# A MULTI-OBJECTIVE BILEVEL OPTIMIZATION MODEL FOR THE RELOCATION OF INTEGRATED AIR DEFENSE SYSTEM ASSETS

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## ABSTRACT

Given a subset of ground-based air defense weapon systems within an Integrated Air Defense System (IADS) that have been incapacitated, we formulate a multi-objective bilevel optimization model to relocate surviving assets to maximize an intruder's minimal exposure across a defended border region, minimize the maximum asset relocation time, and minimize the total number of assets requiring relocation. Our formulation also allows the defender to specify minimum coverage requirements for high-value asset locations and emplaced weapon systems. Adopting the ε-constraint method, we develop a single-level reformulation that enables the identification of Pareto-optimal solutions and identifies trade-offs between the competing objectives.

**Keywords:** Bilevel programming, Multi-objective optimization, Asset relocation, Minimal exposure path, Air defense

## INTRODUCTION

Unlike previously fielded air defense systems, emerging antiaccess/area-denial (A2/AD) IADS assets will be highly mobile, "with some systems demonstrating a 'shoot-and-move' time in minutes rather than hours or days" [12]. Instead of planning only the first salvo of strategic attacks against an enemy IADS, it is important to investigate and understand how an enemy may reposition its assets so that we can predict reactions to intended disruption of an IADS. The objective of this paper is to formulate a multi-objective bilevel optimization model to relocate surviving ground-based elements of an IADS and develop a reformulation that enables direct solution via a commercial solver.

## LITERATURE REVIEW

A majority of facility relocation problems in the literature are applied to the relocation of fire companies [7], ambulances [3], and emergency vehicles [6]. These works have also been extended from single-objective to multi-objective formulations. Sathe & Miller-Hooks [11] set forth a model to locate military units, police forces, and first responders, and to relocate idle units in response to an event, maximizing secondary coverage and minimizing cost. Melachrinoudis & Min [9] presented a multi-objective application involving the relocation and phase-out of a combined manufacturing plant and warehousing facility. The location and relocation of mobile servers in a transportation network were considered by Berman & Rahnama [2], wherein the authors sought to balance coverage, response time, and relocation costs. Recently, Paul et al. [10] provided a multi-objective, maximal conditional covering location problem applied to the relocation of hierarchical emergency response facilities to respond to large-scale emergencies. Incorporating ideas from facility relocation and multi-objective optimization will allow us to understand how an enemy IADS may adjust during conflict.

## MODEL FORMULATION & SOLUTION METHODOLOGY

In this section, we present a baseline formulation for the optimal relocation of IADS assets following an enemy attack. Given a specified set of surviving IADS assets, we determine the optimal layout that maximizes the minimal exposure of an intruder to prevent access across the IADS, minimizes the maximum asset relocation time, and minimizes the total number of assets requiring relocation, while also ensuring adequate coverage of high-value asset locations and a subset of Surface to Air Missile (SAM) batteries.

### Assumptions

We make several assumptions related to the defender's objectives and IADS assets. In addition to adjusting an IADS to inhibit an adversary traversing the border region, we also seek to minimize the maximum time required to relocate assets, as well as to minimize the number of assets requiring relocation. Additionally, we require protection of a specified set of high-value asset locations (e.g., fielded force locations, command and control centers, etc.) and a subset of the located assets (e.g., long-range SAM batteries). A minimum probability of protection will be specified for each high-value asset location and for each IADS asset type. We assume a given allocation of SAM batteries; specifically, our model includes a combination of long-range (e.g., SA-21), medium-range (e.g., SA-22), and short-range (e.g., SA-24) missile batteries. Although these weapons do not comprise the full range of SAM technologies the U.S. could encounter, they are representative of the various threats that countries employing A2/AD strategies are likely to possess and employ [5]. In addition to the aforementioned SAM battery types, the long-range assets will require separate targeting and tracking radars to engage a target. However, to simplify the model, we assume that each SAM battery possesses the required radar coverage to engage enemy targets.

Instead of assuming binary SAM battery coverage (i.e., covered/not covered), we implement a representative, but unclassified, probability-of-kill curve as a function of the distance from target to SAM battery, for each SAM battery type.

Furthermore, we assume for this study the defender's incoming threat consists only of aircraft, vis-à-vis a wide range of threats not limited to, but including, cruise missiles and ballistic missiles. This assumption will determine the coverage capabilities for each SAM battery instead of requiring the model to account for a myriad of target types. Additionally, we assume IADS assets that are attacked by the intruder are completely incapacitated. That is, no partial capability remains for the attacked assets. Incapacitated assets, therefore, cannot be relocated. However, we allow the model to relocate unaffected assets to sites of incapacitated assets.

To formulate instances of our model, we first construct a hexagonal tessellation over the border region of interest. We choose to discretize an IADS border region using a mesh of uniformly-sized regular hexagons, as Yousefi & Donohue [14] demonstrated it to be superior to alternative uniform tessellation means (e.g., square, rhombus, triangle). Intruding aircraft can traverse the arcs of the graph, traveling from an artificial origination node $o$ on the (*w.l.o.g.*) left side of the hexagonal grid to the artificial destination node $d$ on the right. Potential SAM battery locations will exist at the center of each hexagon in the grid.

Lastly, we assume the adversaries know each other’s capabilities, and the intruder has sufficiently capable intelligence to know the location of IADS assets, once emplaced.  Our bilevel program is formulated as a zero-sum, two-player, extensive form, complete-and-perfect information game using the following notation.

**Sets**

$T$: the set of all types of IADS assets available to locate, indexed by $t$.
$S$: the set of all potential sites where SAM batteries can be located, indexed by $s$.
$\bar{S}$: the set of all sites where SAM batteries are initially located (i.e., $\bar{S} = \{S | x_{\bar{s}}^t = 1, \forall \bar{s} \in \bar{S}, t \in T\}$), indexed by $\bar{s}$.
$\hat{S}$: the set of all sites where SAM batteries are located following asset relocations (i.e., $\hat{S} = \{S | x_{\bar{s}s}^t = 1, \forall \bar{s} \in \bar{S}, s \in S, t \in T\}$), indexed by $\hat{s}$, where $x_{\bar{s}s}^t = 1$ indicates a decision to relocate a SAM battery of type $t \in T$ from site $\bar{s} \in \bar{S}$ to site $s \in S$.
$F$: the set of all sites where high-value assets are located, indexed by $f$.
$A$: the set of arcs in the graph that are equidistant from adjacent potential SAM battery sites $s \in S$, and over which an intruding aircraft can traverse, indexed by $a$.
$N$: the set of all nodes at which arcs intersect and through which an intruding aircraft can traverse, indexed by $n$.
$G(N, A)$ : the graph over which an intruding aircraft will traverse, as induced by the set of potential SAM battery sites $s \in S$.

**Parameters**

$w^t$: the exposure weight for asset type $t \in T$.
$e_{ij}^{st}$: the exposure time of an aircraft traversing arc $(i, j) \in A$ to an asset of type $t \in T$ located at site $s \in S$.
$d_{\bar{s}s}$: the Euclidean distance between SAM battery sites $\bar{s} \in \bar{S}$ and $s \in S$.
$r^t$: the transit speed of IADS asset type $t \in T$.
$x_{\bar{s}}^t$: 1 if a type $t \in T$ IADS asset is initially located at site $\bar{s} \in \bar{S}$, and 0 otherwise.
$z_{\bar{s}}^t$: 1 if a type $t \in T$ IADS asset initially located at site $\bar{s} \in \bar{S}$ is incapacitated, and 0 otherwise.
$B^t$: the maximum number of type $t \in T$ IADS assets to locate.
$p_{sp}^t$: the probability that a SAM battery of type $t \in T$ located at site $s \in S$ can cover the point $p$.
$C^f$: the minimum probability of protection required for each high-value asset location $f \in F$.
$C^t$: the minimum probability of protection required for each located SAM battery of type $t \in T$.

**Decision Variables**

$x_{\bar{s}s}^t$: 1 if a type $t \in T$ IADS asset is relocated from site $\bar{s} \in \bar{S}$ to site $s \in S$; 0 otherwise.
$y_{ij}$: 1 if arc $(i, j)$ is in the minimal exposure path; 0 otherwise.
$\psi_{max}$: the maximum time (in hours) required to complete asset moves.

**Formulation**

Given our assumptions and leveraging the aforementioned notation, we formulate the multi-objective, bilevel program **IADS Multi-Objective Asset Relocation Problem (IADS-MOARP)**, denoted **Problem P1**, as follows:

$$\max_{\boldsymbol{x}, \psi_{max}} \quad f(\boldsymbol{x}, \boldsymbol{y}, \psi_{max}) = \big(f_1(\boldsymbol{x}, \boldsymbol{y}), -f_2(\psi_{max}), -f_3(\boldsymbol{x})\big) \tag{1}$$

$$s.t. \quad f_1(\boldsymbol{x}, \boldsymbol{y}) = \sum_{(i,j) \in A} \left( \sum_{\bar{s} \in \bar{S}} \sum_{s \in S} \sum_{t \in T} w^t e_{ij}^{st} x_{\bar{s}s}^t \right) y_{ij}, \tag{2}$$

$$f_2(\psi_{max}) = \psi_{max}, \tag{3}$$

$$f_3(\boldsymbol{x}) = \sum_{\bar{s}\in\bar{S}} \sum_{s\in S\setminus\{\bar{s}\}} \sum_{t\in T} x_{\bar{s}s}^t, \tag{4}$$

$$\left(\frac{d_{\bar{s}s}}{r^t}\right) x_{\bar{s}s}^t \leq \psi_{max}, \qquad \forall \bar{s} \in \bar{S}, s \in S, t \in T, \tag{5}$$

$$\sum_{s\in S} x_{\bar{s}s}^t = x_{\bar{s}}^t - z_{\bar{s}}^t, \qquad \forall \bar{s} \in \bar{S}, t \in T, \tag{6}$$

$$\sum_{\bar{s}\in\bar{S}} \sum_{s\in S} x_{\bar{s}s}^t = B^t - \sum_{\bar{s}\in\bar{S}} z_{\bar{s}}^t, \qquad \forall t \in T, \tag{7}$$

$$\sum_{\bar{s}\in\bar{S}} \sum_{t\in T} x_{\bar{s}s}^t \leq 1, \qquad \forall s \in S, \tag{8}$$

$$\sum_{\bar{s}\in\bar{S}} \sum_{s\in S} \sum_{t\in T} \ln\left(1 - p_{sf}^t\right) x_{\bar{s}s}^t \leq \ln(1 - C^f), \qquad \forall f \in F, \tag{9}$$

$$\sum_{\bar{s}\in\bar{S}} \sum_{s\in S\setminus\{\hat{s}\}} \sum_{t\in T} \ln(1 - p_{s\hat{s}}^t) x_{\bar{s}s}^t \leq \ln(1 - C^t), \qquad \forall \hat{s} \in \hat{S}, t \in T, \tag{10}$$

$$x_{\bar{s}s}^t \in \{0,1\}, \qquad \forall \bar{s} \in \bar{S}, s \in S, t \in T, \tag{11}$$

$$\min_{\boldsymbol{y}} \quad f_1(\boldsymbol{x}, \boldsymbol{y}) \tag{12}$$

$$s.t. \sum_{j:(i,j)\in A} y_{ij} - \sum_{j:(j,i)\in A} y_{ji} = \begin{cases} 1, & i = o, \\ -1, & i = d, \\ 0, & i = N\setminus\{o,d\}, \end{cases} \quad \forall i \in N, \tag{13}$$

$$y_{ij} \geq 0, \qquad \forall (i,j) \in A. \tag{14}$$

The objective function (1) maximizes the total expected weighted exposure of the minimal exposure path (2), minimizes the maximum IADS asset relocation time (3), and minimizes the total number relocated IADS assets (4). Constraint (5) provides lower bounds on the maximum relocation time, $\psi_{max}$. Constraint (6) ensures we can only relocate assets that are initially located and not incapacitated. Constraint (7) determines the number of each type of IADS asset the defender can relocate. Constraint (8) prevents more than one SAM battery from being relocated to the same site. Constraint (9) ensures that all high-value asset locations receive the required coverage. The form of Constraint (9) results from a logarithmic transformation of the constraint

$$1 - \prod_{s\in S} \prod_{t\in T} \left(1 - p_{sf}^t\right)^{x_{\bar{s}s}^t} \geq C^f, \ \forall f \in F,$$

wherein independence is assumed among the probabilities of coverage, $p_{sf}^t$, over SAM battery locations, $s \in S$, and SAM battery types, $t \in T$. (Implied is the assumption that $C^f < 1$, which is appropriate for this probabilistic metric wherein guaranteed coverage is not attainable.) Likewise, Constraint (10) provides for the coverage of SAM batteries by other SAM batteries. Constraint (11) enforces binary restrictions on the IADS asset relocation decision variables. The lower-level objective function (12) seeks to minimize the total expected weighted exposure of the minimal exposure path (2). Constraint (13) induces the flow balance constraints of the minimal exposure path from the intruder's point of origin, $o$, to destination point, $d$. Lastly, Constraint (14) is the non-negativity constraint associated with the minimal exposure path variables.

## Methodology

Instead of solving IADS-MOARP (1)-(14) using a weighted sum or lexicographic approach, we utilize the ε-constraint method to identify a set of non-inferior solutions. We first reformulate Problem P1 (i.e., IADS-MOARP) to **Problem P2** as follows:

$$\max_{x,\psi_{max}} \min_{y} \quad \sum_{(i,j)\in A} \left( \sum_{\bar{s}\in\bar{S}} \sum_{s\in S} \sum_{t\in T} w^t e_{ij}^{st} x_{\bar{s}s}^t \right) y_{ij} \tag{15}$$

$$s.t. \quad \psi_{max} \leq \varepsilon_2, \tag{16}$$

$$\sum_{\bar{s}\in\bar{S}} \sum_{s\in S\setminus\{\bar{s}\}} \sum_{t\in T} x_{\bar{s}s}^t \leq \varepsilon_3, \tag{17}$$

Constraints (5)-(11) and (13)-(14).

In this reformulation, we replaced the objective function (1) with the defender and intruder objectives of maximizing and minimizing the total expected weighted exposure of the minimal exposure path (2), respectively. We utilize Constraints (16) and (17) to respectively bound our second and third objective functions: the maximum asset relocation time and the total number of asset relocations.

Similar to other bilevel math programming works [1][4][8][13], we reformulate the bilevel Problem P2 by replacing the lower-level problem with its dual formulation. Treating the upper-level variables $x_{\bar{s}s}^t$ as parameters, the lower-level minimization problem becomes a shortest path problem in which the expected weighted exposure objective is minimized, subject to Constraints (13) and (14). Replacing the primal, lower-level problem with its dual formulation as represented in Equations (18)-(21),

$$\max_{\pi} \quad \pi_d - \pi_o \tag{18}$$

$$s.t. \quad -\pi_i + \pi_j \leq \sum_{\bar{s}\in\bar{S}} \sum_{s\in S} \sum_{t\in T} w^t e_{ij}^{st} x_{\bar{s}s}^t, \qquad \forall (i,j) \in A, \tag{19}$$

$$\pi_o = 0, \tag{20}$$

$$\pi_i \text{ unrestricted}, \forall i \in N\setminus\{o\}, \tag{21}$$

where $\pi_i$ is the dual variable associated with the $i^{\text{th}}$ Constraint (13), we obtain the following reformulation of Problem P2, denoted **Problem P3**:

$$\max_{x,\psi_{max},\pi} \quad \pi_d - \pi_o \tag{22}$$

$s.t.$ Constraints (5)-(11), (16)-(17), and (19)-(21).

Problem P3 provides a baseline, single-level model to determine the optimal relocation of surviving air defense assets following an attack.

## CONCLUSION

Problem P3 can be solved directly using a commercial solver and iteratively while incrementally decreasing the values of $\varepsilon_2$ and $\varepsilon_3$ to map the efficient Pareto frontier for an instance of Problem P1, thereby examining the tradeoffs between the competing objectives of maximizing the intruder's minimal exposure, minimizing the maximum asset relocation time, and minimizing the total number of asset relocations.

## REFERENCES


[1] E. Amaldi, A. Capone, M. Cesana, I. Filippini, Coverage planning of wireless sensors for mobile target detection, in: 2008 5th IEEE Int. Conf. Mob. Ad-Hoc Sens. Syst. MASS 2008, 2008: pp. 48–57.

[2] O. Berman, M.R. Rahnama, Optimal location-relocation decisions on stochastic networks, Transp. Sci. 19 (1985) 203–221.

[3] L. Brotcorne, G. Laporte, F. Semet, Ambulance location and relocation models, Eur. J. Oper. Res. 147 (2003) 451–463.

[4] B. Colson, P. Marcotte, G. Savard, An overview of bilevel optimization, Ann. Oper. Res. 153 (2007) 235–256.

[5] C.F. Foss, J.C. O'Halloran, IHS Jane's Land Warfare Platforms: Artillery and Air Defence, Jane's Information Group, United Kingdom, 2014.

[6] M. Gendreau, G. Laporte, F. Semet, The maximal expected coverage relocation problem for emergency vehicles, J. Oper. Res. Soc. 57 (2006) 22–28.

[7] P. Kolesar, W.E. Walker, An algorithm for the dynamic relocation of fire companies, Oper. Res. 22 (1974) 249–274.

[8] A. Lessin, Bilevel multi-objective programming and optimization for integrated air defense system disruption, Air Force Institute of Technology, 2017.

[9] E. Melachrinoudis, H. Min, The dynamic relocation and phase-out of a hybrid, two-echelon plant/warehousing facility: A multiple objective approach, Eur. J. Oper. Res. 123 (2000) 1–15.

[10] N.R. Paul, B.J. Lunday, S.G. Nurre, A multiobjective, maximal conditional covering location problem applied to the relocation of hierarchical emergency response facilities, Omega. 66 (2016) 147–158.

[11] A. Sathe, E. Miller-Hooks, Optimizing location and relocation of response units in guarding critical facilities, Transp. Res. Rec. J. Transp. Res. Board. (2005) 127–136.

[12] United States Joint Chiefs of Staff, Joint Publication 3-01: Countering Air and Missile Threats, (2012).

[13] R.K. Wood, Deterministic network interdiction, Math. Comput. Model. 17 (1993) 1–18.

[14] A. Yousefi, G. Donohue, Temporal and spatial distribution of airspace complexity for air traffic controller workload-based sectorization, in: AIAA 4th Aviat. Technol. Integr. Oper. Forum, 2004.