

UNEXPLORED COSTS OF RANDOM ORDER CROSSOVER IN SUPPLY CHAINS

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ABSTRACT

Studies have shown that random order crossover helps reduce the risk of an out-of-stock event when an order that was issued later arrives earlier to offset the late arrival of an order issued earlier. The current school of thought is that, although such crossovers create complexities from a modeling perspective, they are surprisingly beneficial in practice since they decrease the realized lead-time variability. Unlike all previous work, which has focused on the realized lead-time distribution produced by random order crossover and its benefits, this study explores the potential costs of random crossover in supply chains. The results warrant the hypothesis that random order crossover is likely to create more costs than benefits.

INTRODUCTION

Random lead times enable order crossover in which some orders arrive out of sequence, so that an order that was issued later arrives earlier. The earlier arrival raises the inventory level and thereby helps mitigate and an out-of-stock event caused by the late arrival of an earlier order. Since traditional inventory analysis ignores order crossover, it is likely to overstate the realized lead-time variability and, thereby, to inflate inventory costs. Additionally, since the relatively long and variable lead times of global supply chains facilitate random order crossover, while modern systems such as JIT and lean foster multiple open orders, it is important to consider how random order crossover affects supply chain costs in practice.

As documented in the next section of the paper, previous work on random order crossover has focused predominately on characterizing lead-time variability, evaluating safety stocks, and assessing the benefits to inventory policies, levels, and costs. The underlying assumptions are that early arrivals, as well as additional volatility in units received, are without cost, while safety stock and shortage costs accurately represent the cost of late arrivals. Yet, the arrival sequence could trigger penalties in supplier compliance agreements or hurt supplier scorecards. Additionally, the volatility in the quantities ordered and received could affect transportation cost, as well as labor, equipment, and facility capacity costs. This initial study explores the potential cost implications of random order crossover.

LITERATURE REVIEW

Random order crossovers occur when lead times are identically distributed and more than one order is outstanding [11]. Researchers have given considerable attention to this phenomenon in recent years. The works of Robinson et al. [12], Bradley and Robinson [2], and Robinson et al. [13] focus on benefits to safety stock and shortage costs in periodic review (base stock) systems. He et al. [10] and Hayya et al. [7] [9] consider the benefits in continuous review systems. Srinivisan et al. [17] corroborated the benefits of optimal policies with order crossover in relation to approximate policies. Hayya et al. [8] considered the benefits given exponential lead times. More recently, Bischak et al. [1] provided a novel approximation of the realized lead time standard deviation and found that crossover leads, on average, to more accurate

policies and significant cost reductions. Finally, Disney et al. [5] studied the impact of random order crossover on inventory costs and safety stock in a periodic inventory system.

The practitioner literature offers abundant evidence of supplier compliance agreements, especially in the retail sector. These agreements typically require suppliers to meet delivery windows and levy stiff penalties for early and late shipments. For example, in 2010 Wal-Mart imposed a penalty of 3% of the cost of goods sold on shipments arriving outside of a prescribed four-day delivery window, either early or late [4]. A SAP white paper reports that such chargebacks are essential to doing business today [14]. Other reports [3], [6], and [16] provide an overview of the chargebacks and fees found in supplier compliance agreements. Additionally, Schorling [15] offers some guidelines on how to manage chargebacks.

APPROACH

We simulate an inventory model with independent random demands and independent random lead times based on a realistic case study found in Disney et al. [5]. This setting encompasses a simple base stock inventory system with a complete backorder policy and a review period of one week. The distribution of demand that is Normal with a mean of 40 units per week and a standard deviation of 10 units per week. The door-to-door lead-time distribution that is shown in Figure 1 has a mean of 5.49 weeks and a standard deviation of .88 weeks. The simulation assumes 52 weeks per year. The results are based on one thousand iterations of a simulated year.

FIGURE 1. DISCRETE LEAD-TIME DISTRIBUTION

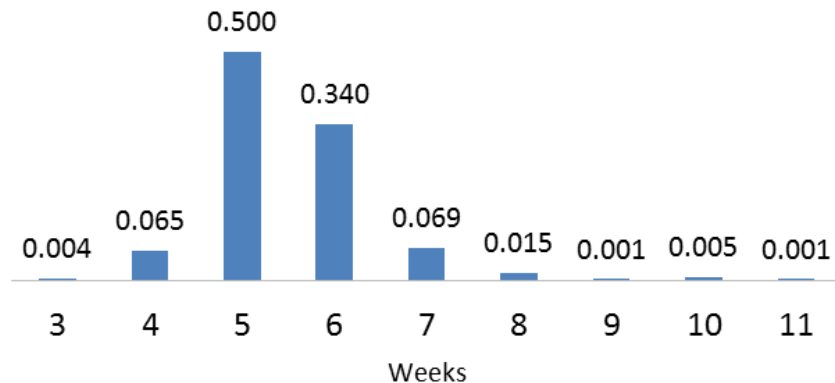


Table 1 provides examples of three arrival patterns of interest. The first includes redundant arrivals in period (see $t=9$ in Table 1). Redundant shipments might not receive a delivery appointment before the planned delivery window, which means the supplier would have to absorb the cost temporary storage, as well as possible detention and demurrage costs. The second pattern encompasses an offsetting order crossover, which could occur in adjacent periods (see $t=11$ and $t=12$ in Table 1) or across multiple periods. Although the arrivals are offsetting, the out-of-sequence order quantities may trigger compliance penalties based on over/short provisions in the agreement. The third pattern includes an order crossover in which one of the two arrival periods has a redundant delivery. Thus, this pattern provides a partial benefit in one period (see $t=14$, in Table 1) coupled with a redundant delivery in the other ($t=15$).

**TABLE X. ILLUSTRATION OF ARRIVAL PATTERNS OF INTEREST FOR ORDERS (O)
PLACED IN PERIOD T-L-1***

t	Random Draw	Possible Lead Times									# Arrivals	Arrival Pattern	
	L	3	4	5	6	7	8	9	10	11			
1	7												
2	6												
3	5												
4	7												
5	5										0		
6	8										0		
7	7										0		
8	4										0		
9	7			O3	O2	O1					3		Redundant*
10	8									0			
11				O5							2		Offsetting**
12					O4						1		
13										0			
14			O8								1		Partial offsetting***
15						O7	O6				2		
16										0			

*Yellow fill highlights possible arrivals for orders placed in period t=1. Since L=7 in t=1, the order (O1) arrives in t+L+1 = 9

RESULTS

As illustrated in Table 2, the realized lead-time distribution in this setting has a standard deviation of .64, which is less than 0.88 standard deviation of the parent lead-time distribution. Thus, given a 90% in-stock probability for one replenishment cycle, the random order crossover would reduce safety stock by about nine units or 16%.

TABLE 2. REALIZED LEAD TIME

Order Placed (t)	Random Lead Time (L)	Order Received (R=t+L+1)	Chronological Sequence (R')	Realized Lead Time (R'-t-1)
1	4	6	6	6
2	6	9	9	6
3	6	10	10	6
4	5	10	10	5
.
.
.
n	n	n	n	n
Mean	5.49			5.49
Std dev	0.86			0.64

Figure 2 summarizes the expected frequency of the arrival patterns after one thousand iterations of a simulated year. On average, 11.9 redundant arrivals would occur per year. If appointments were not granted for early delivery, the burden for detention, demurrage, or temporary storage would fall on the

supplier. By contrast, the combination of offsetting and partially offsetting arrivals would occur about 3.5 weeks per year.

FIGURE 2. EXPECTED ARRIVAL PATTERN FREQUENCY PER YEAR (52 WEEKS)

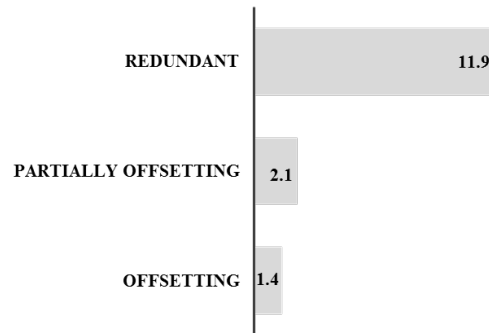


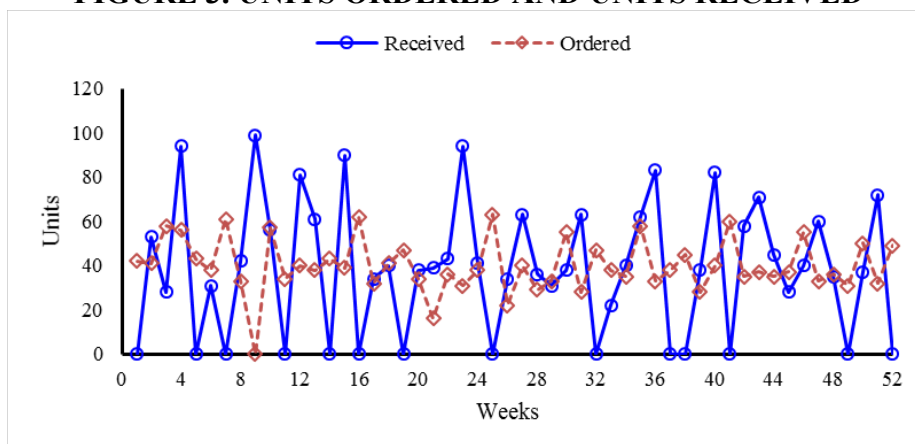
Table 3 summarizes the frequency of arrivals per week, which ranges from zero to four. The demand node, on average, would receive no orders in about 15 weeks of the year and multiple orders in approximately 13 weeks of the year. Figure 3 illustrates the kind of volatility in units ordered and units received that could occur as a result of the order arrival frequencies shown in Table 3. On average, the units ordered would range from about 7 to 62, which suggests that transportation costs would fluctuate between less-than-container transportation charges and full container charges. Meanwhile, without delivery windows, the relatively wide range of units received (from 0 to 121 units) would exacerbate capacity planning for labor, equipment, and facilities.

TABLE 3. EXPECTED ORDER ARRIVALS PER WEEK

Orders Received Per Week	Frequency Per Year	Relative Frequency Per Year
0	14.76	28.39%
1	24.54	47.19%
2	11.22	21.57%
3	1.42	2.73%
4	0.06	0.12%

52 weeks per year

FIGURE 3. UNITS ORDERED AND UNITS RECEIVED



CONCLUSIONS

This study explored the heretofore unexamined potential of random order crossover to impose important costs in supply chains. The exploration focused on a realistic case setting for a global supply chain. The results show random order crossovers created far more instances of redundant (multiple) deliveries per year than offsetting arrivals, which leads to considerable volatility in units received. Such results are likely to trigger significant penalties found in common supplier compliance agreements or exacerbate capacity-related costs. Additionally, since the shipment size directly affects transportation costs, the order volatility implies potentially higher transportation costs. Further, the demand node receives revenue from late shipments, which suppliers must absorb and may make the cost of underage calculation negative for the demand node. Thus, consideration of the realized lead-time demand only addresses one aspect of random order crossing—namely, the benefit to safety stock holding and shortage costs. Redundant deliveries and volatility of units received are other important aspects, for they are likely to influence compliance penalties, transportation economies, and capacity planning. These aspects warrant further study to test the hypothesis that random order crossover creates more costs than benefits in many, if not most, industries.

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