MULTI-INDEX CAPACITATED TRANSPORTATION PROBLEM

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ABSTRACT

In this paper, we discuss a procedure for finding an optimal solution a multi-index capacitated transportation problem with bounds on rim conditions having axial constraints. The procedure achieves a solution by solving an equivalent transformed transportation problem formed by adding two additional rows, two additional columns and two additional commodities. The solution method is very useful in situations when there is a need to transport heterogeneous commodities. The solution procedure is computationally simple and quicker than that of the simplex technique.

INTRODUCTION

In the classical transportation problem (TP) a commodity is transported from each of m sources to each of n destinations. The total sum of the available amounts at the sources is equal to the sum of the demands at the destinations. The goal is to determine the amounts of a commodity to be transported over all routes such that the total transportation cost is minimized. The solid transportation problem arises when we need to transport heterogeneous commodities of products. A 3-dimensional transportation problem with planar constraints was introduced by Haley [3], who also provided a solution procedure. A solid transportation problem equivalent to Schell's "Three Axial Sums" problem was formulated by Haley [4]. The method for solution presented by Haley is an extension of MODI method.

Sometimes, the total capacity of each route is also specified by some external decision maker because of budget/political consideration. The optimal solution of such problem is of practical interest to the decision maker. This gives rise to capacitated transportation problem which was first studied by Wagner [11]. Verma and Puri [10] have discussed distribution problems with lower and upper bounds on the supply availabilities and destination requirements. Mishra and Dass [8] discussed solid transportation problem with lower and upper bounds on rim conditions. Dahiya and Verma [2] studied 2-dimensional problem with bounds on rim conditions.

There may be situations sometimes when one wishes to keep reserve stocks at the sources, say for emergencies, thereby restricting the total flow to a known specified level. Khanna et al. [5] solved classical TP with flow constraint. Khanna and Puri [6] studied time minimization transportation problem and transportation problem with mixed constraint with a specified flow respectively. Thirwani [9] studied fixed-charge bi-criterion TP with enhanced flow. Khurana and Arora found cost time trade-off pairs in indefinite quadratic TP [7]. Bandopadhyaya and Puri discussed impaired flow multi-index TP with axial constraints [1].

PROBLEM FORMULATION

The "Three Axial Sums" problem deals with transporting of various commodities from a set of different warehouses to different markets, whose total demands are specified. Here we discuss the 3-dimensional CTP having bounds on rim conditions. The problem can be written as

P: Minimize $\sum_I \sum_J \sum_K c_{ijk} x_{ijk}$

subject to

$$a_i \le \sum_J \sum_K x_{ijk} \le A_i, \quad i \in I$$
$$b_j \le \sum_I \sum_K x_{ijk} \le B_j, \quad j \in J$$
$$e_k \le \sum_I \sum_J x_{ijk} \le E_k, \quad k \in K$$
$$l_{ijk} \le x_{ijk} \le u_{ijk} \qquad \forall\ i \in I, j \in J, k \in K$$

Also for any feasible solution of **P**, $\sum_I A_i = \sum_J B_j = \sum_K E_k = \text{N}$.

where a_i and A_i are the minimum and maximum availability at the i^{th} origin, respectively, I = 1, 2, …, m , b_j and B_j are the minimum and maximum number demand at the j^{th} destination, $j = 1, 2, \ldots\ldots\ldots, n$ respectively, e_k and E_k are the minimum and maximum availability of the k^{th} commodity, k=1,2,…..,p and c_{ij} is the unit cost of transportation from the i^{th} origin to the j^{th} destination. Let l_{ijk} and u_{ijk} be the lower and upper bounds on the (i, j)th route for the k^{th} type of commodity. The problem of minimizing cost of transportation can be stated analytically as the 3-dimensional CTP with bounds on rim conditions discussed below.

In order to solve the above problem **P**, a related solid problem is formulated, with a dummy supply point, a dummy destination and an extra commodity. The related 3-dimensional transportation problem is given as follows

P$_R$: Minimize $\sum_{I'} \sum_{J'} \sum_{K'} c'_{ijk}\, y_{ijk}$

subject to

$$\sum_{J'} \sum_{K'} y_{ijk} = A'_i, \quad i \in I' = I \cup \{m+1\}$$
$$\sum_{I'} \sum_{K'} y_{ijk} = B'_j, \quad j \in J' = J \cup \{n+1\}$$
$$\sum_{I'} \sum_{J'} y_{ijk} = E'_k, \quad k \in K' = K \cup \{p+1\}$$

where $A'_i = \begin{cases} A_i, & i \in I \\ \sum_J B_j, & i = m+1 \end{cases}$; $B'_j = \begin{cases} B_j, & j \in J \\ \sum_I A_i, & j = n+1 \end{cases}$; $E'_k = \begin{cases} E_k, & k \in K \\ \sum_K E_k, & k = p+1 \end{cases}$

$$l_{ijk} \le y_{ijk} \le u_{ijk} \quad \forall\ i \in I, j \in J, k \in K$$
$$0 \le y_{m+1jp+1} \le B_j - b_j \quad \forall\ j \in J; \quad 0 \le y_{in+1p+1} \le A_i - a_i \quad \forall\ i \in I$$
$$0 \le y_{m+1n+1k} \le E_k - e_k \quad \forall\ k \in K; \quad y_{m+1n+1k} \ge 0 \quad \forall\ k \in K$$

$$\left.\begin{array}{l} c'_{ijk} = c_{ijk} \\ c'_{ijp+1} = c'_{in+1k} = c'_{m+1jk} = M \end{array}\right\} \quad \forall i \in I,\ j \in J,\ k \in K$$
$$c'_{in+1p+1} = c'_{m+1jp+1} = c'_{m+1n+1k} = 0 \quad \forall i \in I',\ j \in J',\ k \in K'$$

The problem $\mathbf{P_R}$ is a *Three Axial Sum* problem which can be reformulated as a transformed multi-index TP by using the following definitions by Haley [4].

$$c''_{ijk} = c'_{ijk}\ (i \le m+1, j \le n+1, k \le p+1),\ c''_{m+2n+2p+2} = 0$$
$$c''_{ijp+2} = 0\ (i \le m+1, j \le n+1),\ \ c''_{in+2p+2} = M\ (i \le m+1),$$
$$c''_{in+2k} = 0\ (i \le m+1, k \le p+1),\ \ c''_{m+2jp+2} = M\ (j \le n+1),$$
$$c''_{m+2jk} = 0\ (j \le n+1, k \le p+1),\ \ c''_{m+2n+2k} = M\ (k \le p+1)$$

Let $R = \underset{i,j,k}{Max}\left(A'_i, B'_j, E'_k\right)$

$$A_{jk} = R\ \left(j \le n+1; k \le p+1\right),\ \ B_{ki} = R\ \left(k \le p+1; i \le m+1\right),\ \ E_{ij} = R\ \left(i \le m+1; j \le n+1\right),$$
$$A_{n+2k} = (m+1)R - e'_k\ \left(k \le p+1\right),\ \ A_{jp+2} = (m+1)R - b'_j\ \left(j \le n+1\right),$$
$$B_{p+2i} = (n+1)R - a'_i\ \left(i \le m+1\right),\ \ B_{km+2} = (n+1)R - e'_k\ \left(k \le p+1\right),$$
$$E_{m+2j} = (p+1)R - b'_j\ \left(j \le n+1\right),\ \ E_{in+2} = (p+1)R - a'_i\ \left(i \le m+1\right),$$
$$A_{n+2p+2} = B_{p+2m+2} = E_{m+2n+2} = R$$

Let $\mathbf{P_M}$ denote the problem with these modified constraints and the objective function as minimize $\sum_{i=1}^{m+2} \sum_{j=1}^{n+2} \sum_{k=1}^{p+2} c'_{ijk}\ y_{ijk}$. Problem $\mathbf{P_M}$ can be solved easily by any regular method [3].

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