An Application of the Multiple Knapsack Problem: The Self-Sufficient Marine

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Expeditionary forces such as Marine squads face a unique challenge: they must be able to survive and carry out required missions for as long as possible without external support, but are constrained by the amount that they can physically carry. Some of the items that they carry provide value by helping the squad conduct activities relevant to the mission, e.g. rifles and radios. Other items provide value by sustaining the squad, e.g. water and MREs (food rations). Determining the optimal load of items for a squad can be an extremely complex problem.

The self-sufficient Marine problem is an extension of a multiple knapsack problem, in which items must be selected to fill *n* knapsacks in an optimal way. Typically, each item has both a value and a cost (usually weight or volume) associated with it. In the context of this problem, each Marine can be viewed as a knapsack, with a constraint on the amount of weight (s) he is able to carry. However, there are a few features of the self-sufficient Marine problem that make it more complicated than the standard multiple knapsack problem.

First, some items can be shared between Marines while still retaining part or all of their value. For example, a two-person tent can be shared by two Marines, but only needs to be carried by one. Some items, such as signal flares, are generally used infrequently, and thus might be able to be shared between Marines. This affects the relationship between the number of an item carried and the amount of value provided to the squad.

Second, some items can be transferred between Marines as needed, while others cannot. In particular, the articles of clothing worn by the Marines cannot be transferred between them. This places an additional restriction on the allocation of items to individuals; one Marine cannot carry the vests for the entire squad.

Third, as mentioned previously, we must draw a distinction between *mission items* and *sustainment items*. As we will observe, some of the optimization models used for self-sufficiency will treat the two types of items differently, depending on how exactly self-sufficiency is defined, and what the relevant constraints are. For example, if the mission has a fixed (required) duration, then there will be a constraint requiring that a sufficient quantity of sustainment items is included.

We develop three different optimization models for self-sufficiency, based on three different interpretations of what exactly self-sufficiency means. The first model is based on a *binary* interpretation; the squad must be able to carry a given set of items, otherwise it is not self-sufficient. This model is fairly straightforward; if a feasible solution exists given the weight constraints, the squad is self-sufficient. The second model is based on a *degree* interpretation; an ideal set of items is given, and we maximize the proportion of the ideal set's value that can be obtained, given a constraint for the required duration of the mission. The third model is based on a *duration* interpretation; given a required set of mission items, we maximize the duration of time for which the squad can be self-sufficient.

We consider two sets of items, based on standard operating procedures for hot and cold weather missions. The relative values provided by each item are assessed from subject matter experts. To gain insight into the relationships between the different components of the problem, we apply the *degree* model to missions of several different durations, and we apply the *duration* model while varying the individual Marines' weight constraints.

The multiple knapsack problem, even without these extensions, is NP hard. However, because the optimization problems we are analyzing are of a manageable size, we are still able to obtain numerical solutions. Based on these solutions, we are able to draw several interesting conclusions that the Office of Naval Research found useful as a baseline analysis for determining loads of self-sufficient Marine squads.

Our application of the degree model reveals that self-sufficiency increases as squad size increases, due to the increase in flexibility of how items are allocated between Marines. However, as required duration increases, self-sufficiency decreases dramatically, due to the need to replace some mission items with sustainment items. We provide numerical results to illustrate this relationship more clearly.

Our application of the duration model confirms that as the individual Marine weight capacity increases, self-sufficiency increases as well. Specifically, an increase in the weight capacity of approximately 8-10 pounds is associated with a 12-hour increase in duration. Again, the numerical results provided illustrate this relationship in more detail.