliciting fuzzy utility differences, two definitions need to be introduced.

**Definition 1: Fuzzy attribute wise utility difference.** A fuzzy attribute wise utility difference (AFUD) is a fuzzy subset that is normal, convex, symmetric, bounded, and has a piecewise continuous compatibility function.

Normal, convex, symmetric, and bounded fuzzy numbers are defined in Appendix I. Throughout the paper, for any fuzzy number $A$, its compatibility function is denoted by its lowercase letter $a$, and its mean value with $\bar{a}$. The minimum and maximum operators are denoted respectively by (^) and (v).

All fuzzy numbers processed in this study may be expressed in terms of basic fuzzy numbers. Because fuzzy attribute wise utility differences are actually canonical fuzzy numbers, then for any fuzzy attribute wise utility difference A, there exists a real number $\sigma_a$ such that $a(u)$ may be written as follows [10]:

$$a(u) = b\left(\frac{u - \bar{a}}{\sigma_a}\right)$$

where $\bar{a}$ is the mean value of the attribute wise utility difference A. The fuzzy number B whose compatibility function is $b(u)$ is called a basic fuzzy number.

**Definition 2: Semi-common shape.** Two fuzzy numbers C and D are in semi-common shape if they can be expressed in terms of a common basic fuzzy number. That is, there exist two real numbers (called, throughout the paper, spread factors) $\sigma_c$ and $\sigma_d$ such that for any u in U:

$$c(u) = b\left(\frac{u-\bar{c}}{\sigma_c}\right) \text{ and } d(u) = b\left(\frac{u-\bar{d}}{\sigma_d}\right)$$

The system helps the decision maker in the process of assessing his/her fuzzy utility differences by proposing the real intervals that will contain the mean value of the attribute wise fuzzy utility difference for pairs of alternatives.

For every fixed attribute and a fixed alternative, the system asks the decision maker to identify the alternative associated with the most important difference, among the set of available alternatives, assuming that he/she is indifferent about the alternatives with respect to the rest of the attributes. Once this alternative is identified, the decision-maker selects a real value within the proposed interval that best represents the fuzzy utility difference of the pair of alternatives. This procedure is described in more details in the heuristic provided in Figure 2.

## AFUD INCONSISTENCY MANAGEMENT

We only consider inconsistencies caused by intransitivity in decision maker responses. Let us denote by $FP$ $(FI)$ the set of fuzzy numbers whose mean values are strictly positive (zero). That is, $FP = \left\{du(x_1, x_2), (x_1, x_2) \in X^2, \overline{du}(x_1, x_2) > 0\right\}$ and $FI = \left\{du(x_1, x_2), (x_1, x_2) \in X^2, \overline{du}(x_1, x_2) = 0\right\}$. The problem of detection and elimination of intransitivity is an NP-complete one [1] whose solution cannot be obtained in polynomial time. Treating this problem, as in that of cycle elimination in a graph ([19] [11]) is inappropriate because 1) it may produce information loss when arcs are eliminated, and 2) because it does not identify the decision-maker's wrong answer that produced the cycle [9].

```
Begin
    For k = 1, M
    Do
    Begin
        Consider attribute y_k
        u_h = 1.00
        u_1 = 0.00
        While there exists x_j such that x_i R x_j do:
        Begin
            Assign a utility difference value between .50 and u_h, as a mean of an interval of
            length .1, to the most important difference between x_i and x_j, given the attribute y_k.
            Assign a utility difference value between u_1 and .50, as a representative of an
            interval of length .1, to the least important difference between x_i and x_j, given the
            attribute y_k.
        End;
    End;
End.
```

**Figure 2: Heuristic used by AFUD Assessor**

This section provides a practical procedure to identify inconsistencies. Once identified, the inconsistencies are explained to the decision maker who is asked to review his/her judgment. This interactive tank is reiterated until inconsistency is eliminated.

Let us assume that the decision maker has examined the alternatives $x_1$, $x_2$ and $x_3$ towards the assessment of utility differences associated with the three alternatives. The decision-maker produced $\overline{du}(x_1, x_2)$, $\overline{du}(x_2, x_3)$, and $\overline{du}(x_1, x_3)$ such that $\overline{du}(x_1, x_2) \in FP$, $\overline{du}(x_2, x_3) \in FP$, and $\overline{du}(x_1, x_2) \in FI$. That is, the decision maker just expressed that he/she prefers $x_1$ to $x_2$, and $x_2$ to $x_3$ but he/she is indifferent between $x_1$ and $x_3$. This judgment is incorrect because of intransitivity. For any triple alternatives $x_1$, $x_2$ and $x_3$, the decision maker's judgment is incorrect if one of the following conditions is satisfied:

1. $du(x_1, x_2) \in FP$, $du(x_2, x_3) \in FI$, and $du(x_1, x_2) \in FI$
2. $du(x_1, x_2) \in FP$, $du(x_2, x_3) \in FI$, and $du(x_1, x_2) \in FP$
3. $du(x_1, x_2) \in FP$, $du(x_2, x_3) \in FP$, and $du(x_1, x_2) \in FI$
4. $du(x_1, x_2) \in FP$, $du(x_2, x_3) \in FP$, and $du(x_1, x_2) \in FP$

The decision maker is only asked to provide fuzzy utility differences for those pairs $(x_i, x_j)$ such that he/she prefers $x_i$ to $x_j$ ($x_i$ R $x_j$ where R reads "is preferred to"). The fuzzy utility differences for the remaining pairs of alternatives are completed by the system using a reciprocity closure procedure.

The reciprocity closure procedure complements the fuzzy utility difference structure by adding for every pair of alternatives $(x_i, x_j)$, for which $x_i$ R $x_j$, and who fuzzy utility difference mean is $\overline{du}(x_i, x_j)$, a fuzzy utility difference $du(x_i, x_j)$ represented by its mean $\delta$, such that $du(x_i, x_j)(+)du(x_j, x_i) = 0$. We know that the equation $du(x_i, x_j) + (A) = 0$, *where* $A$ is a fuzzy number, and 0 is a crisp zero has no solution [5], in F-noninteractive arithmetic. It is possible however to think of a fuzzy number $A$, as defined in Definition 1, that produces the fuzzy number zero (0), when $A$ is added in an F-interactive manner to $du(x_i, x_j)$. This fuzzy number will have a mean $-\overline{du}(x_i, x_j)$ ([10]).

## COMPUTATION OF CFUD

**Definition 3: Fuzzy compound utility difference structure.** A fuzzy compound utility difference structure is defined as a matrix whose entries are the F-interactive sums of the respective AFUDs.

The $(NxN)$ fuzzy compound utility difference structure (CFUD) $P$ is therefore a matrix 1) whose $P_{ij}$ are fuzzy numbers in the sense of Definition 1 and 2) such that all fuzzy numbers entries of the matrix may be expressed by a common basic fuzzy number $B$. That is, for any $i, j$ $1 \leq i, j \leq n$, there exist $N^2$ real numbers $\sigma_{aij}$ such that:

$$p_{ij}(u) = b\left(\frac{u - \bar{p}_{ij}}{\sigma_{aij}}\right)$$

## ADDITION OF CFUDs

Let $P$ and $Q$ be two CFUDs with entries $P_{ij}$ and $Q_{ij}$ respectively. The sum of the two CFUDs $P$ and $Q$ is a matrix $P(+)Q$ whose entries $p(+)q_{ij}$ are defined as follows:

$$p(+)q_{ij}(u) = n\left(\frac{u - \overline{p(+)q_{\imath\jmath}}}{\sigma_{p(+)q_{ij}}}\right)$$

where

$$\overline{p(+)q_{\imath\jmath}} = \bar{p}_{ij} + \bar{q}_{ij}$$

$$\sigma_{p(+)q_{ij}} = \begin{cases} \left(\sigma_{p_{ij}} + \sigma_{q_{ij}}\right)\Phi(1+\theta)\; for\; \theta_* \leq \theta \leq 1 \\ \left|\sigma_{p_{ij}} + \sigma_{q_{ij}}\right|\Phi(1+\theta)\; for -1 \leq \theta \leq \theta_* \end{cases}$$

$$\theta_* = \left\{\frac{-2\sigma_{p_{ij}}}{\sigma_{p_{ij}} + \sigma_{q_{ij}}}\right\}(v)\left\{\frac{-2\sigma_{p_{ij}}}{\sigma_{p_{ij}} + \sigma_{q_{ij}}}\right\} \leq 0$$

$$\Phi(v) = \begin{cases} 1\; if\; v > 1 \\ v \text{ otherwise} \end{cases}$$

These equations follow immediately from [10], since matrix entries are canonical fuzzy numbers, as shown above.

The term $\sigma_{p(+)q_{ij}}$ may be written as:

$$\sigma_{p(+)q_{ij}} = \begin{cases} \sigma_{p_{ij}} + \sigma_{q_{ij}}\; if\; \theta \geq 0 \\ \left(\sigma_{p_{ij}} + \sigma_{q_{ij}}\right)(1+\theta)\; for\; \theta_* \leq \theta \leq 0 \\ \left|\sigma_{p_{ij}} - \sigma_{q_{ij}}\right|\; for\; -1 \leq \theta \leq \theta_* \end{cases}$$

The computation of the sum of $n$ fuzzy numbers in an F-interactive way will produce $3^{n-1}$ lines. This computation cannot be manually computed when n is large. For example, 243 lines are produced for the computation of 6 fuzzy numbers. The computation of the F-interactive sum of n fuzzy numbers may however be eased by the algorithm provided in Figure 3. This is a symbolic algorithm in the sense that new terms are generated and organized in an equation layout, according to the values of relational factors estimated by the decision maker. Only the final form of the equation will have a mathematical meaning. The relational factors are selected from the real interval $[-1, 1]$, for the added attributes. Optimistic people will usually select values close to -1, whereas those who are pessimistic will select values close to 1.

/*This algorithm computes the F-interactive sum of M attribute wise fuzzy utility differences $A_1, \dots, A_M$ whose mean values are $\bar{a}_1, \dots, \bar{a}_M$ and whose spread factors are $\sigma_1, \dots, \sigma_M$. The resulting sum will be a fuzzy number whose mean value is $\sum_{i=1}^{M} \bar{a}_i$ and whose spread factor $\sigma_{A_1} + \cdots + \sigma_{A_M}$ is computed by this algorithm as an equation of $3^{M-1}$ lines/*

Begin
- H=highest rank (equals M-1)
- Compute $\{\theta_{*h}, h = 1, \dots, M-1\}$
- Select $\{\theta_h, h = 1, \dots, M-1\}$
- /* Every selected $\theta_h$, will generate a new term replacing the question mark (?) in the Case-of block */
- While h>1 Do
  - Begin
    - Case CONDITION of
      - $\theta_h \geq 0$: $? + \theta_{h+1}$
      - $\theta_h \leq \theta_{*h} < 0$: $(? + \theta_{h+1})(1 + \theta_h)$
      - $-1 \leq \theta_h \leq \theta_{*h}$: $|? - \sigma_{h+1}|$
    - End of Case;
    - H=h-1
  - End;
- End;

End;

**Figure 3. F-interactive Computation of the sum of M AFUDs**

## COMPUTATION OF FUZZY PREFERENCES

The heuristic proposed in Figure 4 for the computation of fuzzy preferences employs Nakamura's properties linked to the relationship between fuzzy utility differences and fuzzy preferences, expressed as follows:

1. $if\ \bar{\Pi}(x_i, x_j) \geq 0\ and\ \bar{\Pi}(x_m, x_n) \geq 0\ then$
   $if\ \pi(x_i, x_j)(0) > \pi(x_m, x_n)(0)\ then\ .5 \leq r(x_i, x_j) \leq r(x_m, x_n);$
   $if\ \pi(x_i, x_j)(0) = \pi(x_m, x_n)(0)\ then\ \bar{\Pi}(x_i, x_j) \geq \bar{\Pi}(x_m, x_n)\ iif\ r(x_i, x_j) \geq r(x_m, x_n).$
2. $if\ \bar{\Pi}(x_i, x_j) \geq 0 \geq \bar{\Pi}(x_m, x_n)\ then\ r(x_i, x_j) \geq .5 \geq r(x_m, x_n).$
3. $if\ \bar{\Pi}(x_i, x_j) \leq 0\ and\ \bar{\Pi}(x_m, x_n) \leq 0\ then$
   $if\ \pi(x_i, x_j)(0) > \pi(x_m, x_n)(0)\ then\ .5 \leq r(x_i, x_j) > r(x_m, x_n);$

$$if\ \pi(x_i, x_j)(0) = \pi(x_m, x_n)(0)\ then\ \overline{\Pi}(x_i, x_j) \geq \overline{\Pi}(x_m, x_n)\ iif\ r(x_i, x_j) \geq \mathrm{r}(x_m, x_n).$$

We are only interested in the fuzzy utility differences whose mean values are non-negative. We organize the set of those fuzzy numbers in two bins: 1) MPB (mainly positive fuzzy numbers) which contains those fuzzy numbers $A$ for which $a(0) > 0$ and $\bar{a} > 0$, and FPB (fully positive fuzzy numbers) which contains those fuzzy numbers $C$ for which $b(u) = 0$ and $\bar{b} > 0$.

Fuzzy numbers in MPB are sorted in descending order of $a(0)$ whereas those fuzzy numbers in FPB are sorted in ascending order of $\bar{b}$. Because all those pairs of alternatives $(x_1, x_2)$ represented by the fuzzy numbers in MPB and FPB are characterized by $r(x_i, x_j) \geq .5$, then the heuristic organizes them so that elements of MPB come before FPB in the orders enforced earlier. Then, let them share the interval $[.5, 1]$ giving a certainty factor of $.5 + \frac{i}{2(size(MPB \cup FPB)+1)}$ for the fuzzy preference of $(x_1, x_2)$ (in the $ith$ position of the sequence of elements in $MPB \cup FPB$. That is:

$$r(x_i, x_j) = .5 + \frac{i}{2(size(MPB \cup FPB) + 1)}$$

The remaining pairs of alternatives for which fuzzy preferences are not yet computed are either the pairs $(x_1, x_2)$ such that $x_1 = x_2$ or the reciprocal pairs $(x_2, x_1)$. The heuristic assign 1 as a fuzzy preference for the reflexive pairs and the value $r(x_2, x_1) = 1 - (x_1, x_2)$ for the reciprocal pairs [10].

Begin
For i=1 to N-1 do
For j=1 to N do
Begin
1. Compute $\Pi_{ij}$
2. Compute $\overline{\Pi}_{ij}$
3. Compute $\pi_{ij}(0)$
4. if $\overline{\Pi}_{ij} \geq 0\ then$
$if\ \pi_{ij} > 0\ then\ add\ the\ double\ index\ (i, j)\ to\ the\ bin\ MPB$
$else\ add\ the\ double\ index\ (i, j)\ to\ the\ bin\ FPB$
End;
Sort MPB in descending order of $\pi_{ij}(0)$
Sort FPB in ascending order of $\overline{\Pi}_{ij}$
Let $K = size(MPSB) + size(FPSB)$
Arrange the sorted bins to obtain the sequence $< MPB, FPB >$
Assign the confidence factors $.5 + \frac{k}{2k+1}, k = 1, 2, ...$, in this order ta the sequence. That is, for every double index $(i, j)$ in the sequence at the order $k$, $r(i, j) = .5 + \frac{k}{2k+1}$
Create the $(NxN)$ fuzzy preference matrix $R$
Fill $R$ by the entries $r(i, j), j \geq 1$
Fill $R$ by the entries $r(i, i) = 1$
Fill $R$ by the entries $r(i, j) = 1 - r(j, i),\ \ j \leq i$
End.

**Figure 4. Computation of Fuzzy Preferences**

## FUZZY PREFERENCE INCONSISTENCY MANAGEMENT

It is also possible that intransitivity appears again in the compound fuzzy utility difference structure. The same procedure utilized in the second phase may be employed again to identify inconsistencies. All alternatives involved in the intransitivity are returned to the decision maker for new assessment of fuzzy utility differences. This process is reiterated until intransitivity is no longer present in CFUD.

There are three types of transitivity that may be defined on a fuzzy preference relation $R$ on $X \times X$:

- W-transitivity: $R$ is W-transitivity if and only if, for any $x_i, x_j \text{ and } x_k, r(x_i, x_j)(\wedge) r(x_j, x_k) \geq .5 \text{ implies } r(x_i, x_k) \geq .5$
- M- transitivity: $R$ is M-transitivity if and only if, for any $x_i, x_j \text{ and } x_k, r(x_i, x_j)(\wedge) r(x_j, x_k) \geq .5 \text{ implies } r(x_i, x_k) \geq r(x_i, x_j)(\wedge) r(x_j, x_k)$
- S-transitivity: $R$ is S-transitivity if and only if, for any $x_i, x_j \text{ and } x_k, r(x_i, x_j)(\wedge) r(x_j, x_k) \geq .5 \text{ implies } r(x_i, x_k) \geq r(x_i, x_j)(\text{v}) r(x_j, x_k)$

In fact, If a binary fuzzy relation is S-transitive, then $R$ is also M-transitive; and if $R$ is M-transitive, then $R$ is W-transitive [10]. It is therefore sufficient to eliminate inconsistency produced by S-intransitivity. This is a desirable feature that the system builds an S-transitive closure for the fuzzy binary relation on $X \times X$ during the transformation of compound utility differences into fuzzy preference relations.

## CHOICE

A choice vector contains the N-ary fuzzy preference relation of an alternative. The entry $C_i$, of the choice vector may be understood as the confidence factor that the alternative $x_i$, is preferred to the rest of alternatives $\{x_j, j \neq i\}$. $C_i$ is computed as follows:

$$C_i = r(x_i, x_1)(\wedge) \dots (\wedge) r(x_i, x_j)(\wedge) \dots (\wedge) r(x_i, x_N), j \neq i$$

The most preferred alternative $x^*$ is computed as follows: $x^* = x_i$ such that $C_i = Sup\{C_i\}$.

It is important to note that strong dominance and weak dominance relations may be defined on the choice vectors. These relations are not however total order relations but can serve as a means to reduce the size of the set of alternatives. For further information on these relations, one may refer, for example to [2].

## CONCLUSION

This article proposed a decision support environment for fuzzy multi-attribute decision-making. The Fuzzy Multi-attribute Decision Support System (FMDSS) included interactive natural form sessions to elicit reliable subjective judgment, and effective procedures for detection and correction of inconsistencies.

A prototype of the FMDSS can be developed. This article demonstrated how the FMDSS assists the decision maker in all phases of the fuzzy multi-attribute decision process. Nakamura's requirements for additive fuzzy utility differences have been incorporated in the system. The demonstration showed a complete illustration of a fuzzy multi-attribute decision problem.

Full decision support software for individual and group fuzzy multi-attribute decision environments can be developed.

## Appendix
## Definitions of normal, convex, symmetric, and bounded fuzzy numbers

If $A$ is a fuzzy attributewise utility difference with a compatibility difference function $a(u)$, then $A$ is normal if $sup_u\, a(u) = 1$.

$A$ is convex if, for any $u_1, u_2$ in $U,$ for any $\tau$ in $[0, 1]$, $a(\tau u_1 + (1-\tau)u_2) \geq a(u_1)(^\wedge)a(u_2)$.

In addition, $A$ is symmetric when there exists $\bar{a}$ in $U$ such that for any $x$ in $U, a(\bar{a}+x) = a(\bar{a}-x)$.

$A$ is bounded when for any $w > 0$, there exit $p$ and $q$ in $U$ such that $q \geq u \geq p$ for any $u$ such that $a(u) > w$. The real number $\bar{a}$ is called the mean value of the attribute-wise utility difference $A$.

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