FINDING OPTIMAL MAINTENANCE LEVELS

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ABSTRACT

In the case of a breakdown or high-level degradation of equipment, particularly that of agricultural machinery with crucial time-sensitivity due to harvest, maintenance should be done. Minimal repair fixes the problem in the way if no problem occurred. Replacement results in new equipment, and partial repair fixes the problem and with the additional activity decreases its effective age and therefore, increases its useful life. The optimal choice of the maintenance action has to be based on several factors: cost, effective age, failure rate, reliability, and remaining useful life of the equipment. Therefore, multi-objective analysis is needed to find the best alternative. This paper introduces a mathematical model and offers a method to find the best maintenance alternative.

INTRODUCTION

Reliability engineering is one of the most important fields of engineering sciences; specifically, in the case of agricultural machinery and equipment, as maintenance is vital due to the time sensitivity of harvest. There is a huge literature introducing its basic concepts and methods [1] [2] [3] [4] [5]. Failures are usually represented by a random variable X, which is the time to failure. In most applications, the Weibull distribution is assumed with Cumulative Distribution Function (CDF) $F(t)$, probability density function (pdf) $f(t)$, reliability function $R(t)= 1 – F(t)$, which give the probability that the equipment will be in working condition at time t. In addition, the failure rate plays an important role in the State of Health (SOH) of any equipment, which is defined as

$$\rho(t)=\lim_{\Delta t\to 0} \frac{P(t < X < t+\Delta t \mid X > t)}{\Delta t} = \frac{f(t)}{R(t)} \qquad (1)$$

which means the conditional probability that the equipment will break down in the next time interval $(t, t+\Delta t)$ given it is still working at the time of t with small Δt.

Function $F(t)$ increases showing that the probability of failure before time t becomes larger with larger value of t, and in most cases $\rho(t)$ also increases since failures occurs more often with older equipment.

Any maintenance action has significant effects on these characteristics by decreasing the effective age by a constant value u. Minimal repair is the only case when $F(t)$, $f(t)$, $R(t)$, and $\rho(t)$ remain the same, however in the cases of partial repair and replacement this is not the case. Assume maintenance/repair/replacement is done at time T. Depending on the level of our action these functions are shifted by the term u, which is zero in the case of minimal repair, $0<u<T$ for partial repair and $u=T$ in the case of replacement. The end of life of the equipment can be determined by using the CDF when it reaches a certain threshold:

$$F(t-u)=F^{*} \qquad (2)$$

which implies

$$T_{end}=F^{-1}(F^{*})+u \qquad (3)$$

The effective age of the equipment becomes $T-u$ meaning that we make the equipment younger, the failure rate decreases to $\rho(T-u)$, reliability of the equipment changes to $R(T-u)$, and the remaining useful life is clearly $T_{end}-T$.

In the following section of this paper a multiobjective approach will be introduced to find an optimal maintenance level considering the combination of the characteristics of the equipment after the maintenance/repair/replacement is performed.

THE MULTIOBJECTIVE APPROACH

Let again T be the time when the action has to be taken. The unknown decision variable is the level of maintenance: u.

The following objective are therefore considered:

$g_1(u)= R(T-u)$ = increased reliability value, (4)

$g_2(u) = T-u$ = decreased effective age, (5)

$g_3(u) = \rho(T-u)$ = decreased failure rate, (6)

$g_4(u) = F^{-1}(F^{*}) + u - T$ = increased remaining useful life, (7)

$g_5(u)$ = cost, which is increasing in u. (8)

By the nature of these objectives we want to have as large values as possible for $g_1(u)$ and $g_4(u)$ and as small as possible values for $g_2(u)$, $g_3(u)$, and $g_5(u)$. Since these objectives have different meanings, they have to be transformed to the same measure. In such cases, utility functions are selected for each objective function showing the satisfaction of the decision maker with the different values of the objectives. Zero value means that the value is completely unacceptable and unit value (100%) shows total satisfaction. And finally all objectives are replaced with their utility values $U_i(g_i(u))$ and an appropriate multiobjective optimization method is used. A comprehensive summary of the different concepts and methods can be found for example in [6]. The most commonly used method is weighting, where the decision maker assigns importance weights for the objectives and the composite objective

$$g(u) = c_1U_1(g_1(u)) + c_2U_2(g_2(u)) + c_3U_3(g_3(u)) + c_4U_4(g_4(u)) + c_5U_5(g_5(u))$$
(9)

is maximized.

NUMERICAL EXAMPLE

Consider an equipment with Weibull failure time distribution:

$$F(t) = 1\text{-}e^{-\left(\frac{t}{2}\right)^{1.5}} \tag{10}$$

with reliability function

$$R(t) = e^{-\left(\frac{t}{2}\right)^{1.5}}, \tag{11}$$

pdf

$$F(t) = \frac{1.5}{2^{1.5}} t^{0.5} e^{-\left(\frac{t}{2}\right)^{1.5}}, \tag{12}$$

and failure rate

$$\rho(t) = \frac{1.5}{2^{1.5}} t^{0.5}. \tag{13}$$

Assume repair is needed at T=4, and there are three options: minimal repair (u=0), partial repair (u=2), and replacement (u=4). Assume also that the reliability threshold is given as F^* = 99%. Table 1 contains the actual values of the objective functions. Here g_2 and g_4 are given in years, g_5 is given in thousands of dollars.

Table 1 Objective Table

u	$g_1(u)$	$g_2(u)$	$g_3(u)$	$g_4(u)$	$g_5(u)$
0	0.0591	4	1.0607	1.5360	5.3
2	0.3579	2	0.7500	3.5360	15.9
4	1.0000	0	0.0000	5.5360	50.5

Here we used that $F^{-1}(F^*) = F^{-1}(0.99) \approx 5.5360$. The utility values are given in Table 2 which are subjective judgments given by the manager.

Table 2 Utility Values Table

u	$U_1(g_1(u))$	$U_2(g_2(u))$	$U_3(g_3(u))$	$U_4(g_4(u))$	$U_5(g_5(u))$
0	0.1	0.1	0.1	0.1	0.1
2	0.4	0.3	0.5	0.5	0.7
4	1.0	0.9	1.0	0.8	0.1

The given weights are as follows:

$$c_1 = 0.2 \quad c_2 = 0.1 \quad c_3 = 0.2 \quad c_4 = 0.2 \quad c_5 = 0.3 \tag{14}$$

so the composite objective values are computed as the weighted averages of the objectives:

$$g(0) = 0.2(0.1) + 0.1(0.1) + 0.2(0.1) + 0.2(0.1) + 0.3(1.0) = 0.37 \tag{15}$$

$$g(2) = 0.2(0.4) + 0.1(0.3) + 0.2(0.5) + 0.2(0.5) + 0.3(0.7) = 0.52 \tag{16}$$

$$g(4) = 0.2(1.0) + 0.1(0.9) + 0.2(1.0) + 0.2(0.8) + 0.3(0.1) = 0.68 \tag{17}$$

Since the last value is the largest, u=4, the replacement is the best option instead of minimal or partial repair.

CONCLUSIONS

This paper introduced a multiobjective optimization model for finding the best action of maintenance/repair or replacement of a failed or largely degraded equipment. Five objectives were considered, and the weighting method was used with utility values replacing the actual objectives. A numerical example illustrated the methodology.

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