PRODUCT-MIX DECISIONS WITH RELEVANT FIXED COSTS

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ABSTRACT

In modeling product-mix linear programming (LP) problems, a common assumption is that fixed costs will not change for decisions made within a relevant range in which revenues and costs are linear. Consequently, LP is commonly used to solve for an optimal mix of products that maximizes contribution margin (i.e., revenues less variable expenses). However, some fixed costs such as equipment rental may increase as units are produced. Hence, when fixed costs are relevant in the product-mix decision, the objective for linear programming should be to maximize profits and not contribution margin. A case illustrates the use of binary linear programming (BLP) to model relevant fixed costs for a product-mix decision seeking to maximize profits. The case presents contribution income statements to highlight the differing linear programming results for maximizing contribution margin versus maximizing profit.

INTRODUCTION

When performing linear programming, a necessary assumption is that all relationships are linear within the relevant range of the decision. If that assumption is violated, Excel's LP add-in Solver for example will return a message indicating that the conditions for linearity are not satisfied and no feasible solution could be found. Accounting textbooks in their discussion of product-mix decisions also do not consider fixed costs within the relevant range of possible solutions [2]. However, fixed costs can be relevant if they may or may not be incurred with the decision to produce certain products and their optimized quantity. The following Abigail Company case illustrates binary linear programming (BLP) as a technique to account for relevant fixed costs within a linear model. The case is suited for MBA Managerial Accounting and upper-division Cost Accounting courses that discuss linear programming.

LINEAR PROGRAMMING AND ACCOUNTING

Linear Programming and the Utilization of Constrained Resources

Linear programming is used to represent the problem of allocating scarce resources such that an objective function is optimized. Within accounting, common objective functions are to maximize profits or minimize costs. Information provided by accounting is essential for linear programming through manufacturing cost estimates for direct materials, direct labor, and manufacturing overhead. In addition, constraints in the form of physical restrictions or economic conditions on the set of feasible decisions are often accounting related. Projected sales, production inputs for labor hours, direct material needs and equipment utilization are common constraints to an LP model.

Contribution Income Statements and Linear Programming

The reporting of linear programming decisions is especially conducive to a management accounting format of an income statement. A contribution income statement (see Table 1 for example) identifies contribution margin which changes proportionately with changes in the units produced and sold. Profit for a contribution income statement is contribution margin less fixed expenses. Within the relevant range of the decision, it is assumed that fixed expenses remain constant even with changes in the units produced and sold. Contribution income statements facilitate the results of linear programming models by highlighting contribution margin and profit. Note that the contribution income statement confirms that maximizing contribution margin will also maximize profits when fixed costs do not change.

Binary Linear Programming and Fixed Costs

Binary linear programming models utilize integer variables (usually 1 and 0) to indicate logical or dichotomous decisions (e.g., on/off, true/false, yes/no). Similarly, a fixed cost is either incurred or it is not incurred. In contrast to linear relationships for revenues or variable costs, a fixed cost does not change proportionately with changes in activity. Hence, fixed costs are usually not included in LP models and are often thought as not relevant [2].

However, there are LP decisions in which fixed costs will increase. For example, additional equipment rental costs or supervisory costs may be incurred if a product is selected or units produced are increased. These step-fixed costs can now be modeled with binary variables and still satisfy the necessary condition of linearity [1]. For example, if step-fixed costs are incurred for a product then the dichotomous value is 1 and if fixed costs are not incurred then the value is 0. This dichotomous variable may be used to facilitate two concurrent decisions: a) which products to produce with its BLP capability, and then b) how many of each product to produce. The following case will contrast the typical LP product-mix decision that maximizes contribution margin to the BLP decision that adds relevant fixed costs and maximizes profit. The results of both decisions will be presented with contribution income statements to highlight their differences.

ABIGAIL COMPANY: RELEVANT FIXED COSTS OF A PRODUCT-MIX DECISION

Overview and Input Data

Abigail Company produces four products A, B, C and D found in Table 1 (an Excel spreadsheet) that have selling prices of $60, $65, $75 and $90, respectively. The variable costs consist of direct material cost and direct labor cost for operating equipment unique to each product. The direct material cost is $2 per pound and the equipment operators are paid an hourly rate of $10. The amount of pounds and direct labor hours for each product is listed. For example, product A requires 2.5 material pounds and 3.5 labor hours. Hence, the variable cost per unit for A is $40, consisting of $5 direct materials cost and $35 direct labor cost. The $20 contribution margin per unit for product A is $60 selling price less $40 variable cost. Each month Abigail Company must make a decision as to which products A, B, C, or D to sell. That decision is constrained by 6,000 direct material pounds and 5,000 direct labor hours available this month. The cost of equipment rental is $12,000 per month for A, $16,000 for B, $15,000 for C and $14,000 for D.

LP Model and Solution to Maximize Contribution Margin

The initial LP model has 10 units produced and sold for each product as this emphasizes that 10 is a multiplicative-factor for contribution margin, direct material pounds, and direct labor hours. Within Excel, select *Data* and then *Solver*. From the *Solver Parameters* screen, *Set Objective:* total contribution margin of $920, *To:* Max, *By Changing Variable Cells*: decision cells units produced and sold for products A, B, C and D, *Subject to the Constraints:* add constraints for direct material pounds and direct labor hours, *Make Unconstrained Variables Non-Negative:* yes, *Select a Solving Method:* Simplex LP, and then click on *Solve.* Fixed equipment rental costs are not included in the LP Solution model but are included in its contribution income statement.

The LP Solution indicates that 213 units of product A and 1,215 units of product C should be produced and sold in maximizing total contribution margin to **$41,925**. The direct materials pounds and direct labor hours constraints are both met. The LP solution is translated into a contribution income statement with fixed costs for equipment rental of $12,000 and $15,000 for products A and C. With the $27,000 ($12,000 – A, $15,000 –B) total deduction for equipment rental, total profit is $14,925 with product A having a loss of $7,740. The loss for product A suggests that not modelling fixed costs in the LP solution can lead to total profits not being maximized.

TABLE 1: LP MODEL AND SOLUTION TO MAXIMIZE CONTRIBUTION MARGIN

Input Data

	A	B	C	D		
Selling price	$60	$65	$75	$90		
Variable cost	$40	$48	$44	$66		
Contribution margin	$20	$17	$31	$24		
Direct material pounds	2.5	4	4.5	5.5	≤ 6,000	$2.00
Direct labor hours	3.5	4	3.5	5.5	≤ 5,000	$10.00
Equipment rental	$12,000	$16,000	$15,000	$14,000		

Initial LP Model

	A	B	C	D	Total	Maximum
Units produced and sold	**10**	**10**	**10**	**10**		
Contribution margin	$200	$170	$310	$240	**$920**	
Direct material pounds	25	40	45	55	**165**	**≤ 6,000**
Direct labor hours	35	40	35	55	**165**	**≤ 5,000**

LP Solution

	A	B	C	D	Total	Maximum
Units produced and sold	**213**	0	**1,215**	0		
Contribution margin	$4,260	0	$37,665	0	**$41,925**	Maximum
Direct material pounds	532	0	5,468	0	**6,000**	**≤ 6,000**
Direct labor hours	746	0	4,254	0	**5,000**	**≤ 5,000**

Contribution Income Statement for LP Solution

	A	B	C	D	Total
Sales	$12,780	0	$91,125	0	$103,905
Variable cost	$8,520	0	$53,460	0	$61,980
Contribution margin	$4,260	0	$37,665	0	**$41,925**
Equipment rental	**$12,000**	**0**	**$15,000**	**0**	**$27,000**
Profit (Loss)	**$(7,740)**	0	$22,665	0	**$14,925**

BLP Model and Solution to Maximize Profit

With the use of binary linear programming (BLP) to model fixed equipment rental costs, Solver is able to determine concurrently which products to produce and its quantity in maximizing total profit. Solver works with two objective functions at the same time: to produce or not to produce each product using binary variables 1 and 0, and for the chosen products the number of units to produce in maximizing total profit.

In the initial BLP model found in Table 2, each product has a dichotomous 1 value to signify production of all the products. A value of 10 units is the starting point for each product as noted along the diagonal for units produced and sold. The maximum number of units for each product is equal to the lesser of units possible based on constraints for direct materials and direct labor hours. For example, product A has a maximum capacity of 1,429 units, which is the **minimum** of 2,400 units possible for direct materials (6,000 pounds divided by 2.5 pounds per unit) and 1,429 units possible for direct labor hours (5,000 direct labor hours divided by 3.5 direct labor hours per unit). Similarly, product B, product C and product D have maximum capacities of 1,250 units, 1,333 units and 909 units. The related initial BLP contribution income statement is also presented.

To perform the concurrent analysis of which products to produce and their number of units to produce, the dichotomous production variable (1=yes, 0=no) must be a multiplicative-factor of each product's maximum constraint and the equipment rental fixed costs. In other words, the maximum constraint for each product A, B, C and D must include in its formula a reference to produce (1) or not (0). For example, if product A is to be produced the maximum capacity is 1429 = (1)*minimum of (2400, 1429), and if product A is not to be produced the maximum capacity is 0 = (0)*minimum of (2400, 1429). Similarly, equipment rental for product A is incurred with its production $12,000 = (1)*$12,000 or not incurred 0 = (0)*$12,000. The constraints for direct material pounds and direct labor hours are still in effect.

Although they appear separately, the Initial BLP Model becomes the BLP Solution when linear programming is performed using similar selections previously described. From the *Solver Parameters* screen, *Set Objective:* total profit of the contribution income statement which is $(56,080), *To:* Max, *By Changing Variable Cells*: binary decision to Produce? having a value of **1,** and the Units produced and sold along its diagonal having a value of **10**, *Subject to the Constraints:* add the maximum capacity constraints for total units produced and sold for each product, and constraints for direct material pounds and direct labor hours, *Make Unconstrained Variables Non-Negative:* yes, *Select a Solving Method:* Simplex LP, and then click on *Solve.*

From the BLP solution, only product C will be produced and sold. The 1,333 units of product C maximizes profit of $26,333 while meeting its maximum constraint and direct material pounds and direct labor constraints. Although the BLP model had a lower $41,333 contribution margin in comparison to the $41,925 LP model, the $15,000 of equipment costs for just product C led to a BLP model profit of $26,333 in comparison to $14,925 for the LP model. For products A, B and D, note that the Produce? binary value is 0, the Units produced and sold along the diagonal is 0, and the maximum constraint is 0.

The Abigail Company case illustrates the importance of including both relevant fixed costs and the use of BLP to solve for the products to produce and their quantities in maximizing profits. Furthermore, a contribution income statement facilitates the recognition of fixed costs that may or may not be incurred for individual products and their impact on profits.

TABLE 2: BLP MODEL AND SOLUTION TO MAXIMIZE PROFIT

Initial BLP Model

	A	B	C	D			
Produce? 1=yes, 0=No	**1**	**1**	**1**	**1**			
Units produced and sold					Total		Maximum
A	**10**	0	0	0	**10**	≤	**1,429**
B	0	**10**	0	0	**10**	≤	**1,250**
C	0	0	**10**	0	**10**	≤	**1,333**
D	0	0	0	**10**	**10**	≤	**909**
Constraints on resources							
Direct material pounds	25	40	45	55	**165**	≤	**6,000**
Direct labor hours	35	40	35	55	**165**	≤	**5,000**

Initial BLP Contribution Income Statement

	A	B	C	D	Total
Sales	$600	$650	$750	$900	$2,900
Variable cost	$400	$480	$440	$660	$1,980
Contribution margin	$200	$170	$310	$240	$920
Equipment rental	$12,000	$16,000	$15,000	$14,000	$57,000
Profit	$(11,800)	$(15,830)	$(14,690)	$(13,760)	**$(56,080)**

BLP Solution

	A	B	C	D			
Produce? 1=yes, 0=No	0	0	**1**	0			
Units produced and sold					Total		Maximum
A	0	0	0	0	0	≤	0
B	0	0	0	0	0	≤	0
C	0	0	**1,333**	0	1,333	≤	1,333
D	0	0	0	0	0	≤	0
Constraints on resources							
Direct material pounds	0	0	6,000	0	6,000	≤	6,000
Direct labor hours	0	0	4,667	0	4,667	≤	5,000

Contribution Income Statement for BLP Solution

	A	B	C	D	
Sales	0	0	$100,000	0	$100,000
Variable cost	0	0	$58,667	0	$58,667
Contribution margin	0	0	$41,333	0	$41,333
Equipment rental	0	0	$15,000	0	$15,000
Profit	0	0	$26,333	0	**$26,333**

REFERENCES

[1] Albright, S. C., Winston, W. L. and Zappe, C. J. *Data Analysis and Decision Making*. Ohio: South-Western Cengage Learning, 2011.

[2] Horngren, C. T., Datar, S. M. and Rajan, M. V. *Cost Accounting: A Managerial Emphasis*. New Jersey: Prentice Hall, 2011.