SYSTEM'S DESIGN CHOICE BY MULTICRITERIA ANALYSIS

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ABSTRACT

The best system's design strategy is selected by using multicriteria analysis. The criteria are cost, reliability, resiliency and vulnerability of the system. First the criteria are transformed into unitless satisfaction levels, and then multicriteria analysis is performed. Three alternative methods are used and the results compared.

1. INTRODUCTION

In every system, the most important requirement is its safe and reliable operation. There is a huge literature discussing the methodology of quality and reliability engineering (see for example, Elsayed [1]). In characterizing the reliable operation of a system, its reliability, resiliency and vulnerability are usually considered. If T is a user selected time length, the reliability of the system is the probability that the system will not break down in time interval $[0, T]$, which is denoted by $R(T)$ and called the reliability function. Notice that

$$R(T) = 1 - F(T) \tag{1}$$

where$F$ is the cumulative distribution function of time to failure. Clearly $R(T) \in [0, 1]$. If X_t is the state of the system at time t with two possibilities: satisfactory operation (S) or failure (F), then clearly

$$R(T) = P(X_t = S) \tag{2}$$

The resiliency of a system shows how quickly a system is likely to bounce back from failure once failure has occurred. If the recovery is slow, then a long downtime may result in significant loss. Resiliency is usually measured as the probability of recovery from the failure state in a single time period:

$$RES(t) = P(X_{t+1} = S \mid X_t = F) \tag{3}$$

and if a longer time period is concerned, then the overall resiliency can be obtained by the average

$$RS(T) = \frac{1}{T}\sum_{t=1}^{T} RES(t) \tag{4}$$

Vulnerability of a system focuses on the possible consequences of failure events.Let M be the number of different failure modes, $e(m)$ the most unsatisfactory and severe outcome of failure mode m with occurring probability $p(m)$. Then, the system vulnerability is defined as

$$V(T) = \sum_{m=1}^{M} e(m)\, p(m) \tag{5}$$

Notice that the probability values *p(m)* give the occurrence probabilities during the considered time length *T*.

The cost of the construction is the most important economic factor in any system design.

In our system selection procedure these characteristics will be considered as objective functions in a multicriteria analysis.

Notice that the larger reliability and resiliency values are better for the system, however lower vulnerability and cost values are favorable. In the first step, we have to be sure that all objectives have the same direction, for example all of them are maximized. Reliability and resiliency have to be maximized as they are, however the other two objectives have to be changed by considering their negative values.

2. METHODOLOGY

The units in which the different objective functions are measured are different. The value of reliabilityis a probability value, the same is true for resiliency, but vulnerability can be measured by monetary units, downtime or even by the level of danger to workers. The cost is clearly a financial category. In order to compare the objective values, we have to transform them to a common satisfaction level. It can be done by either using appropriate utility functions or by normalizing the objectives. The utility function of each objective can be assessed based on inputs from the actual decision makers. If $f_k(l)$ is the value of objective k ($1 \leq k \leq K$) at alternative l ($1 \leq l \leq L$), then the corresponding normalized objective value becomes

$$\bar{f}_k(l) = \frac{f_k(l) - f_k{}^{min}}{f_k{}^{max} - f_k{}^{min}}, \tag{6}$$

where $f_k{}^{min} = \min\{f_k(l), 1 \leq l \leq L\}$and $f_k{}^{max} = \max\{f_k(l), 1 \leq l \leq L\}$.

After each objective is replaced by the corresponding utility values or by their normalized values, then standard multicriteria optimization method can be used.

Let $\alpha_1, \alpha_2, \ldots, \alpha_K$ show the relative importance factors of the objectives such that $\alpha_K > 0$ for all k and $\sum_{k=1}^{K} \alpha_k = 1$.

In applying the weighting method, a composite objective is constructed by the weighted average of the objectives which is then maximized:

$$f^{(1)}(l) = \sum_{k=1}^{K} \alpha_k \bar{f}_k(l) \rightarrow max \tag{7}$$

This value can be interpreted as an average satisfaction by selecting alternative *l*.

The compromise programming method uses an alternative composite objective

$$f^{(2)}(l) = \left(\sum_{k=1}^{K} \alpha_k \bar{f}_k(l)^p\right)^{1/p} \rightarrow max \tag{8}$$

where $p \geq 1$ is a user selected parameter. In most applications $p = 2$ is selected. In the special case of $p = 1$, $f^{(1)} \equiv f^{(2)}$. The most popular conflict resolution methodology is the Nash bargaining solution, which maximizes the Nash-product

$$f^{(3)}(l) = \prod_{k=1}^{K} \bar{f}_k(l)^{\alpha_k} \tag{9}$$

A very broad variety of multicriteria methods are known from the literature (see for example, Szidarovszky et al., [2]), our study can be easily repeated with any other method.

3. CASE STUDY

Five alternative system designs were considered and the experts have given the requested data for all alternatives as shown in Table 1.

Table 1. Data for the case study

Alternative	Cost (f_1) (10^6 \$)	Reliability (f_2) (%)	Resiliency (f_3) (%)	Vulnerability (f_4) (subjective scale)
1	2.2	85	91	80
2	2.4	92	88	90
3	1.9	90	90	70
4	2.6	93	92	85
5	1.8	86	89	95

First we consider the negatives of the cost and vulnerability data. Before normalizing the objectives, the smallest and largest values are selected:

$f_1^{min} = -2.6$ $\quad$ $f_1^{max} = -1.8$

$f_2^{min} = 85$ $\quad$ $f_2^{max} = 93$

$f_3^{min} = 88$ $\quad$ $f_3^{max} = 92$

$f_4^{min} = -95$ $\quad$ $f_4^{max} = -70$

The normalized objective values are shown in Table 2.

Table 2. Normalized objective values

Alternative	Cost	Reliability	Resiliency	Vulnerability
1	0.5	0	0.75	0.60
2	0.25	0.875	0	0.20
3	0.875	0.625	0.5	1
4	0	1	1	0.40
5	1	0.125	0.25	0

Equal weights were selected, that is, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$, and we applied the methods $f^{(1)}$, $f^{(2)}$ and $f^{(3)}$ using these data. The results are presented in Table 3.

Table 3. Multicriteria results

Alternative	$f^{(1)}$	$f^{(2)}$	$f^{(3)}$
1	0.4625	0.5414	0
2	0.3312	0.4659	0
3	0.7500	0.7756	0.7231
4	0.6000	0.7348	0
5	0.3438	0.5192	0

All methods selected alternative 3 as the best alternative, so this choice is robust. Methods $f^{(1)}$ and $f^{(2)}$ even give the same preference order of the alternatives:

$$3 > 4 > 1 > 5 > 2.$$

The last column shows a drawback of the applications of Nash bargaining. If any objective is minimal with an alternative, then the product composite objective also has zero (minimal) value, that is, the Nash bargaining method cannot distinguish between such alternatives.

4. CONCLUSIONS

System design alternatives were compared and the most satisfying design was selected by using multicriteria decision making. This method usually consists of three steps. First the objective functions have to be normalized, then the composite objective function has to be constructed. And finally, the composite objective is optimized.

In our case study, the selected methods gave the same solution for the best alternative, however in many applications this is not the case. If the results by different methods are different, then the decision maker has to modify the preference information (in our case the importance factors) and repeat the computation. This interactive step is usually repeated until a satisfactory decision is obtained. If there is no agreement between the results after several repeats, then the decision maker has to choose based on his/her belief about the appropriateness of the form of the composite objective.

5. REFERENCES

[1] Elsayed, E.A. (1996)Reliability Engineering. Addison Wesley Longman, Inc., Reading, Mass.

[2] Szidarovszky, F., Gershon, M.E. and Duckstein, L. (1986) Techniques for Multiobjective Decision Making in Systems Management. Elsevier, Amsterdam.