TESTING WHETHER THE MARKET PORTFOLIO IS MEAN-VARIANCE EFFICIENT

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ABSTRACT

The mean-variance (MV) capital asset pricing model (CAPM) predicts that securities plot on the Security Market Line (SML) if and only if the market portfolio is MV efficient. Tests of the model are legion. Most concentrate on whether there is a linear tradeoff between reward (expected return) and risk (beta), i.e., securities plot on the SML. A smaller number focus on whether the market portfolio is MV efficient. The change in emphasis provides insights not available from focusing on relationships in mean-beta space.

Best and Grauer [1] reverse engineered the MV problem to show that there are an infinite number of SML means from a two-parameter family that ensure the market portfolio is MV efficient at a point in time. Best and Grauer [2] showed that if there are positively weighted portfolios they lie in a single segment of the minimum-variance frontier. Analytic and computational results suggest that the segment of the frontier containing positively weighted portfolios decreases as the number of assets in the universe increases. Moreover, small perturbations in the means that ensure there is a positively weighted portfolio on the original minimum-variance frontier make it highly unlikely that there are any positively weighted portfolios on the new frontier. Brennan and Lo [3] defined an "impossible frontier" as a frontier on which all portfolios have at least one negative weight. They proved that for randomly drawn covariance matrices the probability of obtaining an impossible frontier approaches one as the number of assets grows. Levy and Roll [5] painted a very different picture. They adopted a reverse-engineering approach to find the minimum distance between the sample means and standard deviations and a set of equilibrium means and standard deviations required to ensure that the market is MV efficient. And they conducted univariate and multivariate tests of these variations. Surprisingly, slight variations in parameters, well within estimation error bounds, suffice to make the market efficient.

This paper examines the time-series and cross-sectional characteristics of different sets of SML means that ensure the (changing-weights) market portfolio is MV efficient. Focusing on MV efficiency provides insights – both with respect to the form of the tests and the results – not readily available otherwise. Consider the form of the tests first. We specify the intercepts and slopes of the SML means in two ways. First, we hold either the market expected return, market risk premium, market mean excess return to variance ratio, or market mean excess return to standard deviation ratio constant. Second, we minimize the distance between the sample and different sets of SML means at a point in time. Traditional CAPM cross-sectional tests focus on whether the slopes or intercepts of the regressions of sample means on betas are statistically significant. By way of contrast, we employ unconstrained or constrained Ordinary Least Squares (OLS) and Generalized Least Squares (GLS) regressions of sample means on betas to minimize the distance between sample means and sets of SML means at a point in time. The constrained regressions restrict the slopes of the SML to be positive and the intercepts to be greater than or equal to the riskfree rate as required by MV theory. We employ a Hotelling's T^2 test that focuses on the statistical significance of the alphas to test the model at a point in time. In the special cases where the market portfolio's sample mean is equal to its equilibrium mean the Hotelling's T^2 tests are more directly related to the efficient set mathematics than the Gibbons, Ross and Shanken [4] test is.

Time-series plots of the sample means, alternative sets of SML means, and the riskfree rate of interest show that the riskfree rate exceeds the market portfolio's equilibrium mean in many cases, which is inconsistent with MV theory. This is more important than the problem of changing riskfree rates, which most cross-sectional and time-series tests of the CAPM attempt to side step by performing the tests in excess return form. The plots show that the alternative specifications of the equilibrium means are quite different. The results also show that any conclusions we might draw are highly dependent on the metric employed. For example, in the high interest rate period, statistical tests in one dataset reject most of the SMLs, even though other MV criteria indicate some are reasonable and others are not. In low interest rate periods, statistical tests reject all the SMLs even though other MV criteria indicate all are reasonable. Finally, as with the traditional tests, the results are sensitive to the way securities are grouped into portfolios and to estimates of sample means, equilibrium means and betas based on a fixed 120-month window or an expanding window.

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