LINEAR PROGRAMMING AND MULTI-PRODUCT PROFIT PLANS

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ABSTRACT

Cost accounting provides much of the quantitative data in support of key managerial decisions; yet, cost/managerial accountants seldom build or use decision models that are critical in forecasting the financial goals of the organization. This paper demonstrates how knowledge of both manufacturing costs and linear programming (LP) can be used to build a profit plan in meeting the strategic marketing, production and financial goals of an organization. An example for an LP profit maximization problem is used to build a pro-forma income statement.

INTRODUCTION

Linear programming is an optimization technique used to maximize profits for a mix of products given multiple requirements [1]. LP is a decision modeling technique found in nearly all operations management textbooks and it is taught with the use of spreadsheet add-ins (e.g., Excel Solver). Operations science textbooks seldom discuss how cost data for a linear programming problem are obtained and the optimized results are rarely used to prepare pro-forma statements.

Cost/managerial accounting textbooks (e.g., [2] [3]) identify linear programming as a tool for a scarce resource decision. Yet, most accounting textbooks either do not illustrate LP or they present the graphical approach to linear programming. Even when LP spreadsheet add-ins are available, only a few accounting textbooks (e.g., [4]) illustrate linear programming with spreadsheets.

While LP optimization techniques and budgeting rely heavily on each other, they are seldom presented together. The purpose of this presentation is to show both cost accounting and linear programming used jointly to prepare budgeted financial statements. After briefly introducing relevant accounting data for linear programming, an optimal solution to the linear programming decision becomes the starting point for a budgeted contribution income statement. The LP/budgeting example has been used in cost/managerial accounting courses at the undergraduate and graduate levels. Accounting students are excited to see cost accounting data coupled with linear programming leading to a pro-forma financial statement.

ACCOUNTING DATA USED FOR LINEAR PROGRAMMING

For a company that manufactures products, linear programming is a useful decision tool that maximizes profit in the production and sale of multiple product lines. The objective function of an LP model is used to maximize the sum of contribution margins for all products. An optimum mix of products that maximizes the sum of the individual product contribution margins will maximize profit for a company. Contribution margin for a product is the difference between selling price and variable expenses.

Variable expenses vary proportionately with changes in activity (e.g., units produced and sold). When manufacturing and selling products, direct materials, direct labor, variable manufacturing overhead (MOH), and variable selling and administrative (S&A) expenses are examples of variable costs. Only revenues and costs that vary proportionately with the number of units produced and sold are used in determining an optimal combination of products with linear programming. In contrast, fixed costs remain constant and are not included in an LP model as they do not vary with units produced and sold. Maximizing profitability with an LP model examines only variable revenues and expenses.

LINEAR PROGRAMMING AND THE CONTRIBUTION INCOME STATEMENT

An example of a budgeted contribution income statement for Kona Nut Company below is aligned with the objectives of linear programming. Per unit selling price and variable expenses for the four macadamia nut products are presented. The units sold generate contribution margins for each product, with a total for the four products also shown. An assumption for the budgeted contribution income statement is that all units produced will also be sold within the accounting period.

		Whole		Cluster		Crunch		Roasted	
	Total	Total	Unit	Total	Unit	Total	Unit	Total	Unit
Units sold	1,944	1,442		400		50		52	
Sales	10,662	8,652	6.00	1,600	4.00	150	3.00	260	5.00
Variable expenses:									
Direct materials	2,425	1,846	1.28	448	1.12	48	0.96	83	1.60
Direct labor	1,815	1,442	1.00	320	0.80	22	0.44	31	0.60
Manuf. overhead	606	461	0.32	112	0.28	12	0.24	21	0.40
Selling and admin.	1,066	865	0.60	160	0.40	15	0.30	26	0.50
Total	5,912	4,614	3.20	1,040	2.60	97	1.94	161	3.10
Contribution margin	**4,750**	4,038	**2.80**	560	**1.40**	53	**1.06**	99	**1.90**
Fixed expenses:									
Manuf. overhead	1,250								
S&A	1,050								
Operating income	2,450								

From Total Contribution Margin, fixed expenses for manufacturing overhead and selling administrative are deducted in total as their per-unit cost is not constant but varies inversely with changes in activity. The contribution income statement illustrates that maximizing operating income occurs when contribution margin is maximized as fixed expenses will remain constant.

KONA NUT COMPANY: BUDGETING WITH LINEAR PROGRAMMING

Kona Nut Company located on the island of Hawaii makes four different products – chocolate covered whole nuts, chocolate nut clusters, chocolate nut crunch bars, and roasted nuts. Kona has a limited supply of nuts that are bought from local growers. The demand for nut products fluctuates with seasonal tourist levels and production is budgeted on a weekly basis.

Selling and Production Data

The selling and production information is presented in the contribution income statement above and the spreadsheet LP Base model below. The selling price for the four nut products Whole, Cluster, Crunch, and Roasted are $6.00, $4.00, $3.00, and $5.00 per pound; respectively. The minimum sales demand for its popular Whole product is 1,000 pounds. Demand for the Cluster product is estimated between 400 and 500 pounds. Crunch must not exceed 50 pounds and Roasted cannot exceed 200 pounds. The composition of hulled nuts within one pound of a finished product varies, with Whole .60, Cluster .40, Crunch .20, and Roasted 1.00. Chocolate is the balance for the three products.

During this time of the year, hulled macadamia nuts are limited to 1,100 pounds. The hulled nuts cost $1.60 per pound, and chocolate costs $0.80 per pound. Therefore, the variable direct material cost for the Whole product is $1.28, computed as $.96 (.6*$1.60) for nuts plus $.32 (.4*$.80) for chocolate. Similarly, Cluster has a direct material cost of $1.12, Crunch $0.96 and Roasted $1.60.

The four products incur variable costs for direct labor, manufacturing overhead, and selling and administrative. Variable labor costs in running the machines are $12 per hour or $0.20 per minute for the Hull, Roast and Chocolate machines, and $6 per hour or $0.10 per minute for the Package machine. Therefore, the direct labor cost for the Whole product is $1.00, Cluster is $0.80, Crunch is $0.44, and Roasted is $0.60. The variable manufacturing overhead costs (e.g., sugar, salt, oil and garlic) are driven by direct materials costs for the macadamia nuts and chocolate; therefore, it is applied at a rate of 25% of DMC. The variable selling and administrative expense is 10% of the selling price. The total variable costs for the products are Whole $3.20, Cluster $2.60, Crunch $1.94, and Roasted $3.10. Given the selling price for the four products, the contribution margins are Whole $2.80, Cluster $1.40, Crunch $1.06, and Roasted $1.90.

The four products require time on four different machines - Hull, Roast, Chocolate, and Package, except for the Roasted product which does not have chocolate. The number of minutes per pound of finished product required on each machine is listed below (for example, the Whole product requires 2 minutes on the Roast machine). Each machine is available 60 hours or 3,600 minutes per week.

	Whole	Cluster	Crunch	Roasted
Machine use in mins:				
Hull machine	1.00	1.00	1.00	1.00
Roast machine	2.00	1.50	0.50	1.75
Chocolate machine	1.00	0.07	0.20	0.00
Package machine	2.00	1.60	1.00	0.50

LP Model

To emphasize the relationships within the LP Base Model, the decision of 10 units for the four products is presented to highlight it as a multiplicative factor for contribution margin and other production constraints. For example, contribution margin for Whole is 28.00 or 10 x 2.80, Nuts used by Cluster is 4.00 or 10 x .40, the Hull machine minutes used by Crunch is 10.00 or 10 x 1.00, and Sales for Roasted is 10 or 10 x 1.

LP Base Model	Whole	Cluster	Crunch	Roasted	Total		
Decision	10	10	10	10	40		
Max contrib. margin	28.00	14.00	10.60	19.00	71.60		
Nuts used in lbs.	6.00	4.00	2.00	10.00	22.00	<=	1100
Machine use in mins.							
Hull machine	10.00	10.00	10.00	10.00	40.00	<=	3600
Roast machine	20.00	15.00	5.00	17.50	57.50	<=	3600
Chocolate machine	10.00	7.00	2.00	0.00	19.00	<=	3600
Package machine	20.00	16.00	10.00	5.00	51.00	<=	3600
Sales requirements in units							
Whole	10.00				10.00	>=	1000
Cluster		10.00			10.00	>=	400
Cluster		10.00			10.00	<=	500
Crunch			10.00		10.00	<=	50
Roasted				10.00	10.00	<=	200

LP Solution

Although they appear separately, the Base Model becomes the Solution when linear programming is performed. The Solution is generated from the SOLVER add-in to EXCEL with the following steps after selecting *Data* and then *Solver*. From the *Solver Parameters* screen, *Set Objective:* total contribution margin, *To:* Max, *By Changing Variable Cells*: decision cells for products, *Subject to the Constraints:* add constraints as listed, *Select a Solving Method:* Simplex LP, and then click on *Solve*.

LP Solution	Whole	Cluster	Crunch	Roasted	Total		
Decision	1442	400	50	52	1944		
Max contrib. margin	4038	560	53	99	4750		
Nuts used in lbs.	865	160	10	52	1087	<=	1100
Machine use in mins.							
Hull machine	1442	400	50	52	1944	<=	3600
Roast machine	2884	600	25	91	3600	<=	3600
Chocolate machine	1442	280	10	00	1732	<=	3600
Package machine	2884	640	50	26	3600	<=	3600
Sales requirements in units							
Whole	1442				1442	>=	1000
Cluster		400			400	>=	400
Cluster		400			400	<=	500
Crunch			50		50	<=	50
Roasted				52	52	<=	200

The LP Solution above indicates that 1,442 pounds of Whole, 400 pounds of Cluster, 50 pounds of Crunch, and 52 pounds of Roasted generate a maximized $4,750 of contribution margin. A review of all the constraints finds that none are being violated. The optimized solution is then transferred to the budgeted contribution income statement presented previously.

The contribution income statement shows total contribution margin to be $4,750, which is the maximized contribution margin of the LP model. The cost information from the LP Solution is consistent with the costs on the contribution income statement. Hence, both contribution income statement and the LP Solution present the same information. In completing the income statement, fixed expenses for manufacturing overhead and selling and administrative are deducted from contribution margin to obtain operating income.

SUMMARY

The students enjoy this spreadsheet project and have little difficulty completing this exercise. They recognize the importance of cost accounting data in meeting the production, marketing and financial goals of a firm. The spreadsheet LP add-in SOLVER becomes another tool to students in support of management decision-making. Some students will perform scenario analysis by changing critical input variables such as the cost of macadamia nuts or the quantity of available nuts. Other students recognize the number of minutes for the package machine and the roast machine are constraints to increasing profitability; hence, this is an example for the theory of constraints (TOC).

Accounting students can easily couple linear programming with their knowledge of relevant costs, contribution margin and variable costing income statements to solve realistic and complex scarce resource problems for multiple products. Accountants can become active users of decision models. With linear programming, accounting educators can effectively demonstrate how relevant accounting data is used to support management decision-making.

REFERENCES

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