ELASTICITIES (ε) IN WEIGHTED AVERAGE COST OF CAPITAL DIAGNOSTICS

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ABSTRACT

The weighted average cost of capital (WACC) is a critical decision-making tool in managerial finance because it is the benchmark used to determine the intrinsic value of a project. A project with a rate of return greater than the WACC will add economic value to the firm so the WACC must be calculated with a high degree of accuracy.

The firm must understand the relationships between the components of its weighted average cost of capital (WACC). Elasticities (ε) similar to those in microeconomic theory are the appropriate tool to understand these relationships and are excellent instruments for more accurate decision-making.

PROBLEM DEFINITION

A key problem is that the weighted average cost of capital (WACC) is a "moving target" and the firm must constantly re-examine its WACC to determine if the current value of the WACC is accurate. Changes to the WACC occur because of normal business operations and also if the firm changes its capital structure on its own accord as the firm attempts to engineer the optimal capital structure.

The "Modigliani/Miller" Model of capital structure states that the debt in the permanent sources of financing adds value to a firm because debt interest expense is tax deductible, which effectively lowers the cost of debt (k_d) and which subsequently lowers the weighted average cost of capital (WACC). A key problem, however, is that adding debt increases the risk of a firm after a certain point and this added risk increases the WACC because of the "mean-variance" paradigm. A firm will maximize economic value from the correct combination of the costs and benefits of debt financing through the engineering of the lowest feasible WACC.

The weighted average cost of capital (WACC) contains numerous components and it is incorrect to only change one component to determine the change to the overall WACC when conducting diagnostics of the WACC. In general, a change in one component forces a change to one or more other components, which then changes the overall WACC. It is important then to understand the relationship interactions amongst the various components of the WACC to accurately perform diagnostics when components of the WACC are expected to change, such as changes to the cost components or adjustments to the capital structure.

Firms generally are reluctant to change the weighted average cost of capital (WACC) on a frequent basis so the firm may set a firm-wide WACC on an annual basis; however, the firm should change the WACC if the current WACC is not accurate regardless of the timing. While a change to the WACC may be inconvenient to a firm, an accurate WACC is critical for any successful firm.

The general formula for the weighted average cost of capital (WACC) is:

$$WACC = (k_e * w_e) + (k_p * w_p) + [(k_d * w_d) * (1 - t)] \quad (1)$$

where k_e = cost of common equity, w_e = weight of common equity
k_p = cost of preferred stock, w_p = weight of preferred stock
k_d = cost of long-term debt, w_d = weight of long-term debt
t = corporate tax rate

Common equity, preferred stock and long-term debt are the permanent sources of financing for the corporation and $[w_e + w_p + w_d] = 100\%$.

It is straightforward to understand the relationships between w_e, w_p and w_d: if one factor changes then the others must adjust so that the total of these three component always sum to 100%. Changes to w_e are constant and occur through normal business operations since the retained earnings for a firm will change monthly based upon net income, net loss and dividends. If the magnitude of the monthly changes to retained earnings is relatively small, a change to the weighted average cost of capital (WACC) may not be necessary if the current WACC is accurate.

What is unclear is the relationship amongst k_e, k_p and k_d. These relationships are important because a change in one component may institute a change in another component but the magnitude of the change and the direction of the change could be unknown. Also, these relationships are different for each firm so what is effective for one firm may be inapplicable to another firm.

The focus will be on the cost of debt (k_d) and the increase in risk caused by the issuance of additional long-term debt into the capital structure. The "mean-variance" paradigm states that additional risk will increase the rate of return to compensate investors for accepting the additional risk. The weighted average cost of capital (WACC) should increase if additional risk is introduced into the capital structure through the issuance of long-term debt.

ELASTICITIES (ε) IN THE WACC

Elasticities (ε) are the correct metric to implement as a diagnostic tool to test the sensitivities of the various components of the weighted average cost of capital (WACC) and to improve decision-making. Specifically, this paper will focus on the effect that a change in the cost of debt (k_d) will institute a change in the cost of common equity (k_e). This paper will not focus on the cost of preferred stock (k_p) but the analysis of the elasticity (ε) between the cost of debt (k_d) and the cost of preferred stock (k_p) will be similar to the analysis of the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e). The elasticity (ε) equation will be:

$$\varepsilon = \frac{\partial k_e}{\partial k_d} * \frac{k_d}{k_e} = \frac{\%\,\Delta\, k_e}{\%\,\Delta\, k_d} \qquad (2)$$

A change in k_d will institute a change in k_e, which will then change the weighted average cost of capital (WACC). A change in k_d will not be isolated to a single component in the WACC only. A change in k_d will cause a "ripple effect" into other internal components of the WACC before the WACC will be affected. The elasticity (ε) will help to measure the magnitude and direction of the "ripple effect" so that the firm may engineer the optimal capital structure through its permanent sources of financing.

There are multiple metrics for the return on common equity (k_e) but this paper proposes the following common measurement of k_e:

$$k_e = \frac{P_1 + D - P_0}{P_0} \qquad (3)$$

where P_1 = current common stock price P_0 = base common stock price
D = annual common stock dividends

The annual rate of return is most appropriate for this calculation. Mathematical models such as the Capital Asset Pricing Model and the Arbitrage Pricing Theory are theoretically correct but they suffer from measurement problems of their inputs. The common equity model proposed above is accurate if the inputs are correct and the inputs are directly observable through financial publications.

The metrics for the cost of debt (k_d) are more problematic. The firm must calculate the yield to maturity (YTM) for its debt but some of the components for the yield to maturity (YTM) may not be directly observable; however, if the firm's debt is publicly traded and transaction costs can be reasonably estimated, the yield to maturity calculation should be straightforward. Again, the theoretical models such as the "risk-free rate (r_f) + risk premium" equation are theoretically correct but the statistical measurements of the equation's inputs may be difficult or intractable. While there are difficulties the firm must somehow accurately measure k_d to precisely calculate the weighted average cost of capital (WACC).

Again, the elasticity (ε) between k_d and k_e or k_d and k_p is crucial because a change in k_d will not be isolated. A change in k_d will then affect k_e, k_p or both, which will have an effect on the overall weighted average cost of capital (WACC).

INTERPRETING THE ELASTICITY (ε)

Proper interpretation of the calculated elasticity (ε) is important in the diagnostic analysis of the weighted average cost of capital (WACC). The elasticity (ε) can help to more precisely predict the change to the WACC when its components change.

The following table provides a cursory overview to interpret the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e):

ELASTICITY (ε) BETWEEN Ke and Kd			
Elasticity (ε)	Definition	Formula	Interpretation
less than -1	elastic	$\frac{\partial k_e}{\partial k_d} * \frac{k_d}{k_e}$	$\%\Delta k_e > \lvert -\%\Delta k_d \rvert$
-1	unitary	↓	$\%\Delta k_e = \lvert -\%\Delta k_d \rvert$
btwn. -1 & 0	inelastic	↓	$\%\Delta k_e < \lvert -\%\Delta k_d \rvert$
btwn. 0 & 1	inelastic	↓	$\%\Delta k_e < \%\Delta k_d$
1	unitary	↓	$\%\Delta k_e = \%\Delta k_d$
greater than 1	elastic	↓	$\%\Delta k_e > \%\Delta k_d$

The elasticity (ε) would generally be expected to be a positive number but this is not always true. A positive elasticity (ε) implies that the cost of debt (k_d) and the cost of common equity (k_e) will move together in the same direction regardless of the magnitude of the change in direction.

Is a negative elasticity (-ε) possible or reasonable? There are plausible scenarios where a negative elasticity (-ε) may occur. Consider a firm that is financed solely by common equity – the permanent source of financing in the capital structure of this firm is only common equity. In this scenario the weighted average cost of capital (WACC) is equal to the cost of common equity (k_e). If this firm issues long-term debt then the WACC may drop because the cost of debt (k_d) is generally lower than the cost of common equity (k_e) and debt interest expense is deductible for income tax purposes, which effectively lowers the cost of debt (k_d). The cost of debt (k_d) adds economic value to the firm which may decrease the cost of common equity (k_e) and will subsequently decrease the WACC.

In most cases, however, the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) is positive which implies that as additional long-term debt is injected into the capital structure, the cost of common equity (k_e) will increase through the "mean-variance" paradigm mechanisms. If both the cost of debt (k_d) and the cost of common equity (k_e) increase, the weighted average cost of capital (WACC} can only increase. The introduction of additional debt into the capital structure will increase the WACC and decrease the value of the firm.

If the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) is inelastic, the subsequent rise in the weighted average cost of capital (WACC) will be small, but if the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) is elastic, an increase in the cost of debt (k_d) will lead to a relatively larger increase in the WACC.

NUMERICAL EXAMPLE

Suppose a firm is financed with 50% corporate bonds which have a yield to maturity of 6% and are about to mature. The remainder of the firm is financed with common equity with a return on common equity of 12%. This firm has no preferred stock in its capital structure and the corporate income tax rate is 40%. The weighted average cost of capital (WACC) in this scenario will be:

WACC = (50% * 12%) + [(50% * 6%) * (1 – 40%)] = 7.8%

The elasticity (ε) from the cost of debt (k_d) to the cost of common equity (k_e) is 1.2 so the ε between the two cost components is elastic.

The corporate bonds matured and are fully refinanced at 6.6%, which represents a 10% increase over the previous cost of debt (k_d).

The elasticity (ε) will increase the cost of common equity (k_e) by 20%:

$\% \Delta k_e$ = $\% \Delta k_d * \varepsilon$ = 10% * 1.2 = 12%

The updated cost of common equity (k_e) will be:

Updated k_e = 12% * (1 + 12%) = 13.44%

The new weighted average cost of capital (WACC) in this scenario will be:

New WACC = (50% * 13.44%) + [(50% * 6.6%) * (1 – 40%)] = 8.7%

In this example, a 10% increase in the cost of debt (k_d) leads to a 12% increase in the cost of common equity (k_e) which subsequently leads to a 11.5% in the weighted average cost of capital (WACC). The application of the elasticity (ε) allows a fairly precise diagnostic of the change to the WACC because of changes to its internal cost components. The use of an elasticity (ε) in the diagnostics of the weighted average cost of capital (WACC) significantly improves decision-making.

HEURISTICS FOR ELASTICITY (ε)

Measurement inaccuracies may make it difficult to obtain precise values of the components of the weighted average cost of capital (WACC) so that elasticities (ε) cannot be calculated. Surrogate measures of elasticities (ε) can be effective substitutes for the true elasticities (ε) and therefore provide effective decision-making tools.

The heuristics proposed will range from the coarse to the complex. The first set of suggestions will be financial ratios, especially leverage ratios. One could be the times interest earned (TIE) ratio:

$$\text{times interest earned} = \text{earnings before interest \& taxes} \div \text{interest expense} \quad (4)$$

This ratio measures the firm's ability to cover and pay its interest expense. A high value of the times interest earned (TIE) ratio is preferred while a low ratio implies that the firm is not generating sufficient cash flow to cover its interest expense. A low times interest earned (TIE) ratio increases the risk of a firm because of a greater probability of default and bankruptcy. The user can develop a mathematical function that approximates the elasticity (ε) from the times interest earned (TIE) ratio.

The second suggested ratio could be the ratio of long-term debt to equity:

$$\text{long-term debt to equity ratio} = \text{long-term debt} \div \text{equity} \quad (5)$$

A lower value of this ratio is preferred since a high ratio implies that the firm has too much debt in its capital structure. A high ratio indicates a greater level of risk so if the firm requires additional debt funds these funds will generally have a higher cost of debt (k_d). The user can develop a mathematical function that approximates the elasticity (ε) from the ratio of long-term debt to equity.

A statistical measure to approximate the elasticity (ε) is the linear correlation coefficient (ρ). The formula for the linear correlation coefficient (ρ) is:

$$\rho = \frac{n(\Sigma k_e k_d) - (\Sigma k_e)(\Sigma k_d)}{\sqrt{n(\Sigma k_e^2) - (\Sigma k_e)^2}\ \sqrt{n(\Sigma k_d^2) - (\Sigma k_d)^2}} \quad (6)$$

The linear correlation coefficient (ρ) ranges from -1 to 1 so it is a particularly good substitute for the true elasticity (ε) for those firms where the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) is inelastic. The linear correlation coefficient (ρ) would not be appropriate if the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) is elastic because the linear correlation coefficient (ρ) must be greater than -1 and must be less than 1. Because of the limits on its range the linear correlation coefficient (ρ} cannot capture the elastic ranges. A further weakness of the linear correlation coefficient (ρ) is that the linear correlation coefficient (ρ) measures the coordinated movement between the cost of debt (k_d) and the cost of common equity (k_e), not the "cause and effect" metric provided by the elasticity (ε). If these weaknesses are appropriately mitigated, the linear correlation coefficient (ρ) is an excellent estimate of the true elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) but only if the elasticity (ε) is inelastic.

If the mathematical relationship between the cost of debt (k_d) and the cost of common equity (k_e) is linear:

$$k_e = f(k_d)\, dk_d \tag{7}$$

a linear regression model can be used to estimate the elasticity (ε):

$$k_e = \beta_0 + \beta_1 k_d + e \tag{8}$$

where:

$$\beta_n = (k_d' k_d)^{-1} k_d' k_e \tag{9}$$

This will be a robust estimate of the elasticity (ε) especially if there is a strong linear correlation between the cost of debt (k_d) and the cost of common equity (k_e). If $\beta_0 \approx 0$, β_1 is the approximation of the true elasticity (ε). Linear regression can provide the full range of the values of the elasticity (ε), both elastic and inelastic, unlike the linear correlation coefficient (ρ) which is limited to elasticities (ε) which are inelastic. This paper recommends linear regression analysis as the heuristic to estimate the elasticity (ε). There are other heuristics for the elasticity (ε) between the cost of debt (k_d) and the cost of common equity (k_e) so the firm must decide which heuristic is the most accurate if the true elasticity (ε) cannot be calculated. The more accurate the elasticity (ε) the better the diagnostics for the weighted average cost of capital (WACC), which leads to more insightful decisions.

CONCLUSION

The weighted average cost of capital (WACC) is a critical decision tool which firms use to estimate the economic value of a project. Elasticities (ε) are the best analytical tools since elasticities (ε) measure the relationships amongst the internal components of the weighted average cost of capital (WACC). An accurate calculation of the weighted average cost of capital (WACC) is an effective decision-making tool to help determine the increase in the economic value of a firm.

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