

Using Higher-Order Neural Networks to Detect Financial Statements Fraud

Salem Lotfi Boumediene, College of Business, Montana State University Billings
Salma Boumediene, College of Business, Montana State University Billings

ABSTRACT

Fraud detection is a real problem hard to resolve given the multiple types and motivations a fraud can observe. Many financial institutions are deeply affected each year and the fraud loss growth is alarming. This study aims to detect managerial fraud in the Tunisian banks before its occurrence. We evaluate the use of Higher-Order Neural Networks (HONN), for fraud detection. HONN can solve problems that are not linearly separable and it is extensively used for pattern classification, recognition, prediction and approximation. Thus, this article introduces the subject and presents the HONN techniques used for fraud detection to recognize behavioral fraudulent patterns and to detect "suspicious" operations to predict banks that commit fraud. This study shows that performance ratios are the most important variables that detect Financial Statements Fraud of Tunisian Banks as well as the core capital (leverage) ratio.

INTRODUCTION

Artificial Neural networks (ANNs) are widely used in real life applications. Many financial institutions use ANNs for applications such as bankruptcy prediction or fraud detection (Kay et al, 2006). While these ANN have been profitable for these institutions, they proved to be unable to handle discontinuities in the input training dataset and are unable to explain their behavior (Zhang et al, 2002). In order to detect and prevent fraud, many methods have been deployed Spann (2014). Let's take a closer definition to fraud meaning first. Garner (2004) defines fraud as "A knowing misrepresentation of the truth or concealment of a material fact to induce another to act to his or her detriment". Nevertheless, the professional and academic literature defines fraud in the financial statements differently. The American Institute of Certified Public Accountants (AICPA) in the Statement on Auditing Standard (SAS) N°99 -Consideration of Fraud in a Financial Statement Audit- refers to fraud as "an intentional act that results in a material misstatement of the financial statements that are the subject of an audit". In the SAS 99, two types of fraud are considered. First type of fraud are misstatements arising from fraudulent financial reporting such as falsification of accounting records or intentional omission from the financial statements of events, transactions, or other significant information. The second types of fraud are misstatements arising from misappropriation of assets such as theft of assets, embezzling receipts or causing an entity to pay for goods or services that have not been received.

The International Federation of Accountants (IFAC) devoted a whole standard for auditor responsibility relating to fraud. In fact, the International Standard on Auditing (ISA) 240 (2009) defines fraud as "an intentional act by one or more individuals among management, those charged with governance, employees, or third parties, involving the use of deception to obtain an unjust or illegal advantage".

The results of the latest report published by the Association of Certified Fraud Examiners (ACFE) (2014) are alarming. Indeed, the lighthouse observation of this report is that fraud costs 5% of revenues each year. This can be translated, if applied to the 2013 estimated Gross World Product, to a potential projected global fraud loss of nearly \$3.7 trillion.

The Committee of Sponsoring Organizations of the Treadway Commission (COSO) (2010), in its third report published in 2010, showed that for a sample of 347 fraudulent companies, the median fraud is \$12.1 million. For 30 cases of fraud, each case includes anomalies or misappropriation of \$500 million or more.

The study of fraud in financial statements of public companies in Tunisia is needed especially after 2011 revolution. In fact, cases of fraudulent financial reporting, misappropriation of assets or embezzlement are or had been in courts.

This study focuses on the Tunisian banks since it came out that the banking sector had been subject to misuse of funds in form of granting large credits for projects without securing them or at an interest lower than it should be.

This paper proceeds as follows. Section 2 presents a brief review of literature. Section 3 presents the methodology. Section 4 presents the results. Section 5 concludes.

LITERATURE REVIEW

The common motivations for companies to commit financial statements fraud are numerous. Economic incentives are common cases of fraud in the financial statements, as well as psychotic motivations, self-centered, or ideology. All these motivations can play an important role of fraud in the financial

statements. Pressures and economic incentives to match analysts' forecasts are fundamental motivations for listed companies subject to financial fraud. The psychological motivations associated with criminal behavior are very rare in our case. Egocentric motivations are outlined up in the fact that through fraud the person increases his personal prestige. In reality, the desire of managers to fulfill a functional authority in society is this type of motivation. Ideological motivations encourage executives to think that through fraud, they can become market leaders and consequently, improve their position in the society. The managerial fraud and companies' performance have been separated, each one had its own theoretical framework. According to Griffin & Lopez (2005), the research of management illegal behavior had produced a variety of models and definitions.

Fraud in the financial statements is made, if the company has strong incentives, as well as economic reasons to announce a more favorable financial performance than it really should be, in accordance with Generally Accepted Accounting Principles (GAAP). Empirical investigations (Carter & Stover (1991) and Latham & Jacobs (2000)) identified two fundamental variables, managerial ownership and the debt limit, which affect the extent of fraud in the financial statements. These studies have shown that when managerial ownership is between 5-25%, the opportunistic behavior of managers is expected and that the likelihood of engaging in financial statement fraud is higher. Previous research (Carcello & Palmrose (1994); Dechow et al. (1996) and Lys & Watts (1994)) focused on examining measures of financial difficulties in terms of weak financial conditions and weak financial performance as motivational mechanisms. The conclusions reached by this researches, argue that the motivations of commitments of fraud in the financial statements increase when the firm encounters financial difficulties. The researchers found that the chance to engage in the financial statements fraud increase when the company financial conditions and performance deteriorate.

According to the COSO Report (2010), the Securities and Exchange Commission (SEC) provided discussion in the Accounting and Auditing Enforcement Releases (AAERs) about the alleged motivation for the fraud. Among most commonly cited reasons summarized by the SEC in the AAERs include committing the fraud to 1.) Meet external earnings expectations of analysts and others, 2.) Meet internally set financial targets or make the company look better, 3.) Conceal the company's deteriorating financial condition, 4.) Increase the stock price, 5.) Bolster financial position for pending equity or debt financing, 6.) Increase management compensation through achievement of bonus targets and through enhanced stock appreciation and 7.) Cover up assets misappropriated for personal gain.

Studies developed on banks financial statements fraud are very rare, although the 2014 report of the ACFE shows that the banking and financial services are leading organizations victim by generating 17.8% of fraud cases. Moreover, the report shows that the managerial fraud ranks first and second in the ranking of fraudsters. Indeed, there is a strong correlation between the fraudster function within the company that suffered from the fraud and the losses that are caused by the fraud. The median loss caused by the owner / manager is more than three times the loss caused by the managers, and more than nine times the losses caused by the employees. Ramage et al. (1979) noted that financial institutions have different characteristics of errors than other sectors. Palmrose (1988) and St. Pierre & Anderson (1984) showed that about 30% of trials involved banks and loans institutions auditors. Kreutzfeldt & Wallace (1986; 1990) noted that the characteristics of inaccuracies in terms of error rate and false accounts vary across sectors. For example, banks are exposed to significantly higher error rates than other sectors companies in the liquidity accounts. In the same idea, Maletta & Wright (1996) examined 36 commercial banks and 14 savings and loan institutions. They showed that they are assigned the highest error percentage that overstated net income of about 68.8%.

METHODOLOGY

In Managerial fraud detection (Carter & Stover (1991) and Latham & Jacobs (2000)) identified two fundamental variables, managerial ownership and the debt limit, which affect the extent of fraud in the financial statements. Several technologies are used to detect fraud, like Data mining, Expert System, McAteer (2013), etc. Unfortunately, these are not capable detect accurately the fraud before occurrence.

In our last research (Boumediene. and Boumediene, 2015), we used Artificial Neural Networks (ANN) namely the Multilayer Perceptron Network (MLP), to detect managerial fraud before its occurrence. ANN technology is totally based on the human brain working concept, giving the potential to a computer to achieve a human-like performance, faced to a new situation. The technique of learning through past experience, as human brain does for decision making, is applied to ANN technique when given new events. Many studies (Lippmann, 1987 ; Radhika & Shashi, 2009 ; Widrow & Lehr 1990), have shown MLPs are much slower than other ANNs (feed forward network, i.e.) and require rigorous repetitive training process and a long time in order to converge. Moreover, in traditional ANNs, interpreting the results of training in a meaningful way, is not obvious. As the number of hidden layers increases and the number of neurons increases, the ANN becomes harder to understand, acting like a black-box. To overcome these limitations some researchers have proposed the use of Higher Order Neural Networks (HONNs) (Redding et al, 1993).

In order to improve the limitations and overcome the difficulties encountered in our last study, we consider in this paper to detect managerial fraud before its occurrence using the High Order Neural Network (HONN).

In our last study (Boumediene. and Boumediene, 2015), we used the the Multilayer Perceptron Network (MLP). MLP, also known as multilayer feed-forward network, which is widely used for pattern classification, recognition, prediction and approximation. MLP success is due to the significant benefits on resolving several types of problems.

Setting up a reliable ANN is hard and takes time, and the learning process of the ANN necessitates a huge effort. First we decide of the number of hidden layers, and the number of neurons in each layer. Then we affect randomly the weights for the connections between the neurons. A variety of activation functions and training criteria may be used. The output layer uses the softmax activation function, and Cross-Entropy as a training criteria (Boumediene & Boumediene, 2015).

The softmax activation function is a normalized exponential function can be expressed as follows:

$$y_j = \frac{e^{z_j}}{\sum_{i=1}^n e^{z_i}} \quad (1)$$

y_j is the desired output and $0 \leq y_j \leq 1$.

Given C different classes and softmax output, the minimum value represents the entropy of the target values and expressed as follows:

$$E = \sum_p \sum_{k=1}^c t_k^p(x_p) \log(y_k(x_p)) \quad (2)$$

t_k^p is the desired output, the value of the k-th component of the target for the n-th sample.

We chose the supervised learning process which is aimed to fit the ANN to the training set provided

(Boumediene. and Boumediene, 2015). We prepare our learning sample set, providing the inputs and the matching outputs sets, and wait for the training results. We need to provide a consistent amount of data in order to ensure that the ANN is accurately trained and produces the target result, when providing new inputs. Every time, we get an incorrect output, we need to change the connections between the neurons, update the weights following the rule described by Schaul et al. (2013), until the ANN makes the right match and a we get the desired output. Training the ANN until the that the network has converged requires a high number of iterations, and a huge effort of iterative adaptation of the weights, which is time consuming. And more hidden layers we have, and more neurons we have, more complex this operation will be.

The weight function is described by the following equation:

$$f(x) = \sum_i w_i B_i(x) \quad (3)$$

In traditional ANN, it's very hard to interpret the results of training in a meaningful way, especially when the number of hidden layers increases and the number of neurons increases. The ANN acts like a black-box, hard to understand. To overcome these limitations some researchers have proposed the use of Higher Order Neural Networks (HONNs) (Redding et al, 1993).

The period and selection of the sample

The period chosen is based on the year of fraud; it generally extends from 1999 to 2010. For our analysis, we took into account one-year period prior to the occurrence of the fraud for fraudulent and non-fraudulent bank. We consider that a bank commits a fraud when the Financial Market Council (the Tunisian equivalent of the SEC) or the Government Accountability Office announced the occurrence of the fraud or its external auditors issued an adverse opinion to the financial statements.

The data were collected directly from the web sites of the banks or from the printed annual reports available at the library of Central Bank of Tunisia (BCT). The sample consists of 10 Tunisian banks over a period of 12 Years. Table 2 gives details on the descriptive statistics of the sample chosen, including the number of observation, the minimum, the maximum, the mean and the standard deviation of each independent variable, presented in Table 1.

Table 1. Variables in the study

Variable	Definition
Performance ratios	
ASTEMPM_1	Assets per employee
EEFFR_1	Efficiency ratio
IDDIVNIR_1	Cash dividends to net income
IDLNCORR_1	Net loans and leases to core deposit
INATRESSR_1	Loss allowance to loans
INLSDEPR_1	Net loans and leases to deposits
INTEXPY_1	Cost of funding assets
INTINCY_1	Yield on earning assets
NIMY_1	Net interest margin
NOIJY_1	Net operating income to assets
NONIY_1	Noninterest income to earning assets
NONIXY_1	Noninterest expenses to earning assets
ROA_1	Return on assets
ROE_1	Return on equity
ROEINJR_1	Retained earnings to average equity
Growth ratios	
ASTEMPM_1	Assets per employee
EQV_1	Equity capital to assets
ROLLPS5TA_1	Growth ratio 1
Capital ratios	
EQV_1	Equity capital to assets
RBC1AAJ_1	Core capital (leverage) ratio

Table 2. Descriptive Statistics of the sample

	N	Minimum	Maximum	Mean	Std. Deviation
Total Asset	120	902,862,000	6,753,589,000	2,802,904,145	1,461,116,459
Number of Employees	120	781	5,826	1,645.2	832.21
ASTEMPM	120	786,516	4,578,061	1,807,938	787,827
EEFFR	120	-0.0071	0.0403	0.0083	0.0069
IDDIVNIR	120	0.0000	349.90	3.303	31.908
IDLNCORR	120	0.7660	1.5238	1.125	0.1815
INATRESSR	120	-0.0012	0.1178	0.0142	0.0142
INLSDEPR	120	0.7660	34.629	1.806	3.943
INTEXPY	120	0.0175	0.0548	0.0310	0.0062
INTINCY	120	0.0584	0.1231	0.0807	0.0100
NIMY	120	0.0320	0.0787	0.0492	0.0102
NOIJY	120	-0.1027	0.0377	0.0101	0.0151
NONIY	120	0.0097	0.0506	0.0209	0.0059
NONIXY	120	0.0000	0.0041	0.0008	0.0007
ROA	120	-0.1035	0.4349	0.0152	0.0546
ROE	120	-0.0281	9.423	0.1720	0.8533
ROEINJR	120	0.0000	0.2977	0.0617	0.0484
ASTEMPM	120	786,516	4,578,063	1,807,938	787,827
EQV	120	0.0330	0.1748	0.0958	0.0287
ROLLPS5TA	120	0.5168	0.9568	0.8502	0.0693
EQV	120	0.0330	0.1748	0.0958	0.0287
RBC1AAJ	120	4.720	29.348	10.485	4.144
Valid N (listwise)	120				

High Order Neural Network (HONN)

The HONN is one among the types of Artificial Neural Networks, that captures higher combination of nonlinear inputs and contains processing units that can efficiently perform various functions (such as polynomial, smoothing, trigonometric functions) Giles & Maxwell (1987) and can make more complex decision Selviah et al. (1991). HONNs lead to reduced network size, thus faster convergence, and more accurate outputs, compared to other types of more complex ANNs Zhang et al. (2002).

HONN uses a higher correlation of input neurons which often leads to a higher number of learning parameters(weights). A HONN neuron contains summing unit and product units that multiply their inputs (preprocessing of the neuron inputs) as well as the activation functions the and the connections to more than one layer Zhang (2008). Unlike the typical ANN, HONN do not require hidden layers. Thus, its structure becomes simpler and initialization of the weights easier to set Chen and Chang (1996). Unlike the typical MLP or feedforward neural network, HONNs can use just one layer of trainable weight to achieve nonlinear separate sets.

Various types of HONNs are used in different research areas, and are successfully used in in many applications such as prediction, classification, pattern recognition, etc. The two major forms of HONN are Sigma Pi Neural Network (SPNN) and Pi-Sigma neural network (PSNN). PSNN avoids the exponential increase of the number of parameters, by using fewer weights and less processing units than HONNs. Despite of that, they achieve to include indirectly all the capabilities and the strengths of HONNs.

The networks use the sum of product of inputs, whereas PSNN uses the product of the sum of the inputs along with the linear summation of a single hidden layer, and then, the product of processing units at output layer. Kang et al. proposed an online gradient algorithm for PSNN with stochastic inputs with improved computational efficiency.

Rumelhart et al. (1986) developed Sigma Pi neurons by applying the standard BackPropagation algorithm, proving that the generalized standard BackPropagation algorithm can be applied to simple additive neurons.

Many different HONN models have been developed: Polynomial, Trigonometric and others by Zhang & Fulcher (2004). Zhang (2008) also developed six different HONN models including: Polynomial Higher Order Neural Network (PHONN), Trigonometric Higher Order Neural Networks (THONN), SINC Higher Order Neural Network (SINCHONN), Sigmoid Polynomial Higher Order Neural Network (SPHONN), Ultra High Frequency Cosine and Sine Higher Order Neural Network (UCSHONN), SINC and Sine Polynomial Higher Order Neural Network (SXSPHONN).

HONNs allows us to to combine the inputs in a nonlinear way and the HONNs are capable of higher order correlation, and thus making higher quality inference. At the opposite of ANN, HONNs are able to provide some explanation for the produced output and always converge and have better accuracy than Statistical Analysis System Nonlinear (SAS NLIN) models Zhang (2008).

DATA AND METHODOLOGY

A HONN can be defined as a collection of higher order logic unit, called neurons. In a HONN each higher order neuron involves polynomial and acquires aggregation of various higher order weights in a polynomial form. The philosophy behind, includes higher order correlations in the aggregation function. A simple HONN can be described a collection of units represented by the following equation:

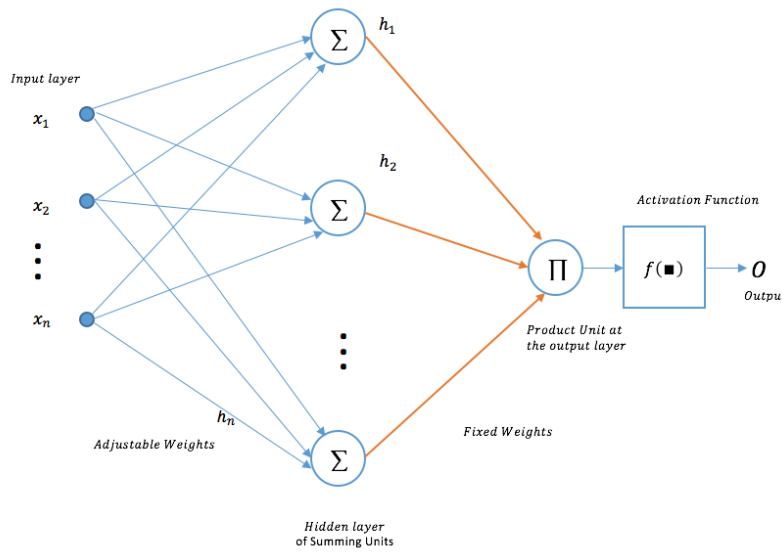
$$y_i = \int [T_0(i) + T_1(i)] + \dots + T_k(i)$$

Where:

- y: the output of the ith high-order neuron unit,
- \int : a sigmoid function,
- $T_j(i)$: the jth order term for unit i,
- k : the order of the unit.
- $T_0(i)$: an adjustable threshold.

For this study, we use the Pi-Sigma Neural Networks (PSNN), which deploys the higher order terms at the output layer. The PSNN is a feedforward neural networks with a single hidden layer. The n^{th} order term is defined by a linear weighted sum over the product of the inputs. The PSNN's output is computed by the product of sum of the input units at the output layer and pass it to a nonlinear function. The weights connected from the input layer to the hidden layer are trainable weights, that will be adjusted during the training and the weights connecting the hidden layer to the output layer are fixed weights, set to 1. Figure 1 depicts a Pi-Sigma Neural Network of k -th order. Bias nodes are not shown for reason of simplicity. The network's order depends on the number of the summing units, i.e., a second order network contains two summing units, a third order network contains three summing units etc. The total number of trainable weights connections for a k -th order PSNN with a dimension= n inputs is expressed as $(n+1)*k$.

Figure 1. Pi-Sigma Neural Network of k -th order



Given the input vector $X = (x_0, x_1, \dots, x_j, \dots, x_n)^T$ of dimension $n + 1$, with an additional bias unit B_j , x_j denotes the j^{th} component of the input vector X . For each j^{th} component, the $(n + 1)k$ dimensional weight vectors are summed at a layer of k summing units. Let w_{ij} be the weight vector, can be expressed by:

$$w_{ij} = (w_{ij0}, w_{ij1}, w_{ij2}, \dots, w_{ijn})^T, i = 1, 2, \dots, k$$

where

k is the corresponding order of the network.

At the hidden layer h_j , the output can be computed by:

$$h_j = B_j + \sum w_{ji} x_i$$

where

B_j is the additional bias unit for the j^{th} component,

w_{ij} represents the weight from the input to the summing unit.

The output O of the PSNN is computed by:

$$O = f \left(\prod_{i=1}^K \sum_{j=1}^N (W_{ji} X_j + W_{i0}) \right)$$

where:

- K is the number of summing unit,
- N is the number of input nodes,
- f is a nonlinear transfer function,
- W_{ji} are adjustable weights,
- W_{i0} are the biases of the summing units,
- X_j is the input vector.

Initially we wanted to compare the performance on the HONN in Fraud detection compared to MLP. The network was used to learn a set of historical data over a period of 12 Years (1999 to 2010), obtained from the printed annual reports of 10 Tunisian banks.

In simulating and predicting fraud data, we propose FHONN (Fraud higher order Neural Network), a two layers Network, the product and the summing unit layers. The weights between the input nodes and the summing unit layer are trainable, whereas the weights between the summing and the product unit layers are unchanged set to 1. As FHONN has only one layer of trainable weights, the training time would be extremely reduced.

Our input vector, $X = (x_0, x_1, \dots, x_j, \dots, x_n)$, is constituted of $n=76$ variables. X 's dimension= $n + 1$, as it includes the bias b_0 for the input layer. b_0 has been initialized to 1. We have only one output in the output layer, expressed in terms of 0/1; 0 in case no fraud is detected and 1 in case a fraud is detected.

We used the supervised training, with the gradient-descent training techniques, which minimize the squared error between the actual outputs of the network and the desired outputs. The learning algorithm used is based on the standard back propagation gradient descent algorithm for estimating the mean squared error (MSE), calculated as follows:

$$MSE = \frac{1}{N} \sum_{i=1}^N (d^i - y^i)^2$$

where y^i is the actual outputs of the network and d^i is the desired outputs.

We trained the network several times, updating the weights each time, in order to get the desired accuracy for the validation sample, and obtain an MSE (mean squared error) for the simulation and prediction close to zero.

RESULTS AND DISCUSSION

Our experiments are made on the original dataset, used in our previous studies. It consists of 120 financial data for of 10 Tunisian banks. The dataset uses 65% of data as training instances, 20% as validation instances and 15% as testing instances which are randomly selected over dataset. In fact, for providing a

good comparison with previous works we use 15% of the dataset for testing our network. In order to confirm our network performance, we need to choose validation samples over the dataset for our training stop epoch. For probing the robustness of the method over the various testing instances, we repeat the method 50 times on each dataset. In contrast to our previous study, no company was excluded for this study. Table 3 presents the case processing summary for the FHONN.

Table 3. Case Processing Summary

Sample	Training	N	Percent
	Testing	78	65%
	Holdout	24	20%
Valid		18	15%
Excluded		120	100.0%
Total		0	
		120	

The following table (Table 4) offers a comparison of the processing summary between the FHONN and the MLP, used in our previous study.

Table 4. Comparison of the processing summary between the FHONN and MLP

		FHONN	MLP
Sample	Training	N	N
	Testing	78	75
	Holdout	24	20
Valid		18	15
Excluded		120	110
Total		0	10
		120	120

The network convergence was pretty fast, 2 epochs were needed to reach an optimal MSE of $1e^{-7}$. At the end, the bias b_0 for the input layer has been adjusted to 0.1216.

The practical results obtained by using FHONN are presented in the classification table (Table 5). For each case, the predicted network response is 1 if the predicted pseudo-probability is greater than 0.5. Figure 2 shows the classification for each sample. The cells on the diagonal of the cross-classification of cases are correct predictions and the cells off the diagonal of the cross-classification of cases are incorrect predictions. We can see in Table 5 over the 21 cases who previously had committed fraud, 99% are classified correctly. Over the 54 cases that had not committed fraud, 99% are classified correctly. Overall, 99% of the training cases are classified correctly, shown in Table 5.

Table 5. Classification

	Predicted Percent Correct	
	FHONN	MLP
Training	99%	92%
Testing	99%	90%
Validation	99%	86%

It's clear from the classification table, that FHONN outperforms MLP. Table 6 shows the FHONN performance.

Figure 2. Classification of each sample

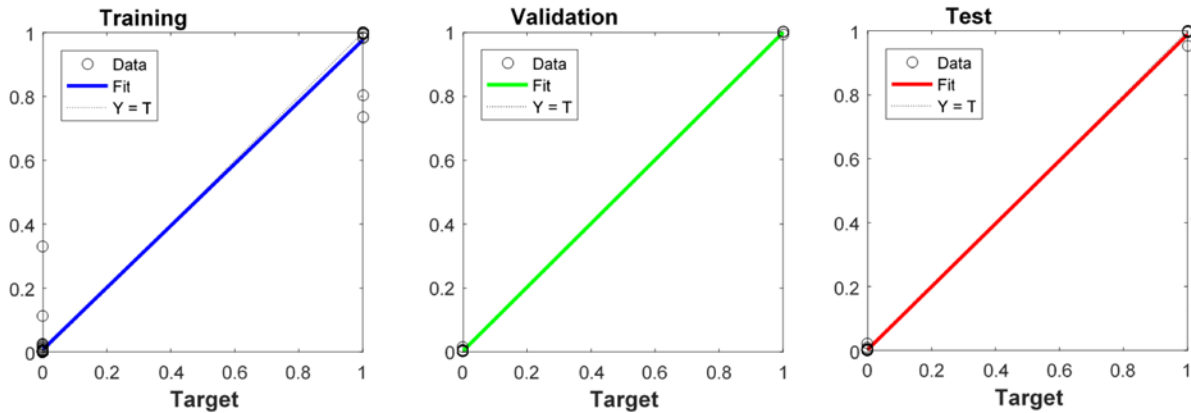


Table 6. FHONN performance

	R^2
Training	0.9931
Testing	0.9997
Validation	0.9999
Overall	0.9951

Table 6 proves that the FHONN is perfectly trained and ready for the final test. For the final test, aimed to confirm the FHONN performance, we selected validation samples over the dataset that will allow us to validate our FHONN. The selected sample consists of 24 financial data that have caused some confusion in fraud detection. We compared the desired output with FHONN's output, and the result is very satisfactory. Table 7 presents the results obtained by our FHONN compared to the output results.

Table 7. FHONN Output compared to the desired results

Validation Sample #	Desired Output	Unformatted FHONN Output	Formatted FHONN Output
1	1	0.80435	1
2	1	0.99946	1
3	0	0.001838847	0
4	1	0.999545624	1
5	0	0.01368201	0
6	1	0.999549134	1
7	0	0.000052261	0
8	1	0.99982729	1
9	1	0.999828506	1
10	0	0.000310547	0
11	0	0.00088648	0
12	0	0.004286824	0
13	0	0.000036745	0
14	0	0.001652697	0
15	0	0.330396034	0
16	0	0.00004182	0
17	0	0.00008024	0
18	0	0.000158242	0
19	0	0.000108747	0
20	0	0.000646023	0
21	0	0.00036711	0
22	0	0.000124985	0
23	0	0.0001308	0
24	1	0.95345766	1

Table 7 shows that FHONN has a high aptitude to simulate extremely accurate outputs and this potential is more noticeable when we compare the formatted output to the desired output. We can conclude that our FHONN has a prediction accuracy of 100%. We can conclude that FHONN have better modeling accuracy than PLM modeling result.

CONCLUSION

The preliminary results suggest that PHONN is stable, reliable and our trained Pi-Sigma network exhibited good generalization capabilities. The convergence is fast and the prediction accuracy is noticeable, therefore PHONN shows considerable promise as a tool helping in fraud detection.

PHONN exhibited high classification accuracy. Unlike the Multi-Layer Perceptron (MLP), the major advantage of HONNs is that only one layer of trainable weights is needed (Park et al., 2000). Compared to neural networks that only uses summation units, i.e., MLP, the HONNs use higher order terms that increase the information capacity of the Neural network. Thus, we don't need to build networks with larger units and more hidden layer to solve the same problem. As shown by Leerink et al. (1995), higher order terms can help in solving complex problems with smaller units' network along with fastest learning and convergence.

FHONN's ability to predict the fraud detection was tested and compared to the Multilayer Perceptron (MLP). Simulation results proved that FHONN's has superior forecast capacity compared to MLP with lower prediction errors.

REFERENCES

- Association of Certified Fraud Examiners (ACFE). "Report to the nations on occupational fraud and abuse", 2016.
- Boumediene, S. L. "Detection and Prediction of fraud in the financial statements of Tunisian banks". *Accounting & Taxation*, 2014, 6(2), 1-10.
- Carcello J. V., Palmrose Z. V. "Author litigation and modified reporting on bankrupt clients". *Journal of Accounting Research*, 1994, 32(Supplement), 1–30.
- Carter, R., Stover R. "Management ownership and firm value compensation policy: evidence from converting savings and loan associations". *Financial Management*, 1991, (Winter), 80–90.
- Chen, C.T., & Chang, W.D. (1996). A feedforward neural network with function shape autotuning. *Neural Networks*, 9(4), 627-641
- Committee of Sponsoring Organizations of the Treadway Commission (COSO). "Fraudulent Financial Reporting: 1998-2007, An Analysis of U.S. Public Companies", 2010.
- Dechow, P. M., Sloan, R. G., Sweeney, A. P. "Causes and Consequences of Earnings Manipulation: An Analysis of Firms Subject to Enforcement Actions by the SEC". *Contemporary Accounting Research*, 1996, 13(1), 1-36.
- Garner, B. (ed.). "Black's Law Dictionary". 8th Ed., s.v. "fraud", 2004.
- Griffin, R. W., & Lopez, Y. P. "Bad behavior" in organizations: A review and typology for future research". *Journal of Management*, 2005, 31(6), 988-1005.
- IFAC. "The Auditor's Responsibilities Relating to Fraud in Audit of Financial Statements". 2009. Retrieved July 25, 2013 from <http://www.ifac.org/sites/default/files/downloads/a012-2010-iaasb-handbook-isa-240.pdf>
- Kang, X. , Xiong, Y., Zhang, C., & Wu, W. (2007) .Convergence of Online Gradient Algorithm with Stochastic Inputs for Pi-Sigma Neural Networks, *Proceedings of the 2007 IEEE Symposium on Foundations of Computational Intelligence (FOCI)*, 564–569, 2007.
- Kay, A. (2006). Artificial neural networks. *Computerworld*. Retrieved on 27 Sept 2016 from <http://www.computerworld.com/article/2591759/app-development/artificial-neural-networks.html>
- Kreutzfeldt, R. W., Wallace, W. A. "Error Characteristics in Audit Populations: Their Profile and Relationship to Environmental Factors". *Auditing: A Journal of Practice & Theory*, 1986, 6 (1), 20-43.
- Kreutzfeldt, R. W., Wallace, W. A. "Control Risk Assessments: Do They Relate to Errors?". *Auditing: A Journal of Practice & Theory*, 1990, 9(Supplement), 1-26.
- Latham C., Jacobs F. "Monitoring and incentives factors influencing misleading disclosures". *Journal of Managerial Issues*, 2000, 12(2), 169–187.
- Leerink, L. R., Giles, C. L., Horne, B. G. & Jabri, M. A. (1995). *Learning with product units*, *Advances in Neural Information Processing Systems 7* (pp. 537-544). Cambridge, MA: MIT Press.

Lippmann, R.P. (1987). An introduction to computing with neural nets, *IEEE ASSP Magazine*, 4–22.

Lys, T., Watts, R. “Law suits against auditors”. *Journal of Accounting Research*, 1994, 32(Supplement), 65-93.

Maletta, M. J., Wright, A. “Audit Evidential Planning: An Examination of Industry Error Characteristics”. *Auditing: A Journal of Practice and Theory*, 1996, 15(Spring), 71-86.

McAteer, M. J. Financial Ratios as Predictors of Financial Fraud in community Banks: The Analytic Model of Management Fraud”. Retrieved July 10, 2013 from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1430984.

Palmrose, Z. “An analysis of auditor litigation and audit service quality”. *The Accounting Review*, 1988, LXIII(1), 55-73.

Ramage, J. G., Krieger, A. M., Spero, L. L. “An Empirical Study of Error Characteristics in Audit Populations”. *Journal of Accounting Research*, 1979, 17(Supplement), 72-107.

Ravisankar, P., Ravi, V., Rao, G. R., Bose, I. “Detection of Financial Statement Fraud and Feature Selection Using Data Mining Techniques”. *Decision Support Systems*, 2011, 50(2), 491-500.

Radhika, Y., & Shashi, M. (2009). Atmospheric temperature prediction using support vector machines, *Int. J. Comput. Theory Eng.*, 1 (1), 55–58.

Redding, N.J., Kowalczyk, A., & Downs, T. (1993). Constructive high-order network algorithm that is polynomial time. *Neural Networks*, 6(7), 997- 1010.

Rumelhart, D., Hinton, G., & McClelland, J. (1986). Learning internal representations by error propagation. In Rumelhart, D., & McClelland, J. (Eds.) *Parallel distributed processing: explorations in the microstructure of cognition, Volume 1: Foundations*. Cambridge, MA: MIT Press.

St. Pierre, K., Anderson, J. A. “An analysis of the factors associated with lawsuits against public accountants”. *Accounting Review*, 1984, LIX(2), 242-263.

Schaul T., & LeCun Y. (2013). Adaptive learning rates and parallelization for stochastic, sparse, non-smooth gradients. *Proceedings of the International Conference on Learning Representations (ICLR)*.

Schaul T., Zhang S.& LeCun Y. (2013). No more Pesky Learning Rates. *Proceedings of the International Conference on Machine Learning (ICML)*.

Spann D.D. (2014). *Fraud Analytics: Strategies and Methods for Detection and Prevention*, John Wiley & Sons, 176 pages.

Widrow, B., & Lehr, M.A (1990). 30 years of adaptive neural networks: perceptron, Madaline and backpropagation, *Proc. IEEE*, 78 (9), 1415–1422.

Zhang, M., Xu, S.X., & Fulcher, J. (2002). Neuron-adaptive higher order neural-network models for automated financial data modeling, *IEEE Transactions on Neural Networks*, 13(1), 188–204.

Zhang, M. (2008). *Artificial higher order neural networks for economics and business*. Hershey: IGI

Global.