LEARNING CURVES AND COST BEHAVIOR: ANALYSIS AND ESTIMATION

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ABSTRACT

Most managerial/cost accounting textbooks perform a logarithmic transformation of a learning curve into a log-linear relationship prior to data analysis. The rationale for this approach is that students are familiar with linear regression techniques and their analyses of the transformed log-linear learning model are equally valid. However, students need not perform this transformation into linear equivalents when enhanced statistical capabilities of spreadsheets will easily examine curvilinear data. This paper illustrates a straightforward curvilinear approach to data analysis for learning curves.

INTRODUCTION

Accounting textbooks discuss learning curves by describing the underlying power function and contrasting the cumulative average-time and the individual unit-time models of it [2, 3]. The two learning curve models are then used with assumed learning rates to predict labor hours for increasing levels of production. Textbook problems for learning curves simply require students to estimate labor hours using the learning models and a given a learning rate. The analysis of historical learning data is seldom performed to determine which model to adopt and its learning rate.

The study of learning curves is prevalent in managerial/cost accounting textbooks. Yet, in contrast to linear regression used for examining cost behavior, it is unusual to perform curvilinear analysis of learning data when estimating labor costs. Furthermore, rather than performing curvilinear analyses, most textbooks perform a logarithmic transformation of the power function into a log-linear model. Hence, the original curvilinear data is converted into log-based equivalents for the purpose of using linear regression techniques. Students are seldom shown how curvilinear learning data can be analyzed directly without this transformation; and, they may conclude that all nonlinear costs can be analyzed using linear regression. With enhanced statistical capabilities of spreadsheets, data analysis for the learning curve is easily accessible and simple to use.

LEARNING CURVES

The learning curve relationship is a power function described as the constant percentage model. The learning power function features a systematic decrease in labor hours by a constant percentage each time the volume of production increases geometrically [1].

$$(A, \text{ or } I_n) = aX^b$$

The choice of a dependent variable would depend on whether the cumulative average-time learning model (A) or the individual unit-time learning model (I_n) best modeled the learning effects. The dependent variable and independent variables are listed below.

A	=	the average cumulative labor hours for X number of units.
I_n	=	the number of labor hours required to produce the last nth unit.
a	=	the number of labor hours required to produce the first unit.

X	=	cumulative number of units produced.
b	=	learning exponent, which is always negative.

The negative learning exponent b is equal to (log **r**)/(log **f**), where **r** is the rate of learning represented by the constant percentage decrease in hours, and **f** is the factor increase in output (usually 2 for a doubling of units). For example, an 80% learning rate with a doubling of units has a learning exponent b equal to –0.3219, which is (log .80)/(log 2).

A common logarithmic transformation of the power learning function is described as a log-linear model and is performed below. The model is a linear regression where Log (A) or Log (I_n) is the dependent variable, Log (a) is the y-intercept, b is the slope coefficient, and Log (X) is the independent variable.

$$(A, \text{ or } I_n) = aX^b$$

$$\text{Log } (A, \text{ or } I_n) = \text{Log } (aX^b) = \text{Log } (a) + \text{Log } (X^b) = \text{Log } (a) + b \text{ Log } (X)$$

EXAMPLE OF CURVILINEAR DATA ANALYSIS FOR LEARNING

Background Information

The building of complex and specialized products requires significant amounts of labor hours by highly-skilled technicians. Only a few units will be produced and learning by the technicians will be important and expected for estimating costs. ABC Company builds such products and keeps accurate job-order records, with an emphasis on labor hours. A reason for keeping a highly skilled labor force at ABC Company is the learning acquired from previous projects.

ABC Company has been asked to submit a bid to manufacture eight Z1 orbital lasers for the Department of Defense (DOD). The design specifications of the Z1 orbital laser were made available to ABC Company, and the parts to be assembled would be delivered to ABC Company from other DOD subcontractors. The DOD engineers estimate 6,000 direct labor hours for the first Z1 orbital laser. ABC Company cost accountants anticipate that the eight Z1 lasers could be built with the same amount of learning experienced with the building of eight previous Y1 orbital lasers for the DOD.

Labor Hour Data Analysis

From the labor hour data for the Y1 orbital laser, a learning curve model will be used to estimate labor hours required to build the eight Z1 units. In particular, the selection of the cumulative average-time learning model (A) or the individual unit-time learning model (I_n) with their learning rates must be made.

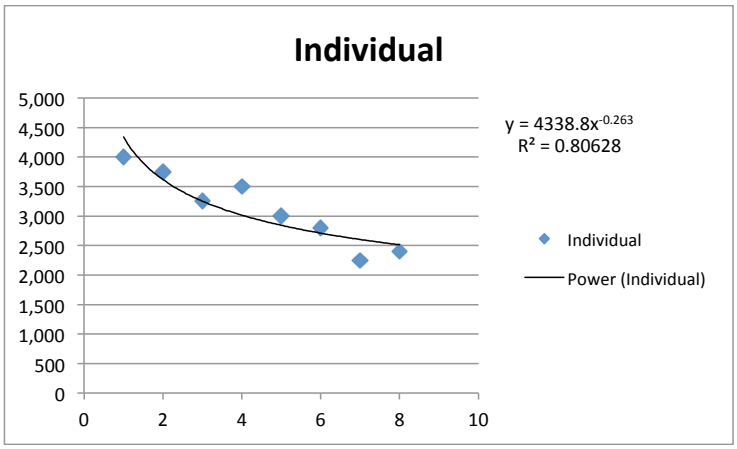
From the job-order costing records of ABC Company, the individual labor hours incurred for each of the eight Y1 orbital lasers are given in Panel A of Exhibit 1. In addition, the related cumulative average-time data are given. By selecting within Excel the Insert and then Scatterplot commands, the plots of the direct labor hours for the Individual and Average models are generated in Panel B and Panel C of Exhibit 1. Furthermore, by right-clicking on a data point within the plot, the Trendline of the Power function curve, Equation, and R-squared value are added to the graphs.

Exhibit 1: Learning Curve Data and Models for Y1 Orbital Lasers

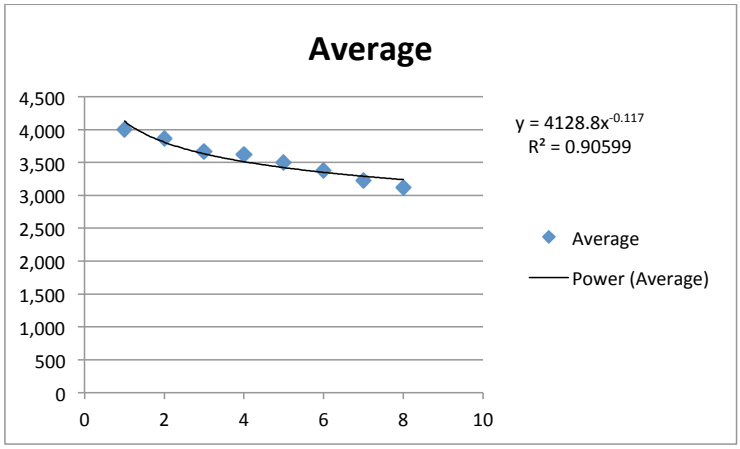
Panel A: Y1 individual unit-time and cumulative-average data

Unit	Individual	Total	Average
1	4,000	4,000	4,000
2	3,750	7,750	3,875
3	3,250	11,000	3,667
4	3,500	14,500	3,625
5	3,000	17,500	3,500
6	2,800	20,300	3,383
7	2,250	22,550	3,221
8	2,400	24,950	3,119

Panel B: Y1 individual unit-time model



Panel C: Y1 cumulative average-time model



The data analysis supports the use of the cumulative average-time model $A = 4{,}128.8X^{-0.117}$ because of its larger R-square value of 0.906 versus the individual unit-time R-square of 0.8063. From the cumulative average-time model, the learning rate is the anti-log of the negative exponent or $10^{-0.117*Log(2)}$ or 92.21%. This derived model for past learning will be used below in the Z1 orbital laser estimates for direct labors.

Forecasting Direct Labor Hours With A Learning Curve

With the given initial estimate of 6,000 hours for the first Z1 orbital laser, the cumulative average-time learning curve with a 92.21% learning rate when units are doubled estimates 4,704 direct labor hours as the average to complete the eight Z1 orbital lasers. The total direct labor hours for eight orbital lasers is 37,632 (4,704x8) as shown below.

$$A = aX^b = 6{,}000*8^{(Log(0.9221)/Log(2))} = 6{,}000*8^{-0.117} = 4{,}704$$

Unit	Average	Total
1	6,000	6,000
2	5,533	11,066
3	5,276	15,828
4	5,101	20,404
5	4,970	24,850
6	4,865	29,190
7	4,778	33,446
8	4,704	37,632

The total estimated labor hours of 37,632 is more than 20% lower than if no learning was assumed (i.e., 48,000 = 6,000x8). Furthermore, the individual unit-time model would estimate 41,228 labor hours, which is 3,596 labor hours higher. The incorrect selection of the individual unit-time model often occurs because the historical Y1 data is given as individual unit times by their job cost sheets.

SUPPLEMENTARY LOGARITHMIC ANALYSIS

From log-based equivalents of the historical data, the log-linear regression line for this curvilinear relationship can be derived. While the log-linear approach is common in textbooks performing data analysis, this same regression line is the logarithm of the cumulative average-time model as shown below. In comparison to the cumulative average-time model $A = 4{,}128.8X^{-0.117}$, the log-linear regression line and its graph shown below have the same R-square value, the same slope/learning rate of -0.117, and the y-intercept 3.616 is the log of 4,128.8. The purpose of this exercise is to show the same learning relationship modeled as a true power curve and as a transformed log-linear regression line.

$$Log\ A = Log\ 4{,}128.8X^{-0.117} = Log\ (4{,}128.8) + Log\ (X^{-0.117}) = 3.616 - 0.117\ Log\ (X)$$

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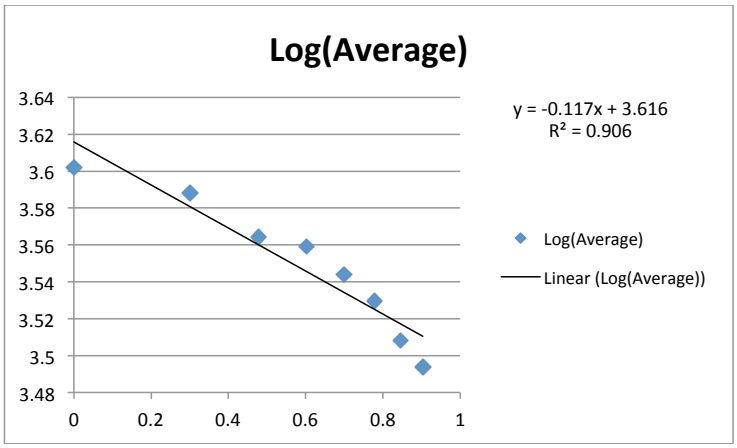

SUMMARY

The benefits of performing curvilinear data analysis were identified for the example ABC Company that had highly skilled technicians assembling expensive and a limited number of units. It identified a learning model and related learning rate of a past project that was used to estimate learning for the assembly of a similar product in the future. The curvilinear analysis was done without performing a logarithmic transformation of the original data. Curvilinear data analysis is another tool to better understand cost behavior.

REFERENCES

[1] Belkaoui, A., *The learning curve: A management accounting tool.* Westport, CT: Greenwood Press, 1986.

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[3] Horngren, C. T., Datar, S. M. & Rajan, M. V. *Cost accounting: A managerial emphasis.* New Jersey: Prentice Hall, 2011.