MONITORING THE PROCESS QUALITY FOR MULTISTAGE SYSTEMS WITH CORRELATED CHARACTERISTICS

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ABSTACT

This study aims to provide a new approach for detecting the smaller shifts of multistage systems with correlated characteristics. Considering both the auto-correlated process outputs and the correlations occurring between neighboring stages in a multistage manufacturing system, we propose a new multivariate linear regression model to describe their relationship. Then, the multistage multivariate residual control charts are constructed accordingly. An overall run length concept is also adopted to evaluate the detecting performances of our proposed control charts. Finally, a numerical example with cascade data is used to demonstrate the usefulness of our proposed control charts in the Phase II monitoring.

Keywords: multistage systems, multivariate residual control charts, multivariate linear regression model, overall run length, Phase II monitoring

INTRODUCTION

Normally, modern manufacturing systems have become very sophisticated and possess correlated multiple quality characteristics. Such a multistage system requires multiple stages or stations to complete its final product or service. Although Pan *et al.* [5] proposed multistage residual exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts for monitoring the process quality of multistage manufacturing systems with a single quality characteristic under small sustained process shifts, a new multivariate control chart model suitable for detecting the process changes in multistage systems with correlated characteristics is still lacking. Thus, it becomes necessary to extend their research to a multivariate case. This study aims to develop the multistage residual multivariate EWMA (MEWMA) and multivariate CUSUM (MCUSUM) control charts for monitoring the correlated quality characteristics of multistage manufacturing systems.

LITERATURE REVIEW

In a multistage manufacturing system, the quality characteristics of interest are often highly dependent. To monitor quality of a process with multivariate variables, Asadzadeh *et al.* [1] proposed cause-selecting control charts for monitoring multistage processes when the outgoing quality characteristics followed normal distributions. Recently, Davoodi and Niaki [3] proposed a maximum likelihood method to estimate the step-change time of the location parameter in multistage processes. Bera and Mukherjee [2] proposed an integrated solution approach using multivariate regression, desirability function and a heuristic search strategy to solve multistage multiple response optimization problem. Pan *et al.* [5] considered a multiple-step process in which the observations in the 1st step/stage data can be fitted with the following autoregressive model of order 1 (AR(1)) model without loss generality:

$$Y_{1,j} = (1-\phi)\xi_{Y_{1,0}} + \phi Y_{1,j-1} + \varepsilon_{1,j} \ , \ j = 1,2,\ldots \tag{1}$$

Starting from the 2nd stage, the process variable for each stage of a multistage system can be written $\hat{Y}_{i,j} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1}Y_{i,j-1} + \hat{\beta}_{i,2}Y_{i-1,j}, i = 2,\ldots,\mathrm{W}$.The residual of the jth sample taken from the ith stage is defined as $e_{Y_{i,j}} = Y_{i,j} - \hat{Y}_{i,j} \ , i = 2,\ldots,\mathrm{W}$.

DEVELOPMENT OF MULTISTAGE RESIDUAL CONTROL CHARTS

The structure of a multistage system model with correlated quality characteristics

Let $\mathbf{Y}_{\mathrm{i,j}}$ be the quality characteristic vector for jth sample taken from ith stage. The structure of our multistage system model with correlated quality characteristics can be shown in Figure 1.

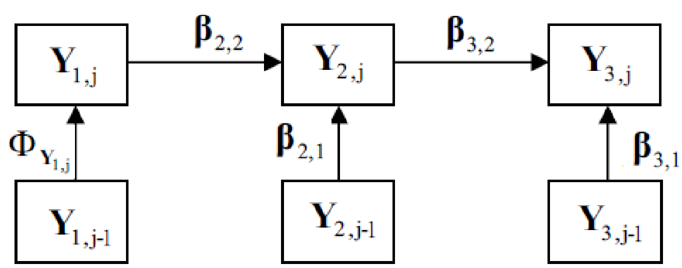

FIGURE 1. Structure of a multistage system model with correlated quality characteristics

Since the multivariate vector autoregressive model is commonly used to describe the multivariate correlated data, most researchers use it to represent variation transmissions in multistage processes with correlated multiple quality characteristics. In this study, we also assume the first stage data are fitted with the first order vector autoregressive (VAR(1)) time series model:

$$\mathbf{Y}_{1,j} = (\mathbf{I}-\mathbf{\Phi})\ \xi_{\mathbf{Y}_{1,0}} + \mathbf{\Phi}\mathbf{Y}_{1,j-1} + \boldsymbol{\varepsilon}_{1,j} \ , \ j = 1,2,\ldots \tag{2}$$

, where $\mathbf{Y}_{1,j}$ is the quality characteristic vector for the jth sample taken from the 1st stage and $\mathbf{Y}_{1,j-1}$is the quality characteristic vector for the $(j-1)$th sample taken from the 1st stage. $\boldsymbol{\xi}_{\mathbf{Y}_{1,0}}$ is the process mean vector of the 1st stage, $\boldsymbol{\Phi}$ is autoregressive parameter matrix. $\boldsymbol{\varepsilon}_{1,j}$ is assumed to be an independent normal random vector with mean vector $\mathbf{0}$ and variance matrix $\Sigma_{\boldsymbol{\varepsilon}_1}$. The estimated time series model can be expressed as

$$\hat{\mathbf{Y}}_{1,j} = \left(\mathbf{I} - \hat{\boldsymbol{\Phi}}\right) \boldsymbol{\xi}_{\mathbf{Y}_{1,0}} + \hat{\boldsymbol{\Phi}}\mathbf{Y}_{1,j-1}. \tag{3}$$

The residual vector of the jth sample taken from the 1st stage is defined as $\mathbf{e}_{\mathbf{Y}_{1,j}} = \mathbf{Y}_{1,j} - \hat{\mathbf{Y}}_{1,j}$. Note that the quality characteristic vector $\mathbf{Y}_{2,j}$ depends not only on the vector $\mathbf{Y}_{2,j-1}$ but also on the vector $\mathbf{Y}_{1,j}$. Starting from the 2nd stage, the quality characteristic vector for each stage of a multivariate system with multiple characteristics can be written as the following multivariate linear regression model.

$$\begin{cases} \mathbf{Y}_{2,j} = \boldsymbol{\beta}_{2,0} + \mathbf{Y}_{2,j-1}\boldsymbol{\beta}_{2,1} + \mathbf{Y}_{1,j}\boldsymbol{\beta}_{2,2} + \boldsymbol{\varepsilon}_{2,j},\ \mathrm{j} = 1,2,... \\ \mathbf{Y}_{3,j} = \boldsymbol{\beta}_{3,0} + \mathbf{Y}_{3,j\text{-}1}\boldsymbol{\beta}_{3,1} + \mathbf{Y}_{2,j}\boldsymbol{\beta}_{3,2} + \boldsymbol{\varepsilon}_{3,j},\ \mathrm{j} = 1,2,... \\ \vdots \\ \mathbf{Y}_{\mathrm{W}\text{-}1,j} = \boldsymbol{\beta}_{W-1,0} + \mathbf{Y}_{W-1,j-1}\boldsymbol{\beta}_{W-1,1} + \mathbf{Y}_{W-2,j}\boldsymbol{\beta}_{W-1,2} + \boldsymbol{\varepsilon}_{W-1,j},\ \mathrm{j} = 1,2,... \\ \mathbf{Y}_{\mathrm{W},j} = \boldsymbol{\beta}_{W,0} + \mathbf{Y}_{W,j-1}\boldsymbol{\beta}_{W,1} + \mathbf{Y}_{W-1,j}\boldsymbol{\beta}_{W,2} + \boldsymbol{\varepsilon}_{W,j},\ \mathrm{j} = 1,2,... \end{cases}$$

where $\mathbf{Y}_{i,j}$ is the random vector for the jth sample taken from the ith stage and $\mathbf{Y}_{i,j-1}$ is the random vector for the $(j-1)$th sample taken from the ith stage. $\boldsymbol{\beta}_{i,0}$ is a constant vector, $i = 2, \ldots, \mathrm{W}$. $\boldsymbol{\varepsilon}_{i,j}$ is assumed to be an independent normal random vector with mean vector $\mathbf{0}$ and variance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}_i}$, $i = 1, \ldots, \mathrm{W}$. $\boldsymbol{\beta}_{i,1}$ and $\boldsymbol{\beta}_{i,2}$ are the regression coefficient metrics for describing the within stage and between stages relationship of quality characteristic vectors respectively. Hence, the least square estimator of quality characteristic vector for the jth sample taken from the ith stage is given by

$$\hat{\mathbf{Y}}_{i,j} = \hat{\boldsymbol{\beta}}_{i,0} + \mathbf{Y}_{i,j-1}\hat{\boldsymbol{\beta}}_{i,1} + \mathbf{Y}_{i-1,j}\hat{\boldsymbol{\beta}}_{i,2}, i = 2, ..., \mathrm{W}. \tag{4}$$

The residual vector of the jth sample taken from the ith stage is defined as

$$\mathbf{e}_{\mathbf{Y}_{i,j}} = \mathbf{Y}_{i,j} - \hat{\mathbf{Y}}_{i,j},\ i = 2, ..., \mathrm{W}. \tag{5}$$

Simulation results for multistage residual MEWMA control charts

After conducting 10000 simulation runs, the average overall out-of-control run length (AOORL) is used to evaluate the detecting performances of multistage residual control charts with those of Phase II control charts while the average overall in-control run length (AOIRL) is fixed at 370. Without loss of generality, a multistage system is set as three (3) stages and the sample size is set as 3000 for each stage. In addition, the covariance metrics for each stage are assumed to be homogeneous and the amount of process mean vector shift for each stage are also assumed to be the same when a multistage system is out of control. To evaluate the performance of our proposed multistage residual MEWMA control chart, we vary $\phi = 0.25, 0.50, 0.75$ and define the shift of process mean vector as

$$\delta = (\boldsymbol{\mu}' \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu})^{1/2} \tag{6}$$

where μ is the standardized mean vector shift. Considering various process mean vector shifts δ = 0.25, 0.5,0.75,1.0, the simulation results of AOORL values for multistage residual MEWMA control charts under various combinations of autocorrelations, control parameters and the process mean vector shift are summarized. Our simulation results indicate that, for a given process mean vector and control parameters, AOORL value increases as the autocorrelation increases. In other words, the detecting abilities in terms of AOORL values show that the multistage residual MEWMA control chart performs better when the autocorrelation is smaller.

Simulation results for multistage residual MCUSUM control charts

As AOORL values are used to evaluate and compare the detecting abilities of multistage residual control charts, the simulation results of AOORL values for multistage residual MCUSUM control charts under various combinations of autocorrelations, control parameters and the process mean vector shifts are summarized. Our simulation results also indicate that, for a given process process mean vector shift and control parameter, AOORL value for multistage residual MCUSUM control chart increases as the autocorrelation increases, which means the detecting ability of the multistage residual MCUSUM control chart performs better when the autocorrelation is smaller.

NUMERICAL EXAMPLE

To demonstrate the practical application of our proposed multistage residual MEWMA and MCUSUM control charts, the cascade data provided by Montgomery [4, p. 514] with 40 observations is used for illustration purpose. There are nine input variables and two output variables in the cascade data. In this numerical example, we assume that the output variables are quality characteristics of interest and the 40 observations are taken from two stages (20 observations for each stage). The Pearson's correlation coefficients between two quality characteristics at stages I and II are -0.07 and 0.19 respectively.

Comparison between Phase II MEWMA and multistage residual MEWMA control charts

In this example, Phase II MEWMA and multistage residual MEWMA control charts using control parameter $(r, H) = (0.05, 10.54)$ are shown in Figure 2 and Figure 3 respectively if the process mean vector shift is set as $\delta = 0.50$. As one can see from Figure 2, Phase II MEWMA control chart cannot detect the mean vector shift until the 2nd stage and its overall out-of-control run length (OORL) value is

20. On the other hand, the multistage residual MWEMA control chart using the same control parameters is able to detect this mean vector shift at the 2nd stage and its OORL value is 11 (see Figure 3).

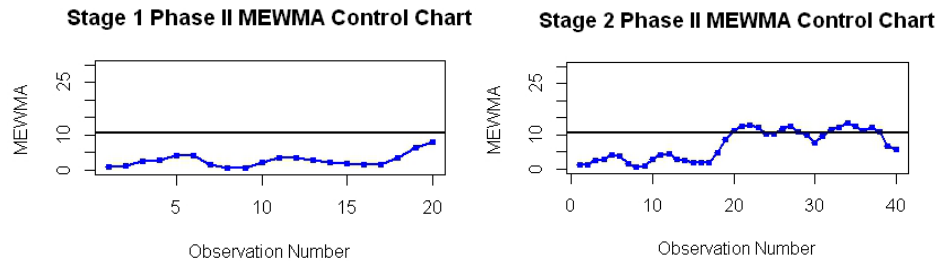

FIGURE 2. The Phase II MEWMA control chart using control parameter (r, H)= (0.05, 10.54) if the process mean vector shift is set as$\delta = 0.50$.

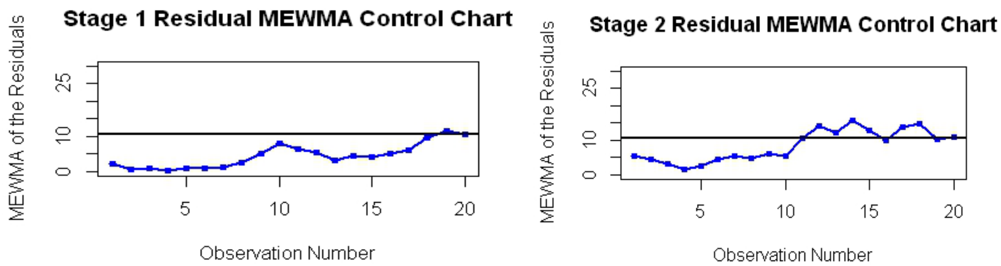

FIGURE 3. The multistage residual MEWMA control chart using control parameter (r, H)= (0.05, 10.54) if the process mean vector shift is set as $\delta = 0.50$.

The comparison results in terms of OORLs for Phase II and multistage residual MEWMA control charts under various mean vector shifts are summarized in Table 1. Table 1 indicates that the detecting abilities of our proposed multistage residual MEWMA control charts outperform those of Phase II MEWMA control charts.

TABLE 1. Comparison of OORLs between Phase II MEWMA and Multistage Residual MEWMA Control charts under various mean vector shifts (δ)

		δ			
	(r, H)	0.25	0.50	0.75	1.00
Phase II MEWMA	(0.05, 10.54)	--	20(II*)	19(I*)	11(I*)
	(0.10, 11.63)	--	20(II*)	19(I*)	11(I*)
	(0.15, 12.16)	--	19(II*)	19(I*)	10(I*)
	(0.20, 12.54)	--	19(II*)	19(I*)	10(I*)
	(0.25, 12.70)	--	19(II*)	19(I*)	10(I*)
Multistage MEWMA	(0.05, 10.54)	--	19(I*)	10(I*)	8(I*)
	(0.10, 11.63)	--	19(I*)	10(I*)	8(I*)
	(0.15, 12.16)	--	12(II*)	9(I*)	8(I*)

	(0.20, 12.54)	--	10(I*)	9(I*)	8(I*)
	(0.25, 12.70)	--	10(I*)	9(I*)	8(I*)

Note: (I*) indicates that multistage control chart can detect the mean shift at the1st stage.

Comparison between Phase II MCUSUM and multistage residual MCUSUM control charts

The Phase II MCUSUM control chart using control parameter $(h, k) = (11.50, 0.25)$ is shown as in Figure 4 when the process mean vector shift is set as $\delta = 0.50$. As shown in Figure 4, Phase II MCUSUM control chart cannot detect the mean vector shift until the 2nd stage. Its OORL value is 23.

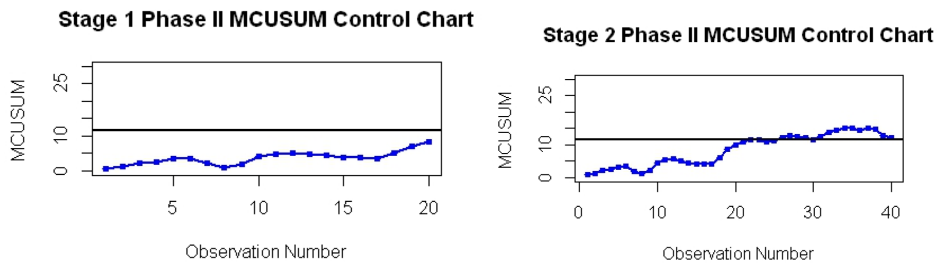

FIGURE 4. The Phase II MCUSUM control chart using control parameter (h, k)=(11.50, 0.250) when the process mean vector shift is set as $\delta = 0.50$

If the process mean vector shift is set as $\delta = 0.50$, the multistage residual MCUSUM control chart using the same control parameter is shown in Figure 5 . Note that the multistage residual MCUSUM control chart cannot detect this mean vector shift at the 1st stage. However, the multistage residual MCUSUM control chart will be able to detect the mean vector shift at the 2nd stage and its OORL value is 17.

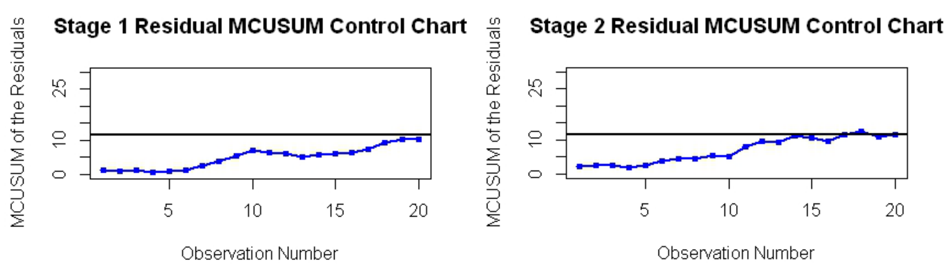

FIGURE 5. The multistage residual MCUSUM control chart using control parameter (h, k)=(11.50, 0.250) when the process mean shift is set as $\delta = 0.50$

In addition, the comparison results in terms of OORLs for Phase II and multistage residual MCUSUM control charts under various process mean vector shifts are summarized in Table 2. Based on the comparison results shown in Tables 1 and 2, we can conclude that the detecting abilities of our proposed multistage residual MEWMA and MCUSUM control charts outperform those of Phase II MEWMA and MCUSUM control charts.

TABLE 2. Comparison of OORLs between Phase II MCUSUM and multistage residual MCUSUM Control charts under various mean vector shifts (δ)

		δ			
		0.25	0.50	0.75	1.00
Phase II	(h,k)	(16.98, 0.125)	(11.50, 0.25)	(8.70, 0.375)	(6.98, 0.50)
MCUSUM		--	23(II*)	19(I*)	11(I*)
Multistage	(h,k)	(16.98, 0.125)	(11.50, 0.25)	(8.70, 0.375)	(6.98, 0.50)
MCUSUM		--	17(II*)	16(I*)	9(I*)

Note: (I*) indicates that multistage control chart can detect the mean shift at the1st stage.

CONCLUSIONS

In this study, we provide a new approach for detecting the small sustained process shifts in multistage systems with correlated multiple quality characteristics. After developing a multivariate linear regression model, the multistage residual MEWMA and MCUSUM control charts are constructed accordingly. An overall run length (ORL) concept proposed by Pan *et al.*[5] is also adopted in the simulation studies to evaluate the detecting performances for our proposed multistage residual MEWMA and MCUSUM control charts. The numerical example further shows that the detecting abilities of our proposed residual MEWMA and MCUSUM control charts outperform those of Phase II MEWMA and MCUSUM control charts. Hence, it further indicates that our proposed control charts provide a better decision making tool for correctly detecting the small sustained process shifts in multistage systems with correlated quality characteristics. Hopefully, this new approach can lead to the direction of continuous improvement for any product or service within a company.

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